

# ***Modeling and searching for optimal location of cargo vessel when unloading by helicopter in order to solve economic problems of sea transportation along Northern Sea Route***

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**Abstract — The poor development of transport infrastructure in the Arctic is the cause of significant time losses and increased costs when delivering goods to points located on the coast and islands of the Arctic Ocean and its seas. Often, a helicopter is a way of unloading a ship, which is the only way to deliver cargo to the necessary places. The search for the optimal geographical location for unloading a vessel using a helicopter is an urgent task, since such way of unloading a vessel is very expensive. The mathematical model of finding a geographical location for maximally profitable transportation of cargo from a cargo vessel using a helicopter to the necessary points of unloading is developed in the work. Using the example of a developed model, it has been shown that it is possible to obtain economic benefits when transporting goods in the Ob Bay.**

**Keywords — Ob Gulf, Transport task, Northern Sea Route, Optimal place, Economic benefit, Cost minimization**

## I. INTRODUCTION

Weak development of transport infrastructure in the Arctic is the cause of significant time losses when delivering goods to points located on the coast and islands of the Arctic Ocean and its seas. As the analysis of the situation [1] has shown, the only way to deliver goods to these points is sea transport. Especially it concerns the delivery to hard-to-reach

hydrometeorological stations, for the majority of which sea vessels are the only way and without alternative possibility of supplying them with food, fuel and change of polar workers [2]. The development of the Arctic zone of the Russian Federation requires the solution of a whole range of problems. One of the most important directions in this case is carrying out cargo transportation along the Northern Sea Route to supply the Arctic territories with the necessary equipment, materials, etc. The creation and management of transport infrastructure requires the consideration of important natural and economic characteristics inherent in this region. It is necessary to develop original approaches and methods of mathematical modeling in solving transport problems. As is well known, the transport problem in the classical formulation was solved in the 1940s by the Soviet mathematician, Nobel laureate in economics L.V. Kantorovich [3]. Later, progress in the field of linear programming, databases and computer calculations allowed designing intelligent systems that manage transport flows. However, increasing detail and taking into account the specifics in each specific case make it necessary to search for new mathematical tools and methods. Therefore, until now, research in this area is relevant and in demand both from a fundamental and applied point of view. A special place in this case has physical methods and approaches that allow

using analogies to successfully solve transport problems using an adapted mathematical apparatus. Methods of mathematical physics, statistical physics, thermodynamics, etc. are actively used in the theory and practice of logistics transport. Examples are the similarity of transport flows to fluid flows using hydrodynamics, gravitational and entropy models of the relationships between consumers and suppliers, diffusion analogies of cargo transportation, etc. In this paper, we present an original method for solving the transport problem of loading and unloading a vessel to several port points with the aim of choosing the optimal method and location of vehicles. Feasibility, time costs and the possibility of the vessel to approach the shore for unloading are taken into account.

The lack of a sufficient number of ports and port stations, on the one hand, small depths in the coastal zone and the condition of the coasts, on the other, make it impossible to unload the ship to shore at most points. For this, helicopters based on the ship are used, and also unloading is the reloading to small vessels and pontoons. At the same time, when it is required to deliver goods and people to closely located points, the problem arises of the location of a cargo ship in the water area in such way that the use of helicopters is most effective, i.e. minimum in time, and hence, in price.

## II. MATHEMATICAL MODEL

Consider the problem of finding a geographical location for the most profitable transportation of cargo from the ship, using a helicopter, to the necessary points of unloading. Obviously, this task is connected with the transport task [3-8], where one point of production is a ship with a cargo, and  $n$  of consumption, points are geographically fixed places. In this problem, the following data are needed:  $a$  - the volume of production (one point of production is a ship with cargo),  $b_j$  - the volume of consumption in point  $j$ ,  $c_{1,j}$  - the cost of transporting a unit of product from the ship to the  $j$  point with

the chosen route, the total production of  $a = \sum_{j=1}^n b_j$  equals total consumption (all necessary cargo must be taken out of the vessel). Let us denote  $p_{1,j}$  - the volume of transport from the vessel to the point of the  $j$ , in addition, the condition  $\sum_{j=1}^n p_{1,j} = a$  and  $p_{1,j} = b_j$ . To solve this problem, it is

necessary to consider the cost function  $z = \sum_{j=1}^n c_{1,j} p_{1,j}$ , and it is

necessary to choose such geographical location of the vessel so that the  $z$  function is minimal. In the search for such geographical location, the cost of moving a ship can be neglected, since the price of cargo transported by the vessel from one point to another is much less than the price of the same cargo being transported to the same points by the helicopter method (the shipment must be much larger than a one-time transportation by a helicopter). Using the methods of linear programming [9-14], the solution of this problem is quite a difficult problem, because geographically fixed points

of departure, when minimizing  $z$ , you need to select a lot to find the desired solution. For the solution, we find an analytic model for the problem under consideration.

Since, for our task, it is natural to assume that  $c_{1,j} = \frac{S_{1,j}}{m}$ , where  $S_{1,j}$  - is the cost, and  $m$  - is the mass of the cargo being transported, we will assume that  $S_{1,j} = kl_{1,j}$ , where  $l_{1,j}$  - is the traversed path of the helicopter from the vessel to point  $j$  for the transportation of the necessary cargo of mass  $m$ , where  $k$  - is the proportionality coefficient. As a result, we get

$$z = \frac{k}{m} \sum_{j=1}^n l_{1,j} p_{1,j}. \quad (1)$$

Let us choose the Cartesian coordinate system (in the framework of the problem being solved, due to the limited distances, it is entirely permissible to abandon the geographic coordinates), in which it is necessary to specify the coordinates of the points, for example, in Fig.1. It is natural to assume that for a helicopter the minimum path from the ship to the unloading point is straight, therefore in this coordinate system:  $l_{1,j} = \sqrt{(x - x_j)^2 + (y - y_j)^2}$ .

The minimization of expression (1) can be done analytically by finding the extremum of this function. The problem under consideration can be complicated, for example, in the case where the position of the vessel is limited to a certain geographical area. Such situation is possible when the draft of the vessel does not allow going beyond these limits because of the depth, or movement beyond the borders is impossible due to the shoreline. In this case, the solution must be sought inside this area.

As a result, the coordinates of the vessel's best position, when there are no restrictions on the geographic area, can be found by solving numerically the system of equations

$$\begin{cases} \sum_{j=1}^n \frac{x - x_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} p_{1,j} = 0 \\ \sum_{j=1}^n \frac{y - y_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} p_{1,j} = 0 \end{cases}. \quad (2)$$

In case of restriction, it is necessary to minimize function (1), taking into account that  $f(x, y) \leq 0$ , where  $f(x, y)$  is the function defining the domain within which the solution is being searched.

## III. PRACTICAL IMPLEMENTATION OF THE MODEL

Let us consider an example of practical importance. It is necessary to find the position of the ship in the Ob Bay so that the costs for transporting goods by helicopter to Tambey, Sjojaha and Antipayuta were minimal (Figure 2). In this task, there are restrictions on the area of the vessel's location because of the shoreline. Let us consider some examples of

transportation to the mentioned destinations. The optimum position points of the vessel were found in each case according to the procedure described above by minimizing equation (1) taking into account the shoreline.

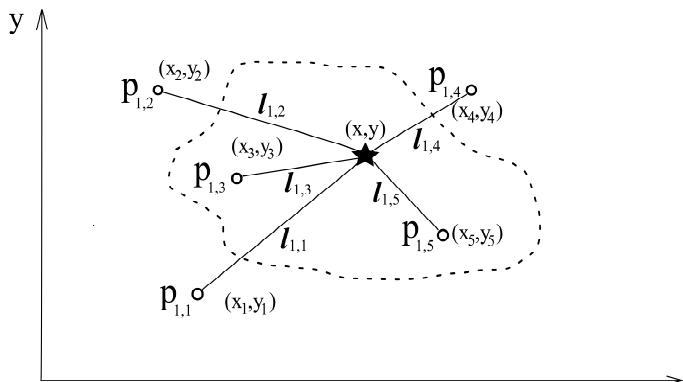


Fig.1 An example of the location of five points of consumption (circles), given by Cartesian coordinates and the best position of the ship (asterisk) for unloading by the helicopter, taking into account the restrictions on the position of the ship (the ship is inside the area bounded by a dashed line)

The cost calculation was based on the average helicopter rent price, which is equal to 150000 rubles per hour, according to the average speed of the helicopter during operation - 150 km / h. And the average carrying capacity for transportation at a time - 2 tons (the approximate performance of the Mi-8T helicopter is taken). It turns out that  $k = 2000 \text{ Rub} / \text{km}$ . The calculation data are given in Table 1.

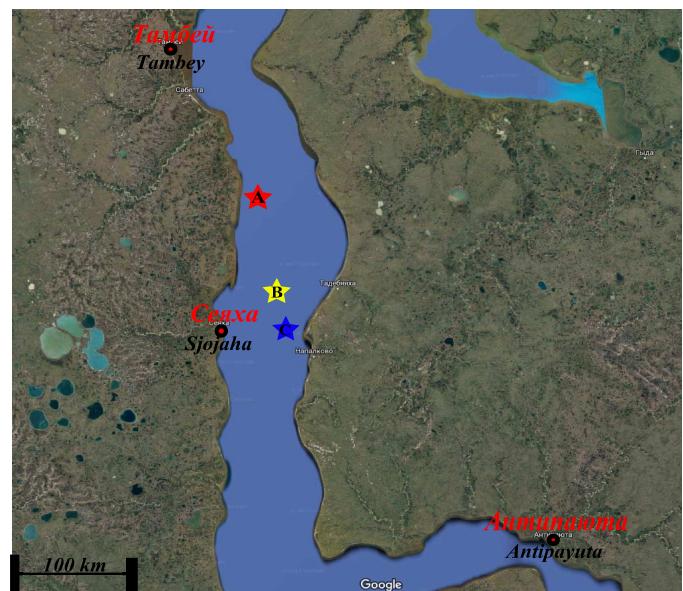


Fig.2. Map of the "Ob Bay" with the places of the best berthing of the vessel while unloading the ship

Also, to compare the economic benefits, we present the table of transportation costs  $z$ , from points A, B, C (with the data given in Table 1) and points located near Tambey, Sjojaha and Antipayuta. The results of the comparison are presented in Table 2. As a percentage for the ratio

$\frac{z(j)}{z(A,B,C)}$  where  $z(j)$  is the cost of transportation from point  $j$ , and  $z(A,B,C)$  the cost of transportation from points A,B,C respectively.

TABLE I. COST OF VESSEL UNLOADING BY THE HELICOPTER AT THE OPTIMAL POINT FOR DIFFERENT NEEDS OF PORT POINTS

Freight volume	Tambe y	Sjojaha	Antipa yuta	Optimal place for unloading	Z (value, mln. Rub.)
$P_{1,j}$ (Tons)	24	24	24	Sjojaha	14.35
$P_{1,j}$ (Tons)	48	24	24	Tambey	19.32
$P_{1,j}$ (Tons)	24	48	24	Sjojaha	14.35
$P_{1,j}$ (Tons)	48	48	24	Sjojaha	20.49
$P_{1,j}$ (Tons)	76.8	24	60	Point A in Fig. 2	38.55
$P_{1,j}$ (Tons)	48	24	48	Point B in Fig. 2	28.33
$P_{1,j}$ (Tons)	24	24	40.8	Point C in Fig. 2	19.93

TABLE II. TABLE OF COMPARISON OF ECONOMIC BENEFITS

Ship packing points	Tambey	Sjojaha	Antipayuta
A	1.4	4.2	30.6
B	14.7	1.2	24
C	43.2	1	7.2

Let us analyze the obtained data. Obviously, in the problem with equal volumes of cargo transportation to all points, the optimum anchorage point of the vessel when unloading is near Sjojaha. The reason is that for the same volume (equal to the weights of the vertices) of transportation from the ship to the point of unloading, the desired point in the triangle becomes the Fermat point [15]. This point is at the minimum total distance from the vertices of the triangle.

From geometric considerations it is clear that for a solution angle at one of the vertices of a given triangle is equal to or greater than  $120^\circ$ , the Fermat point degenerates into this vertex, which explains the position of the unloading point in the first case of Table. 1 near Sjojaha. It should be noted that other remarkable points of the triangle (centroid or center of mass, the center of the circumscribed circle, etc.) do not give the desired optimal position in the case of equal vertex weights. In the case of different weights, the optimal point will no longer be a Fermat point and can be found only by solving system (2).

Despite the fact that it is possible to find, using the method of calculation presented, the best geographical location for unloading a ship, there are a number of problems. It is not always possible to reach a given unloading place to unload a

ship. This is due to various factors that can not always be taken into account, for example, these are bad weather conditions, shallow water, change of the fairway, and others. These obstacles can arise directly from the movement of the vessel to the point of unloading. In this case, a rapid response to the current situation is necessary. Delays in response entail losses. In addition, it is necessary to search for a new unloading point, but in the current situation it is difficult to find. In this case, the person (manager), who makes the decision to unload, needs prompt assistance in the new modeling of the situation. In this case, contour maps (a card-chart) of economic benefit will help. They consist of a geographical area, which includes the optimal unloading point, where the economic indicators of the possible point of unloading the vessel are indicated. As an example, let us show three options for unloading in the Ob Bay, at the same discharge points, as the authors did earlier in Table 1. The results of the calculations are shown in Figures 3-5. In the presented drawings on the color scale, one can see where it is possible to unload the vessel with minimal losses. The darker the area is, the more profitable it is. It is also convenient and easy enough to analyze and draw conclusions on the choice of the place of unloading using isolines (black lines in the figures) - lines of the same economic benefit. These lines mean that, moving along the chosen line i.e. Isolines, the cost of unloading will be the same, despite the different geographical location of the ship unloading point. Figure 4 is particularly interesting, where the isolines are not converging to one point (the optimum unloading point). Judging by Fig. 4, economic losses, if the vessel is not available at the best point for unloading, may be minimal on a fairly wide geographical location of the vessel.

One can see that analysis of the search for a place to unload the vessel is simple enough, if it is impossible to stay at the optimal point. Such cards are easy to use on the ship by the person (manager) responsible for unloading the vessel. It should be noted that finding the optimal place, as well as economic losses for unloading, is not a trivial task, if it is impossible to be in this geographical location.

The task can be inverted and the calculation of the optimal loading point, the number of port points and their position can vary. In modeling, it is possible to take into account, as additional conditions, the depth isobars near the shore and the ice situation at the unloading site.

Thus, with the example of an analytical solution of the problem of loading and unloading a ship into several port points, the proposed methods and approaches for mathematical modeling of an important stage of cargo transportation allow us to go beyond the limits of standard linear programming and a numerical search of possible combinations.

In the future, the development of this approach and the inclusion of this particular task in more large-scale ones are proposed. Analogously, it is possible to analyze and find the locations of the optimal locations for transshipment points,

taking into account the natural conditions and the specifics of the goods and means of delivery for further transportation of necessary deep into the Arctic territories, or delivering to shore to ensure the life of settlements, mining, etc.

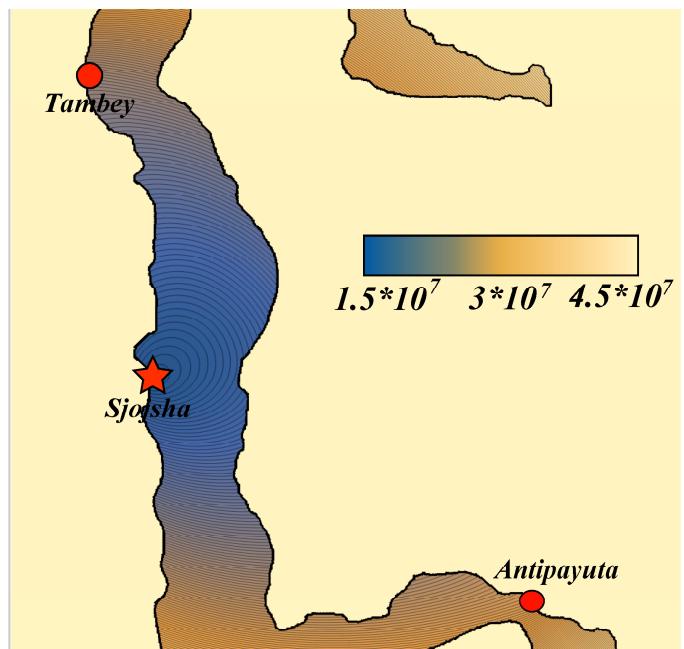


Fig.3. The contour map of economic benefits when unloading a vessel by helicopter to Sjojaha points (the optimum place for unloading) is 24 tons, Tambey 24 tons, Antipayut 24 tons. The numerical values on the color scale correspond to the costs in rubles.

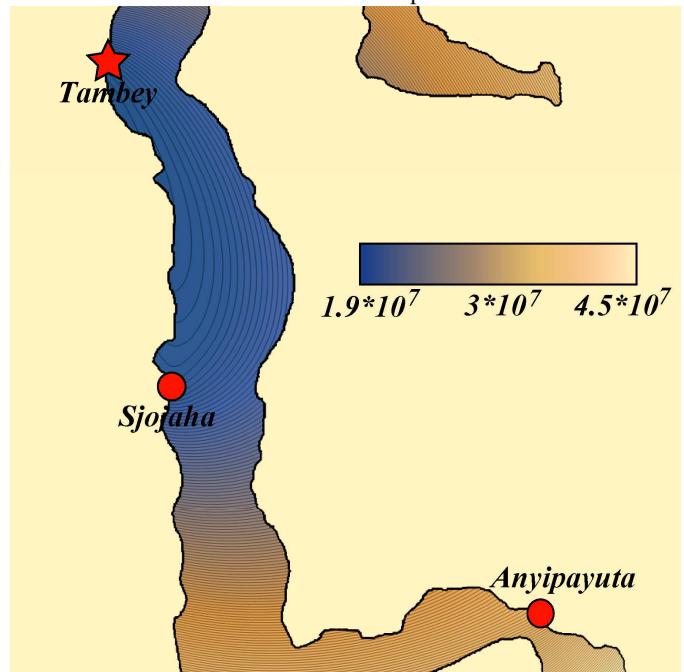


Fig.4. Contour map of economic benefits when unloading the vessel by helicopter to Sjojaha points -24 tons, Tambey (optimal place for unloading) -48 tons, Antipayut-24 tons. The numerical values on the color scale correspond to the costs in rubles.

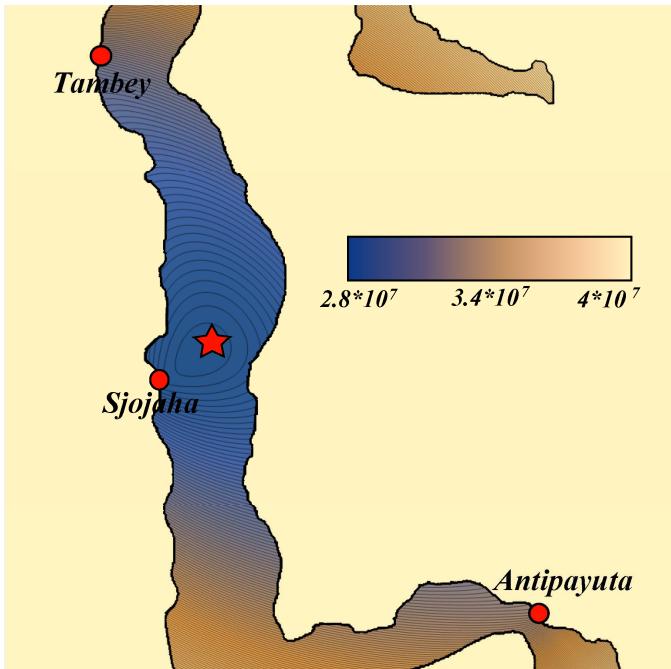


Fig.5. Contour map of economic benefits when unloading the vessel by helicopter to Sjojaha points -24 tons, Tambey -48 tons, Antipayut -48 tons. Point B of Table 1 is the optimal place for unloading the vessel. The numerical values on the color scale correspond to the costs in rubles.

#### IV. CONCLUSIONS

Summing up, it can be said that the problem of the optimum point of the cargo ship's landing when unloaded by helicopter or other means can be quickly solved by linear programming and multiparametric minimization of the analytical systems of equations obtained. In this case, complex and volumetric calculations are not required, as they are assumed in the standard solution of transport problems. The economic gain in choosing the optimum point of helicopter discharge in this case reaches up to 50% in relation to other options, using a helicopter and a cargo vessel. The gain in value is even greater if in the calculations one takes into account that the ship's day of the vessel type "Mikhail Somov" (usually used for this task) costs at least 950 thousand rubles. If one takes into account that the ship can not approach the shore due to shallow depths and the helicopter is inevitable, the costs of non-optimal options increase significantly. In addition, the work time increases from two to three days, which can be of critical importance in conditions of limiting the period of navigation in the Arctic and bad weather conditions. The results obtained can be used not only for the Ob Bay, but also elsewhere in the Arctic zone of the Russian Federation. The approaches to modeling and optimizing the

process of loading / unloading large-tonnage carriers with the help of additional aids can be developed by imposing additional conditions and taking into account other factors. This will enable these results to be included in the larger tasks of organizing transportation, for example, along the Northern Sea Route.

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