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Image Reconstruction Based on Fourier Transform Algorithm

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Abstract. To study the problem of CT image reconstruction, we first analyze the projection process. According to Radon transform and ray attenuation law ,we establish an image reconstruction model based on Fourier transform algorithm and divide the reconstruction process into three parts: transformation, overlay and inverse transformation. Finally, we calculate, simulate and obtain the reconstructed image of the corresponding medium.

Introduction

Different samples of the radiation absorption rate and transmittance are different .The CT system is based on this principle and uses a highly sensitive instrument to measure the sample. The measured data is then input into a computer. After the data is processed by the computer, a cross-sectional or three-dimensional image of the sample to be measured is obtained and the internal structure and morphology of the sample are found.

Image reconstruction process analysis

CT uses a beam to scan a layer with a certain thickness. The detector receives X-rays transmitted through the layer to convert it into visible light, then converts it into an electrical signal by photoelectric conversion and then converts it into a digital signal by an analog / digital converter Computer processing. The process of image formation is like dividing the selected layer into several cuboids of the same volume, called voxel.

The scanning information is calculated to obtain the X-ray attenuation coefficient or absorption coefficient of each voxel, and then arranged into a digital matrix. Each digit in the digital matrix is converted to a small square of pixels ranging in size from black to white via a digital / analog converter and arranged in a matrix to form a CT image.

Therefore, the CT image is also called image reconstruction.

The X-ray absorption coefficient of each voxel can be calculated by different mathematical methods. This paper uses the Fourier transform method for image reconstruction.

We define the variables that we will use next:

Table 1 Symbol Table-Variables

Symbol	Definition
D	Detection unit spacing
v	Attenuation coefficient distribution function value
I_0	The initial ray intensity
I	Ray intensity
l	Penetrating path
p	Attenuation coefficient line integral value
Rv	Radon transform of v
F_2	Two-dimensional Fourier transform
θ	Rotation angle

The radiation emitted by the CT system will be scattered and absorbed by the material in its path so that the energy will decay. The attenuation coefficient distribution function is introduced to characterize the degree of attenuation. Assuming this function is v (x, y), assuming that the intensity of the ray before decay is I0, decaying to I and the path length of the ray penetrating the object is l, then Deduced the attenuation before and after the relationship between the intensity of radiation [1]:

$$I = I_0 e^{-\int_L v(x,y)dl}$$

Since I is the sum of the information received by the probe board, p (l) can be defined as the integral of the attenuation coefficient line

$$p(l) = \int_L v(x,y)dl = \ln \frac{I_0}{I}$$

The intensity of the radiation emitted by the CT system is a known parameter, and the intensity of the attenuated radiation can be obtained from the data of the detection plate. Therefore, through the CT system rotation obtained by different angles, the value of p(l) on each path can be obtained by the data on the probe board .Assuming that the attenuation coefficient is only related to the density of the object being penetrated, and the density can be represented by the image grayscale, the image can be easily restored according to the image grayscale. Therefore, the image reconstruction process first obtains the attenuation coefficient distribution function v (x, y), The problem is reduced to the introduction of v (x, y) based on the set of p (l) [1]

Radon transform

We assume that the scanning tomography is infinitely thin and that the fault attenuation coefficient distribution is determined by all of its linear integral sets. Therefore, if the linear integral of the two-dimensional distribution function can be obtained, the two-dimensional distribution function can be obtained [2]. This process can be achieved by Radon inverse transform:

In view of the rotation of the CT system probing data, the attenuation coefficient function can be converted into a polar form:

$$r = \sqrt{(x^2 + y^2)}$$

$$\varphi = tg^{-1}(\frac{y}{x})$$

among them

$$x = r\cos\varphi$$

$$y = r\sin\varphi$$

For Radon transform its use Rv to represent, Radon transform can be defined as follows [2]

$$\mathrm{Rv}(\theta, \mathrm{t}) = \int r dr \oint d\varphi v(r,\varphi)\sigma[r\cos(\theta-\varphi)-t]$$

On this basis, Radon proposed inverse transformation formula [2]

$$v(r,\varphi) = \frac{1}{4\pi}\oint d\theta \int_{-\infty}^{\infty} \frac{dt}{r\cos(\theta-\varphi)-t}\frac{\partial}{\partial t}p(\theta,t)$$

Among them, p (θ, t) is a continuous transform spectrum; the following will be based on the transformation principle to establish the image reconstruction model.

Image Reconstruction Model Based on Fourier Transform Algorithm

Because the data collected by the CT system is actually discrete, according to the analysis of the image reconstruction process, we use the direct Fourier transform [2] to reconstruct. According to the analysis of the image reconstruction process, we use Radon transform principle, the center slice The principle is based on the establishment of an image reconstruction model based on Fourier transform algorithm, the specific reconstruction process is:

Transform. Since the one-dimensional Fourier transform of the projection of an image at a certain angle is the slice of the two-dimensional Fourier transform of the object density distribution function, one-dimensional Fourier transform of the projection at each angle ;

Superposition. According to the principle of central slice, the transformed two-dimensional Fourier transform of each transformed value is obtained;

The inverse transform. Finally obtains the two-dimensional distribution of the attenuation coefficient by inverting it, so that imaging can be performed.

Formula transformation

According to the projection theorem, we can first write the Cartesian coordinates of Radon transform

$$p^{p}(\theta,t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}v(x,y)\delta(y\cos\theta-x\sin\theta-t)dxdy$$

Then the projection theorem can be expressed as:

$$F_{t}P^{p}(\theta,t)=F_{2}v(-\rho\sin\theta,\rho\cos\theta)$$

Further deduction available The final formula of the algorithm is expressed as:

$$\mathrm{v}(x,y)=F_{2}^{-1}F_{2}v(\rho_{x},\rho_{y})e^{j2\pi(x\rho_{x}+\rho_{y})}d\rho_{x}d\rho_{y}$$

Description

Different tissues absorb and transmit X-rays differently. Using this feature, the CT system scans the sample using different radiation scanning methods. According to the law of ray attenuation, the optical signal is converted into electrical signals, and then converted into digital signals. After processing the measured data by Fourier algorithm, the pixels of different gray scales are used to represent and reconstruct the CT images of the measured samples. CT is more and more widely used in modern life. Especially in medical field, imaging requirements are getting higher and higher.

Summary

The imaging of the CT system relies on the image reconstruction algorithm and can establish the image reconstruction model by relying on the central slice theory [3,4], and then program the transform-superposition-inverse transform process to obtain the matrix of medium absorption intensity by the receiving matrix of the probe plate, and further Can be converted to gray value matrix imaging.

Through analyzing the projection process, studying its physical process and exploring its mathematical expression model, we establish an image reconstruction model based on Fourier transform algorithm according to Radon transform and ray attenuation law. The process of image reconstruction is divided into three parts: transform, overlay and inverse transform. By using the known law of ray attenuation and combining with Fourier algorithm, the reconstructed image process is modeled to derive the mathematical expression of the physical process.

Reference

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