Research on shared-savings contract of online shopping supply chain
based on risk aversion

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Abstract
In the real life, most of the suppliers and retailers are risk averse. This paper discusses the supply
chain collaboration incentive problem based on the secondary supply chain composed of a single risk
aversion supplier and a single risk aversion retailer. Firstly, we study the impact of the risk aversion
coefficient of the supplier and the retailer on the wholesale price and the retail price under the
stackelberg model. Then, demonstrate the relationship between the effort degree and the price decision
of the supplier and the retailer under the shared-savings model based on the effort degree. Finally, the
shared-savings contract can help to coordinate the whole online shopping supply chain.

Keywords: risk averse; Stackelberg model; shared-savings contract

1 Introduction
Online shopping supply chain is a system of multi-member participation, including suppliers,
oneonline retailers, the third-party logistics providers, customers and other channel members.
This chain includes a range of activities such as the supply of goods from the manufacturer,
customer online ordering, electronic payment, express delivery, receipt, until confirmation of
receipt, payment and evaluation feedback. In real life, most of the suppliers and online
retailers are risk averse. However, there are unknown risk factors of product quality, price,
supply volume and market demand, anyone of which makes the enterprise or retailer at huge
losses in the risk aversion coefficient because of their different take different measures. In this
case, the participants will take different measures because of their own risk aversion
coefficient. For example, participants with smaller risk aversion coefficients will take positive
and innovative reforms, while participants with larger risk coefficients will use more
conservative measures. At this time, if the idea of online shopping supply chain participants
transforms from their own interests to the whole supply chain profit maximization, the overall
efficiency and profitability will be improved. Therefore, supply chain coordination and cooperation of the parties involved in the online shopping is an important factor in supply chain management. The development of revenue shared-savings contract is an effective way to achieve supply chain coordination. The establishment of shared parameters and the corresponding incentive mechanism promote the establishment of sharing members to share the risk and actively cooperate with each other to improve the supply chain profits.

Risk averse enterprise refers to the enterprises that take measures to avoid the risks, including suppliers and retailers. At present, many scholars have studied the supply chain coordination of risk averse. Schweitzer and Cachon (2000) were the first to use the newsboy model to verify that the quantity of goods ordered by the loss aversion was significantly less than the risk neutral, and inversely proportional to the degree of loss aversion coefficient1. Shen Hancai, Xu Jin and Pang Zhan (2004) studied the problem of the number of orders for a single manufacturer of loss aversion and verified similar conclusions2. Zhang (2005) studied a supply chain system that considered the cost of out-of-stock penalties, which included a single risk-neutral supplier and a single loss averse retailer, and studied the use of repurchase contracts to coordinate supply chains3. Li Jicai (2013) first established a shared-savings contract model between multiple risk-averse retailers, and then obtained the only "wholesale price-benefit sharing coefficient" ratio that harmonized the supply chain revenue sharing contract4. Hou (2013) used the game model and the optimization theory to study the equilibrium price and quality level of risk-averse participants under the supply chain competition5. Zhang (2014) analyzed the price competition between the two supply chains composed of a risk-neutral manufacturer and a risk-averse retailer under stochastic demand, and analyzed two supply chains in decentralized decision-making, centralized. This paper analyzes the pricing strategy of supply chain members under decentralized decision-making and centralized decision-making mode, and compares the equilibrium price and expected profit in each mode6. Shu Lei, Wu Feng (2015) established the single stage purchasing model based on exponential utility function, then analyzed some influential parameters, such as the supply price, the degree of risk aversion, and demand7. On the basis of the newsboy model, Xiong Hengqing, Huang Yong, Wan Jie (2016) researched the impact of manufacturer's risk aversion on ordering time of supply chain, which was a secondary supply chain composed of a risk aversion manufacturer and a risk neutral retailer8. Liu Ying, Mu Yinping (2016) used the mean-variance utility function to study the use of risk-averse
suppliers and retailers under different flexible contracts to maximize the utility of retailers procurement combination and the utility of suppliers production and pricing decisions.

Some scholars have pointed out that the revenue shared-savings contract in the supply chain can effectively solve the problem of overall revenue. In order to solve the stochastic demand condition, Wang Yuyan (2009) analyzes the coordination effect of the revenue sharing contract on the closed-loop supply chain for a closed-loop supply chain composed of a manufacturer and retailer. Hou Linlin and Qiu Yuanhua (2010) discussed the equilibrium and coordination of supply chain system consisting of a supplier and a large number of retailers competing with each other in the revenue sharing contract. Li Xinran (2013) analyzed the closed-loop supply chain for individual retailer responsible for recycling, and explained that the revenue-sharing contract was able to deal with the "double marginalized" problem in the supply chain. Wang Qinpeng, zhao Daozhi (2014) designed the revenue sharing contract to coordinate the relationship between retailers and suppliers, and determined the optimal order of retailers and suppliers to reduce carbon emissions levels of decision-making issues, to realize the pareto improvement. Although there are many researches on the risk-aversion supply chain and the sharing of revenue in supply chain, but most of them are studied separately. This paper explores the integration mechanism of the two aspects, and establishes the utility function of suppliers and retailers in the supply chain. At the same time, it also studies the utility of income in the case of shared-savings contract, and it is expected that this research will provide a theoretical basis for the decision-making of risk aversion members in the supply chain.

2 Problem description

This article considers a supply chain consisting of one supplier and one retailer, where both the supplier and the retailer are risk averse. The retailer buys a certain amount of merchandise at the wholesale price and sells it to the customers at a retail price. The variables and symbols present in the model are represented as follows:

- P: market retail price for unit products (retailer decision variable);
- W: wholesale price of the unit price provided by the supplier to the retailer (supplier decision variable);
- C: supplier's unit product cost of production;
- Q: the total market demands faced by the retailer, the mean is \( a \), the variance is \( \delta^2 \);
- \( \kappa \): the price elasticity coefficient of the retailer selling the product;
- \( \lambda_s \): the supplier's risk aversion coefficient;
- \( \lambda_r \): the risk aversion coefficient of the retailer;
- \( e_s \): the degree of effort of the supplier;
- \( e_r \): the effort of the retailer;
- \( c_r(e_r) \): the cost of retailers'
efforts at $e_r$; $c_s(e_s)$— the cost of suppliers' efforts at $e_s$; $\Delta \tilde{r}(e_s, e_r)$— online shopping supply chain optimization can save the amount of the unit inventory and transportation cost saving amount; $\alpha$— the ratio of inventory and transportation costs shared by retailers; $\Pi$— profits; $s$, $r$ indicate suppliers and retailers respectively; $U$— Utility

The basic assumptions are as follows:

$\Delta \tilde{r}(e_s, e_r)$ is continuous and second order differentiable on $[0,1] \times [0,1]$; And meet $\Delta \tilde{r}(0, e_r) = \Delta \tilde{r}(e_s, 0) = 0$; The supplier and the retailer must also pay efforts to save costs, as long as any party without effort, the other party will not reduce the cost; at the same time as long as the supplier and the retailer joint efforts will reduce the total cost of transportation and inventory:

$$\frac{\partial \Delta \tilde{R}(e_s, e_r)}{\partial e_s} > 0, \quad \frac{\partial \Delta \tilde{R}(e_s, e_r)}{\partial e_r} > 0;$$

However, as the participants' efforts increase, the savings in inventories and transportation costs will decline:

$$\frac{\partial^2 \Delta \tilde{R}(e_s, e_r)}{\partial e_s^2} < 0, \quad \frac{\partial^2 \Delta \tilde{R}(e_s, e_r)}{\partial e_r^2} < 0.$$

The demand for retail products in the market is inversely proportional to the retail price of $P$, the demand function is set up as follows:

$$Q = a - \kappa p$$

According to choi and Xiao's study, retailers with risk characteristics will consider two values: expected expected return and expected expected variance $^{[15]} [16]$. Therefore, this paper uses the mean-variance method to establish the utility function of the retailer. The utility function of the risk aversion retailer:

$$U(\Pi_r) = (a - \kappa p)(\rho - w) - \lambda_r (\rho - w)^2 \delta^2$$

The utility function of the risk aversion supplier:

$$U(\Pi_s) = (a - \kappa p)(w - c) - \lambda_s (w - c)^2 \delta^2$$

### 3 Stackelberg model

The Stackelberg model refers to the leader and sub participant in the supply chain, that is, the higher decision-making power in the supply chain is called the main party, and the party in the passive state is called follower. In Stackelberg, the main party developed their own decision-making to maximize the utility from their own interests, and then according to the
main decision-making, follower developed the optimal program. In the end, the main party improve themselves to make decisions again according to the response from party repeatedly. At the same time, in this model, the supplier is in the main position, the retailer is follower. Given the retail price $p$, the supplier's decision variable is the wholesale price $w$, and the optimal wholesale price is obtained according to the supplier utility function. The retailer then determines the optimal retail price according to the optimal wholesale price, which makes $u(\Pi_r)$ reach the maximum. The model is expressed as follows:

$$\begin{align*}
\text{Max} U(\Pi_S) & \left( w - c \right) \left( a - \kappa p \right) - \lambda_s \left( w - c \right)^2 \delta^2 \\
\text{s.t.} P^* & = \arg \max U(\Pi_R) \\
\text{Max} U(\Pi_R) & = \left( a - \kappa p \right) \left( p - w \right) - \lambda_r \left( p - w \right)^2 \delta^2
\end{align*}$$

When the wholesale price $W$ is certain, it is easy to get a convex function about $p$. Therefore, the unique optimal price is obtained from $\frac{\partial U(\Pi_R)}{\partial p} = 0$, maximizing the utility of the supplier. when $p$ is substituted into (4), the model becomes a function of $w$, and the overall optimal wholesale price $w^*$ of the model is obtained as follow:

$$w^* = \frac{ak + 2a\lambda_r \delta^2 + \kappa^2c + 2\kappa c \lambda_r \delta^2 + 4\kappa \lambda_S \delta^2 c + 4\lambda_r \lambda_S \delta^4 c}{2\kappa^2 + 4\kappa \lambda_r \delta^2 + 4\kappa \lambda_S \delta^2 + 4\lambda_r \lambda_S \delta^4}$$

$$p^* = \frac{a + \kappa w + 2\lambda_r \delta^2 w}{2\kappa + 2\lambda_r \delta^2}$$

We can derive proposition 1 from (7) and (8).

### 4 Supply chain coordination model based on shared savings

In consideration of suppliers and retailers under the condition of risk aversion, we set up a shared-savings contract, establish incentive mechanism to promote the enthusiasm of each participant, so as to maximize the interests of the whole supply chain. Shared-savings contract refers to the fact that both supplier and retailer work together to save inventory and transport, and to make up for cost savings and even profitability with hard work. The profit function of the retailer is the sum of the original profit function and the cost savings, and then subtracts
the cost of the effort caused by the cost savings. So the retailer's profit function is expressed as:

$$\Pi^*_R = (\alpha - \kappa p)(p - w) + \alpha \Delta R(e_s, e_r) Q - C_R(e_r)$$

The retailer utility function is expressed as:

$$U(\Pi^*_R) = \Pi_R + \Delta R(e_s, e_r) Q - C_R(e_r)$$

Supplier profit function:

$$\Pi^*_S = (w - c)(\alpha - \kappa p) + (1 - \alpha) \Delta R(e_s, e_r) Q - C_s(e_s)$$

Supplier utility function:

$$U(\Pi^*_S) = (w - c)(\alpha - \kappa p) - \lambda_S(w - c)^2 \delta^2 + (1 - \alpha) - C_s(e_s) \Delta R(e_s, e_r) Q$$

Since the supplier dominates and the retailer occupies the position of follower in the model, and the retailer's decision variable is the retail price determined by the supplier for the wholesale price. When $w$ is determined, it is easy to deduce that $\Pi(\Pi^*_R)$ is a concave function of $P$ by Eq. (10), then the unique $p^{**}$ can be obtained so that the retailer utility gets the maximum.

$$p^{**} = \frac{\alpha + \kappa w + 2\lambda_r w \delta^2 - \alpha \kappa \Delta R(e_s, e_r)}{2\kappa + 2\lambda_r \delta^2}$$

After knowing the optimal retail price of the network retailer, the supplier decides the wholesale price accordingly. And then get the value of $Q$, namely

$$Q = \frac{\alpha \kappa + 2a \lambda_r \delta^2 - \kappa^2 c - 2a \lambda_r w \delta^2 + \alpha \kappa^2 \Delta R(e_s, e_r)}{2\kappa + 2\lambda_r \delta^2}.$$ Then the $Q$ is substituted into the utility function of the supplier under the shared-savings contract. According to the formula

$$\frac{\partial U(\Pi^*_S)}{\partial w} = 0,$$ we can obtain the optimal wholesale price:

$$w^{**} = \frac{\kappa a + 2a \lambda_r \delta^2 + \kappa^2 c + 2\kappa c \lambda_r \delta^2 + 4\kappa c \lambda_s \delta^2 + 4\lambda_r \lambda_s c \delta^4 + T}{2\kappa^2 + 4\kappa \lambda_r \delta^2 + 4\kappa \lambda_s \delta^2 + 4\lambda_r \lambda_s \delta^4 + T}$$

$$T = \left(2\alpha \kappa^2 - \kappa^2 - 2\lambda_r \kappa \delta^2 + 2\alpha \lambda_r \kappa \delta^2\right) \Delta R(e_s, e_r).$$
We can now continue to analyze the optimal level of suppliers and retailers. In the case of \( p \) and \( q \), the retailer determines the optimal level of effort to maximize the utility function.

\[
\max_{e_s, e_r, e \in [0,1]} \left[ \Pi^*(e_s, e_r) \right] = (a - \kappa p)(p - w) - \lambda_r (p - w)^2 \delta^2 + \alpha \Delta R(e_s, e_r) Q - C_r(e_r)
\]

\[\text{ST: } U(\Pi^*_r) \geq U(\Pi'_r)\] (15)

According to the function, the optimal solution needs to be satisfied:

\[
\frac{\partial \bigcup (\Pi^*_r)}{\partial e_r} = \alpha \frac{\partial \Box R(e_s, e_r)}{\partial e_r} Q - \frac{dC_r(e_r)}{de_r} = 0
\]

Namely:

\[
\frac{dC_r(e_r)}{de_r} = \alpha \frac{\partial \Delta R(e_s, e_r)}{\partial e_r} q
\]

The upper left side is the marginal cost of the retailer's effort, and the marginal revenue earned by the retailer on the right side, which is affected by the savings and the sharing ratio in the total cost of inventory and transportation.

\[
\frac{\partial^2 \bigcup (\Pi^*_r)}{\partial e^2_r} = \alpha \frac{\partial^2 \Box R(e_s, e_r)}{\partial e^2_r} Q - \frac{d^2C_r(e_r)}{de^2_r}
\]

(17)

(18)

We may know from the front hypothesis: \( \frac{\partial^2 \Delta R(e_s, e_r)}{\partial e^2_r} < 0 \), \( \frac{d^2C_r(e_r)}{de^2_r} > 0 \), so \( \frac{\partial^2 \bigcup (\Pi^*_r)}{\partial e^2_r} < 0 \).

Namely, \( \exists e_r \in [0,1] \) making \( \bigcup (\Pi^*_r(e_r, e_s)) \) maximized, and its optimal solution:

\[e^*_r = \arg \max e_r \Pi^*_r(e_r, e_s).\]

Similarly, the best way to solve the optimal level of network providers is similar, namely, \( e^*_s = \arg \max e_s \Pi^*_s(e_s, e_r) \).

Proposition: In the shared-savings contract of online shopping supply chain, suppliers and retailers can achieve the Nash equilibrium \( (e^*_s, e^*_r) \) of the game, and the equilibrium value always exists, which makes the maximum utility of the two parties.

5 Conclusions

This paper studies the problem of shared saving of risk averse supplier and retailer. Firstly, the utility function of the mean - variance method is established to represent the utility of the
retailer and the supplier respectively. Then according to the coordination problem between the supplier and the retailer under the stackelberg model, the cooperative sharing between the supplier and the retailer is analyzed. The study found that shared-savings contract allowed participants to pay more benefits than their own efforts to improve the profitability and operational efficiency of the participants. When supplier and retailer are risk-averse, wholesale prices and retail prices are inversely proportional to their risk aversion. At the same time, the higher the level of efforts of supplier and retailer, the lower the optimal wholesale price and the best retail price in the shared cost savings contracts based on the effort level.

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