Controlling output of chaotic economic three-dimensional object with incomplete description into target state*

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Abstract—Application of the method of analytical design of aggregated regulators for constructing systems of control over nonlinear objects under the conditions of uncertainty (with incomplete description) is discussed. The controlled objects are presented as a system of ordinary nonlinear differential or difference equations with chaotic behavior in case of certain combinations of parameters. The formulation of control problem for a poorly formalized object is given and the synthesis control algorithm is presented and theoretically validated. Illustrative example of application of the proposed algorithm is provided along with its numerical simulation data. The results obtained would be useful in developing an intelligent control system and making decisions in real time in control policy of various application objects in robotics, in control of electromechanical objects, economic entities, etc.)

Keywords—Multidimensional nonlinear object, target manifold, adaptation to manifolds, bounded noise with unknown description

I. INTRODUCTION TO THE PROBLEM

It is known that control of multidimensional nonlinear chaotic objects is a very difficult problem [1-10].

According to Neumann, it is hardly possible to create a content of general theory of non-linear systems.

It is also known that all real systems are nonlinear, multidimensional and have complex transients. In addition, there are also critical and chaotic regimes in case of certain combinations of nonlinear objects parameters.

The objects discussed here are represented by systems of ordinary non-linear differential or difference equations with chaotic behavior. They are models of deterministic chaos. In open-loop states, mathematical models of chaotic objects are thought to be unstable that means the following: there are such initial conditions and parameter values of these objects when the property of self-regulation is no longer valid for them.

Here we use the physical invariants concept [3, 4] and

fundamental property of natural self-organization in dissipative systems. The essence of synergetic control theory [3] boils down to the following provision: the purpose of a control system is to achieve a predetermined state (target attractors, invariant manifolds).

This paper deals with an application of the technique of nonlinear adaptation on manifolds for an object represented by a system of differential (difference) equations and a part of their description is unknown.

II. BASIC ASSUMPTIONS

A. Mathematical Model for Object

The controlled object is represented by a system of ordinary differential equations:

$$\dot{x} = f(x, \theta, u), x \in R^n, u \in R^m, \theta \in R^k, m \leq n, k \leq n, \quad (1)$$

where x is a vector of states, u is a vector of control variables, f is a nonlinear vector function; some components of the vector f are unknown.

The controlled object can be also represented by a system of differential equations with a partially unknown description:

$$x[i+1] = f(x[i], \theta, u[i]), x \in R^n, u \in R^m, \theta \in R^k, m \leq n, k \leq n, \quad (2)$$

where x is a vector of states, u is a vector of control variables, f is a nonlinear vector function; some components of the vector f are undetermined.

Here we construct a new approach of application of analytical design of aggregated regulators (ADAR) and nonlinear adaptation on manifolds [3, 9] to design a control system for a non-linear 3-dimensional economic object with incomplete description. We also assume that $n = 3, m = 2$.

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B. Basic Requirements to the Controlled Object

The control synthesis algorithm with program-defined adaptation to a given manifold assumes that the following standard requirements to be fulfilled [3]. Let us list these provisions once again because of their importance:

- a) there is an asymptotically stable target system which matches the desired physical tasks;
- b) all solutions of the initial system have to be limited;
- c) an initial model of the object's state has to be stabilizable;
- d) there is an attractive target manifold described by $\psi(X)=0$ with respect to the object's initial system of equations, where $\psi(X)=(\psi_1(X), \psi_2(X))$ is a certain continuous differentiable vector function that points the desired physical properties of the target state, which is referred to goal macro-variable, which dimension coincides with that of the control vector, and the quality functional of the synthesized control system is given by

$$\Phi = \int_0^{\infty} \sum_{i=1}^m (\phi_i^2(\psi_i(t)) + \omega_i^2 \psi_i^2) dt \rightarrow \min. \quad (3)$$

- e) functions $\phi_l(\psi_l)$, $l=1,2$ are single-valued, continuous and have differentiable functions for all ψ_l such as $\phi_l(0)=0$, $\phi_l(\psi_l)\psi_l > 0$, $\forall \psi_l \neq 0$.

Remark 1. Weighting factors ω_l , $l=1,2$ are the parameters of regulator settings; they are proportional to the duration of motion of an object image point to the target manifold (target attractor) $\psi(X)=0$.

Vector function $\psi(x)$ is called target macro-variable if x is an object state. It should be noted that dimensions of $\psi(x)$ vector and control function vector have to be the same.

C. Mathematical apparatus of the classical algorithm for object constructing control

Now, to make the paper more readable, we first consider all fundamentals ADAR for a three-dimensional object (without the loss of generality).

Lemma 1. Let there exists a variational problem (Φ, ψ) :

$\Phi = \int_0^{\infty} F(t, \psi, \dot{\psi}) dt \rightarrow \min$, $F(t, \psi, \dot{\psi}) = \phi^2(\psi) + \omega^2 \dot{\psi}^2$ with a restriction, $\psi(t)=0$. Suppose a function $\psi(t)$ satisfies the assumptions of d) condition and $\phi(\psi)=\psi$; then Euler–Lagrange equation for the problem (Φ, ψ) would be given by the equation $\omega \ddot{\psi} + \psi = 0$.

The proof of Lemma 1 relies directly on the results of theoretical mechanics and follows from [11]. This proposition

plays a main role in constructing control with predetermined target properties of an object.

Hence, the global minimum of the quality functional (3) is achieved with those values when ψ_i , $i=1,2$, which satisfy the respective functional equations in the forms similar to $\omega \ddot{\psi}_i + \psi_i = 0$. It means that vector control $u=(u_1, u_2)$ should be sought among the solutions of equations $\omega_i \ddot{\psi}_i + \psi_i = 0$, $i=1,2$.

D. Statement of the Problem

Suppose x is an object state, x satisfies the following system of ordinary differential equations:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, x_3) + u_1, \\ \dot{x}_2 &= f_2(x_1, x_2, x_3), \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + u_2, \end{aligned} \quad (4)$$

where $x \in R^3$ is a vector of states, $u \in R^2$ is a control variable, $f \in R^3$ is a continuous (nonlinear) vector function; components f_1, f_3 of the vector $f \in R^3$ are *unknown* bounded functions.

It is also assumed that estimates $\hat{f}_1, \hat{f}_3$ of unknown functions f_1, f_3 are known.

Under the conditions of B.a)-B.d) subsection we have the following control problem.

It is required to perform control in the object state space that transfers an object from the given initial state $x_0 \in R^3$ into the neighborhood of the target manifold $\psi(x)=0$ and supplies the minimum value to the quality functional of the form (3) considering $\underline{2}$ -dimensionality of control action.

III. PROBLEM SOLUTION

A. Algorithm of Synthesis of Object Vector Control with Incomplete Description

Let a target state set for the controlled object (4) be given in the form:

$$\psi_1(x_1, x_2, x_3) = 0, \quad \psi_2(x_1, x_2, x_3) = 0. \quad (5)$$

In particular, it can be assumed that

$$\psi_1 = \psi_1(x_1) = x_1 - x_{10}, \quad \psi_2 = \psi_2(x_2) = x_2 - x_{20}, \quad (6)$$

where x_{10}, x_{20} are given target values of variables x_1, x_2 . It means the stabilization of variables in the neighborhood of the target set of states $\psi(x_1, x_2) = 0$, $\psi = (\psi_1, \psi_2)$.

Next, to make the algorithm more readable, we consider the process of control constructing with macro-variables expressed as (6).

Basic provisions of the algorithm for constructing control of an object with incomplete description are follows.

1. Use the estimates $\hat{f}_1, \hat{f}_3$ instead of unknown functions f_1, f_3 . Now note that functions $\hat{f}_1, \hat{f}_3$ can be quite rough estimates of functions f_1, f_3 , but it is required that the values $|f_j - \hat{f}_j|, j=1,3$ were bounded.

Hence, within this step of the algorithm, we have the following system:

$$\begin{aligned}\dot{x}_1 &= \hat{f}_1(x_1, x_2, x_3) + v_1 + u_1^A, \\ \dot{x}_2 &= f_2(x_1, x_2, x_3), \\ \dot{x}_3 &= \hat{f}_3(x_1, x_2, x_3) + v_2 + u_2^A,\end{aligned}\quad (7)$$

where $v = v(t) = (v_1, v_2) \in R^2$ is a vector of additional control functions intended to compensate the uncertainty f_1, f_2 arising from the replacement of unknown components by their known estimates $\hat{f}_1, \hat{f}_2$; $u^A = (u_1^A, u_2^A)$ is a control vector according to the method of ADAR [3] for an object:

$$\begin{aligned}\dot{x}_1 &= \hat{f}_1(x_1, x_2, x_3) + u_1^A, \\ \dot{x}_2 &= f_2(x_1, x_2, x_3), \\ \dot{x}_3 &= \hat{f}_3(x_1, x_2, x_3) + u_2^A.\end{aligned}\quad (8)$$

2. Formulate a variational problem given by (Φ, ψ) as follows:

$$\Phi = \int_0^\infty \sum_{i=1}^2 \left((\psi_i(t))^2 + (\omega_i \dot{\psi}_i)^2 \right) dt, \quad \omega_i > 0, \psi = (\psi_1, \psi_2) = 0, i=1,2.$$

Vector control $u^A = (u_1^A, u_2^A)$ is constructed according to the equations $\omega_i \dot{\psi}_i(t) + \psi_i(t) = 0, i=1,2$, considering the system of (8).

Lemma 2. Suppose the controlled object has a description of the (8) form and there is an analytical description of the control target (6). Then $u^A = (u_1^A, u_2^A)$ control exists, $\psi = 0, \psi \in R^2$ occurs and a control system for an object (8) is asymptotically stable.

The proof of Lemma 2 is based on the method of ADAR and briefly consists of the following.

First, we substitute into the equations $\omega_i \dot{\psi}_i(t) + \psi_i(t) = 0, i=1,2$ the corresponding expressions from the system (8):

$$\omega_1 (\dot{\hat{f}}_1 + u_1) + \psi_1 = 0, \quad \omega_2 (\dot{\hat{f}}_3 + u_2) + \psi_2 = 0.$$

Thus, we obtain

$$\begin{aligned}u_1^A &= -(\omega_1^{(1)})^{-1} \psi_1 - \dot{\hat{f}}_1, \\ u_2^A &= -(\omega_2^{(1)})^{-1} \psi_2 - \dot{\hat{f}}_3.\end{aligned}\quad (9)$$

Synthesis of control $u^A = (u_1^A, u_2^A)$ is over.

3. Now we should define the variable control $v = v(t) = (v_1, v_2)$. The further constructing of a control system for an object (7) is based on the method of the second theorem of Lyapunov and consists of the following.

We substitute $u^A = (u_1^A, u_2^A)$ from (9) into the system (7), we obtain a system in the decomposed form:

$$\begin{aligned}\dot{x}_1 &= -\omega_1^{-1} \psi_1 + v_1, \\ \dot{x}_2 &= \mu(x_2 + x_3) - \beta x_1 x_3, \\ \dot{x}_3 &= -\omega_2^{-1} \psi_2 + v_2.\end{aligned}\quad (10)$$

We choose the Lyapunov function in the form:

$$V(t) = 0.5 \sum_{i=1}^2 (\psi_i^2 + v_i^2).$$

We define the derivative of the Lyapunov function taking into account the equations of the system (10):

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^2 (\psi_i \dot{\psi}_i + v_i \dot{v}_i) = \sum_{i=1}^2 \left(\psi_i \sum_{j=1}^3 \frac{\partial \psi_i}{\partial x_j} \dot{x}_j + v_i \dot{v}_i \right) = \\ &= \psi_1 \left(-(\omega_1^{(1)})^{-1} \psi_1 + v_1 \right) + \psi_2 \left(-\omega_2^{-1} \psi_2 + v_2 \right) + \sum_{i=1}^2 v_i \dot{v}_i = \\ &= v_1 \psi_1 - \omega_1^{-1} \psi_1^2 + v_2 \psi_2 - \omega_2^{-1} \psi_2^2 + \sum_{i=1}^2 v_i \dot{v}_i = \\ &= v_1 (\psi_1 + \dot{v}_1) + v_2 (\psi_2 + \dot{v}_2) - \omega_1^{-1} \psi_1^2 - \omega_2^{-1} \psi_2^2.\end{aligned}$$

From the last expression, we get a particular form of controls v_1, v_2 that ensure the negativity of the Lyapunov function:

$$\dot{v}_i = -\psi_i, \quad i=1,2. \quad (11)$$

Synthesis of control is over. Hence, we have the following.

Theorem. Let $\psi(x) = 0, x \in R^3, \psi \in R^2$ be a smooth 2-dimensional manifold, where x is a state vector of the

controlled object, let (6) be its analytical description, and technical requirements a)-e) of B subsection exist. Then there is asymptotically stable of a control system (9), (11) when $\psi(x) = 0$ is an attractive manifold.

IV. EXAMPLES OF APPLIED PROBLEMS

Nonlinear models of economic entities of different scales and business application areas (production, distribution, retail and/or wholesale businesses, banking institutions, etc.) are becoming increasingly more appealing to the researchers, as they approach real physical models and described by multivariable nonlinear indices responsive to the positive and negative feedback loops.

Denote by $\alpha, \beta, \gamma, \delta, \lambda, \mu$ any positive numbers. Suppose functions $\hat{f}_1, \hat{f}_2, \hat{f}_3$ have the forms:

$$\begin{aligned}\hat{f}_1 &= \alpha x_2 x_3 - \gamma x_1, \\ \hat{f}_2 &= \mu(x_2 + x_3) - \beta x_1 x_3, \\ \hat{f}_3 &= \delta x_2 - \lambda x_3,\end{aligned}\quad (11)$$

where parameters $\alpha, \gamma, \delta, \lambda$ are unknown, but only their estimated values are known. This phenomenon is often encountered in practice when the model is "fitted".

As it is well known, the description of (11) defines the Lorentz model [12] with many applications.

The purpose of this example is to design a control system for a model of a small business enterprise derived from the Lorentz model, which in its turn accounts for a number of processes, including temporal variations in economic performance indicators and ecological indices.

For the sake of illustration, we consider here the operation and performance of a small-scale IT service company (that provides services for software engineers and programmers in the field of new application design and specialized software development, etc.).

According to the descriptions (7), (11) we take the following object of control:

$$\begin{aligned}\dot{x}_1 &= \alpha x_2 x_3 - \gamma x_1 + u_1^A + v_1, \\ \dot{x}_2 &= \mu(x_2 + x_3) - \beta x_1 x_3, \\ \dot{x}_3 &= \delta x_2 - \lambda x_3 + u_2^A + v_2,\end{aligned}\quad (12)$$

where $x = (x_1, x_2, x_3)$ is a vector of the variables interpreted as quantitative characteristics of financial investments/withdrawals of funds with respect to such company performance indicators as staffing level, capital stock, and the amount of advanced technologies, respectively, quantities $\delta, \alpha, \beta, \gamma, \lambda, \mu$ are the proportionality factors, and $u^A = (u_1^A, u_2^A)$, $v = (v_1, v_2)$ are vectors variables of ADAR-control and a vector of additional control, respectively.

In accordance with the above algorithm, we obtain an object control system in the form (8), (11), (12):

$$\begin{aligned}\dot{x}_1 &= \hat{f}_1(x_1, x_2, x_3) + u_1^A + v_1, \\ \dot{x}_2 &= \mu(x_2 + x_3) - \beta x_1 x_3, \\ \dot{x}_3 &= \hat{f}_3(x_1, x_2, x_3) + u_2^A + v_2, \\ \dot{\psi}_1 &= -\eta_1 \psi_1, \quad \eta_1 > 0, \\ \dot{\psi}_2 &= -\eta_2 \psi_2, \quad \eta_2 > 0,\end{aligned}\quad (13)$$

where, in accordance with (9)

$$\begin{aligned}u_1^A &= -\omega_1^{-1} \psi_1 - \alpha x_2 x_3 + \gamma x_1, \\ u_2^A &= -\omega_2^{-1} \psi_2 - \delta x_2 + \lambda x_3.\end{aligned}\quad (13)$$

Numerical simulation was carried out considering the influence of random smoothed noises on the system.

Fig. 1. Example of graphs of macro-variables: a) ψ_1 and b) ψ_2 .

Fig. 2. Example of graphs of control variables: a) u_1 and b) u_2 .

Fig. 3. Graphs of coordinates of the controlled object: a) x_1 , b) x_2 c) x_3

The simulation was carried out with the following values of parameters:

$$\mu=1.68, \beta=3.34, \gamma=0.99, \alpha=1.33, \delta=1.05, \lambda=3.52.$$

Modeling results confirm that the acquired control laws ensure an asymptotically stable character of transients.

CONCLUSION

The construction of a control system with compensation of uncertainty in the description of a nonlinear 3-dimensional object (poorly formalized object) is based on the synthesis of classic methods such as the method of control in the sliding mode, backstepping method, the method of analytical

construction of aggregated regulators and the Lyapunov method.

A vector control system is constructed on the basis of a nonlinear mathematical model and ensures to perform a control problem when stabilization of coordinates of an object in the neighborhood of an analytically defined manifold exists.

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