Study on Robust Fault Detection For Linear Discrete Time-Varying Systems With Measurement Missing

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Keywords: Robust fault detection filter, Missing measurements, H_∞ filtering, Linear discrete time-varying systems, Linear matrix inequality.

Abstract: In this study, the robust fault detection problem is investigated for a class of linear discrete time-varying systems with multiple measurement packet dropouts. The missing measurements are modeled as a linear function of the stochastic variable satisfying Bernoulli random binary distribution. A robust fault detection filter based on H_∞ filtering is designed to make residual error system be exponentially mean-square stable and satisfy a prescribed disturbance attenuation level. A sufficient condition is derived in terms of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness of the proposed method. The simulation result has showed the effectiveness and applicability of the obtained approach.

Introduction

In recent years, the robust fault detection method based on H_∞ filtering^[1-7] has attracted much attention of researchers and has gained rather rich research results. However, most of these researches are aimed at the linear constant systems, and the researches on linear time-varying systems are relatively few. The problem of robust fault detection and isolation for a class of discrete time linear parameter varying systems with time-varying delay has been studied in [4]. The problem of H_∞ fault estimation for a class of linear discrete time-varying systems with l_2 -norm –bounded disturbance has been discussed in [5].

At present, with the general application of network in industrial production, the phenomenon of time-delay and missing measurements has been widespread in the process of data transmission. In the case of incomplete measurement information, the problem of fault detection has become the focus of current research. In [6] and [7], the problem of the filtering and fault detection with random time-delays and measurement packet dropouts has been discussed respectively. The design of fault detection filter for linear discrete-time systems with random time-delays and measurement packet dropouts has been studied in [8]. In [9] and [10], the fault detection of a class of linear time-varying systems with multiple packet dropouts and multiple missing measurements has been studied respectively. But the above literatures are not aimed at linear discrete time-varying systems except for the [9] and [10].

In this paper, the robust fault detection problem is investigated for a class of linear discrete time-varying systems with multiple measurement packet dropouts. The missing measurements are described by a linear function of the stochastic variable satisfying Bernoulli random binary distribution. By using H_∞ control theory, the robust fault detection filter (RFDF) design can be formulated as an H_∞ filtering problem. A sufficient condition for the existence of the desired fault detection filter is established in terms of linear matrix inequalities (LMI), and a numerical example is provided to illustrate the effectiveness and applicability of the proposed design method.

Problem formulation

Consider the following class of linear discrete time-varying systems:

$$\begin{cases} x(k+1) = A(k)x(k) + B_d(k)d(k) + B_f(k)f(k) \\ y(k) = C(k)x(k) + D_d(k)d(k) + D_f(k)f(k) \end{cases} \quad (1)$$

where $x(k) \in R^n$ is the state vector; $y(k) \in R^q$ is the signal to be estimated; $d(k) \in R^p$ is the unknown disturbance input; $f(k) \in R^r$ is the fault to be detected; $A(k)$, $B_d(k)$, $B_f(k)$, $C(k)$, $D_d(k)$ and $D_f(k)$ are known real constant matrices with appropriate dimensions.

The insertion of network cable between measurement node and fault detection filter causes measurement missing phenomena. Furthermore, the measurement can be described by

$$\psi(k) = \delta(k)y(k) + (1 - \delta(k))\psi(k-1) \quad (2)$$

where $\psi(k) \in R^q$ is the measured output, and the stochastic variable $\delta(k) \in R$ is a Bernoulli distributed white sequence with

$$\begin{cases} \text{Prob}\{\delta(k)=1\} = E\{\delta(k)\} = \delta \\ \text{Prob}\{\delta(k)=0\} = 1 - E\{\delta(k)\} = 1 - \delta \end{cases} \quad (3)$$

and $\delta \in R$ $\delta \in (0, 1]$ is a known positive scalar.

The core module of the fault detection based on H_∞ filtering is the generation of residuals. In the case of system Eq. 1 is said to be asymptotically stable, consider the following *RFDF* for system with multiple measurement packet dropouts:

$$\begin{cases} \hat{x}(k+1) = A_F(k)\hat{x}(k) + B_F(k)\psi(k) \\ r(k) = C_F(k)\hat{x}(k) \end{cases} \quad (4)$$

where $\hat{x}(k) \in R^n$ is the state estimate, $r(k) \in R^r$ is the so-called residual that is compatible with the fault vector, and $A_F(k) \in R^{n \times n}$, $B_F(k) \in R^{n \times p}$ and $C_F(k) \in R^{p \times n}$ are filter parameters to be determined.

Define the augmented state vector $\xi(k) = [x^T(k) \ \hat{x}^T(k)]^T$, Denote $\omega(k) = [d^T(k) \ f^T(k)]^T$, $r_e(k) = r(k) - f(k)$, the augmented system formed by system Eq. 1, Eq. 2 and the filter Eq. 4 can be expressed by

$$\begin{cases} \xi(k+1) = A_{\xi 1}(k)\xi(k) + A_{\xi 2}(k)\xi(k-1) + B_{\xi}(k)\omega(k) \\ r_e(k) = C_1(k)\xi(k) + D_1(k)\omega(k) \end{cases} \quad (5)$$

where

$$\begin{aligned} A_{\xi 1}(k) &= A_1(k) + \delta(k)A_2(k), \quad A_1(k) = \begin{bmatrix} A(k) & 0 \\ 0 & A_F(k) + I \end{bmatrix}, \quad A_2(k) = \begin{bmatrix} 0 & 0 \\ B_F(k)C(k) & -I \end{bmatrix}, \\ A_{\xi 2}(k) &= A_3(k) + \delta(k)A_4(k), \quad A_3(k) = \begin{bmatrix} 0 & 0 \\ 0 & -A_F(k) \end{bmatrix}, \quad A_4(k) = \begin{bmatrix} 0 & 0 \\ 0 & A_F(k) \end{bmatrix}, \\ B_{\xi}(k) &= B_1(k) + \delta(k)B_2(k), \quad B_1(k) = \begin{bmatrix} B_d(k) & B_f(k) \\ 0 & 0 \end{bmatrix}, \quad B_2(k) = \begin{bmatrix} 0 & 0 \\ B_F(k)D_d(k) & B_F(k)D_f(k) \end{bmatrix}, \end{aligned}$$

$$C_1(k) = \begin{bmatrix} 0 & C_F(k) \end{bmatrix}, \quad D_1(k) = \begin{bmatrix} 0 & -I \end{bmatrix}.$$

The design problem stated above will be referred to as the robust filtering problem with networked time-delay and missing measurements. More specifically, we aim to design the filter Eq. 4 such that the following requirements are satisfied simultaneously: for a given $\gamma > 0$, the system Eq. 5 is exponentially mean-square stable and

$$E \|r_e(k)\|_2 < \gamma E \|\omega(k)\|_2 \quad (6)$$

where $\gamma > 0$ is a prescribed scalar.

The final important task of fault detections is the residual evaluation. Based on the designed fault detection system, the residual evaluation function $J(k)$ and the threshold J_{th} [3] are determined as follows:

$$\begin{cases} J(k) = \left\{ \sum_{k=k_0}^{k_0+N} r^T(k) r(k) \right\}^{1/2} \\ J_{th} = \sup_{d(k) \in I_2, f(k)=0} E \{ J(k) \} \end{cases} \quad (7)$$

where k_0 denotes the initial evaluation time instant, N denotes the length of time window.

Based on the above, the fault can be detected by using the following logical relationship:

$$\begin{cases} J(k) > J_{th} \Rightarrow abnormal \Rightarrow fault \\ J(k) \leq J_{th} \Rightarrow normal \end{cases} \quad (8)$$

Design of robust fault detection filter

Theorem 1: For a given scalar $\gamma > 0$, the system Eq. 5 is exponentially mean-square stable and achieves the H_∞ -norm constraint Eq. 6 for all nonzero $\omega(k)$, if there exist positive definite matrices $P(k)$ and Q satisfying equation Eq. 9.

$$\begin{bmatrix} -P(k) + Q & * & * & * & * & * \\ 0 & -Q & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & -(1-\delta)P(k+1) & * & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & 0 & -\delta P(k+1) & * \\ C_1(k) & 0 & D_1(k) & 0 & 0 & -I \end{bmatrix} < 0 \quad (9)$$

where $\Xi_{41} = (1-\delta)PA_1(k)$, $\Xi_{42} = (1-\delta)P(k+1)A_3(k)$, $\Xi_{43} = (1-\delta)P(k+1)B_1(k)$,

$\Xi_{51} = \delta P(k+1)(A_1(k) + A_2(k))$, $\Xi_{52} = \delta P(k+1)(A_3(k) + A_4(k))$, $\Xi_{53} = \delta P(k+1)(B_1(k) + B_2(k))$.

Remark 1: The proof of Theorem 1 would require to be combined with definition in [12] and lemma 1 in [13]. The specific proof process would be omitted.

Corollary 1: For a given scalar $\gamma > 0$, the system Eq. 5 is exponentially mean-square stable and achieves the H_∞ -norm constraint Eq. 6 for all nonzero $\omega(k)$, if there exist positive definite matrices $W(k)$, $Y(k)$, $Z_1(k)$ and $Z_3(k)$, matrices $Z_2(k)$, $M(k)$, $N(k)$ and $S(k)$ satisfying Eq. 10.

$$\begin{bmatrix} \psi_{11} & * & * & * & * & * \\ 0 & \psi_{22} & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} & * & * \\ \psi_{51} & 0 & \psi_{53} & 0 & \psi_{55} & * \\ \psi_{61} & 0 & \psi_{63} & 0 & 0 & -I \end{bmatrix} < 0 \quad (10)$$

where $\psi_{11} = \begin{bmatrix} -X(k) + Z_1(k) & * \\ -X(k) + Z_2(k) & -Y(k) + Z_3(k) \end{bmatrix}$, $\psi_{22} = \begin{bmatrix} -Z_1(k) & * \\ -Z_2(k) & -Z_3(k) \end{bmatrix}$,

$$\psi_{41} = \begin{bmatrix} (1-\delta)X(k+1)A(k) & (1-\delta)X(k+1)A(k) \\ (1-\delta)(Y(k+1)A(k) + M(k) + X(k) - Y(k)) & (1-\delta)Y(k+1)A(k) \end{bmatrix},$$

$$\psi_{42} = \begin{bmatrix} 0 & 0 \\ -(1-\delta)M(k) & 0 \end{bmatrix}, \quad \psi_{43} = \begin{bmatrix} (1-\delta)X(k+1)B_d(k) & (1-\delta)X(k+1)B_f(k) \\ (1-\delta)Y(k+1)B_d(k) & (1-\delta)Y(k+1)B_f(k) \end{bmatrix},$$

$$\psi_{44} = \begin{bmatrix} -(1-\delta)X(k+1) & * \\ -(1-\delta)X(k+1) & -(1-\delta)Y(k+1) \end{bmatrix},$$

$$\psi_{51} = \begin{bmatrix} \delta X(k+1)A(k) & \delta X(k+1)A(k) \\ \delta(Y(k+1)A(k) + S(k)C(k) + M(k)) & \delta(Y(k+1)A(k) + S(k)C(k)) \end{bmatrix},$$

$$\psi_{53} = \begin{bmatrix} \delta X(k+1)B_d(k) & \delta X(k+1)B_f(k) \\ \delta(Y(k+1)B_d(k) + S(k)D_d(k)) & \delta(Y(k+1)B_f(k) + S(k)D_f(k)) \end{bmatrix},$$

$$\psi_{55} = \begin{bmatrix} -\delta X(k+1) & * \\ -\delta X(k+1) & -\delta Y(k+1) \end{bmatrix}, \quad \psi_{61} = [N(k) \quad 0], \quad \psi_{63} = [0 \quad -I].$$

Proof: Partitioning matrices $P(k)$ and $U(k) = P^{-1}(k)$ into the following block forms:

$$P(k) = \begin{bmatrix} P_1(k) & P_2(k) \\ P_2^T(k) & P_3(k) \end{bmatrix}, \quad U(k) = P^{-1}(k) = \begin{bmatrix} U_1(k) & U_2(k) \\ U_2^T(k) & U_3(k) \end{bmatrix},$$

where $P_1(k) \in R^{n \times n}$ and $U_1(k) \in R^{n \times n}$ are symmetric positive definite matrices, $P_2(k)$ and $U_2(k)$ are two non-singular matrices.

Define the invertible matrices as follows:

$$J_1(k) = \begin{bmatrix} U_1(k) & I \\ U_2^T(k) & 0 \end{bmatrix}, \quad J_2(k) = \begin{bmatrix} I & P_1(k) \\ 0 & P_2^T(k) \end{bmatrix},$$

which implies that $P(k)J_1(k) = J_2(k)$, and let $J_1^T(k)QJ_1(k) = \begin{bmatrix} U_1(k)Z_1(k)U_1(k) & U_1(k)Z_2^T(k) \\ Z_2(k)U_1(k) & Z_3(k) \end{bmatrix}$.

Performing congruence transformation to Eq. 9 by the transformational matrix $\Xi_1(k)$, then pre- and post-multiplying the resulting LMI of the above transformation by the transformational matrix $\Xi_2(k)$ and defining the following matrix variables: $U_1^{-1}(k) = W(k)$, $P_1(k) = Y(k)$, $P_2(k+1)A_F(k)U_2^T(k) = M(k)W^{-1}(k)$, $C_F(k)U_2^T(k) = N(k)W^{-1}(k)$, $P_2(k+1)B_F(k) = S(k)$, we can arrive at the result (10) in Corollary 1, where

$$\Xi_1(k) = \text{diag} \{ J_1(k), J_1(k), I, J_1(k+1), J_1(k+1), I \},$$

$$\Xi_2(k) = \text{diag} \{ W^{-1}(k), I, W^{-1}(k), I, I, I, W^{-1}(k+1), I, W^{-1}(k+1), I, I \}.$$

This ends the proof.

Moreover, the desired filter parameters of Eq. 4 can be given by

$$A_F(k) = (X(k) - Y(k))^{-1} M(k), B_F(k) = (X(k) - Y(k))^{-1} S(k), C_F(k) = N(k) \quad (11)$$

Remark 2: Let $g = \gamma^2$. The sub-optimal robust filtering problem for discrete-time uncertain systems with one-step communication delay and measurement missing can be represented as follows:

$$\begin{aligned} & \min_{W(k), Y(k), Z_1(k), Z_2(k), Z_3(k), M(k), N(k), S_1(k)} g \\ & \text{s.t. Eq.10} \end{aligned} \quad (12)$$

The parameters of the sub-optimal filters can be determined by Eq. 12, and the sub-optimal H_∞ attenuation levels are given by $\gamma_{opt} = \sqrt{g^*}$, where g^* are the sub-optimal solutions of the corresponding convex optimization problems.

Simulation example

In this section, a numerical example is proved to illustrate the effectiveness and applicability of the proposed method. Consider the system described by (1) with parameters as follows:

$$A = \begin{bmatrix} 0.1e^{-k/100} & 0.9^k \\ 0.85 & -0.3\sin(k) \end{bmatrix}, \quad B_d = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.3\sin(k) \\ 0.4 \end{bmatrix},$$

$$C = [0.2 \quad 0.3], \quad D_d = 0.3, \quad D_f = 0.8.$$

For $k=0, 1, \dots, 200$, the unknown input $d(k)$ is supposed to be random noise which is uniformly distributed over $[-0.5, 0.5]$, and the fault signal $f(k)$ is given as:

$$f(k) = \begin{cases} 1, & k = 60, L, 140 \\ 0, & \text{else} \end{cases}$$

By using equation (7), the threshold $J_{th} = 0.0039428$ can be obtained. The simulation results indicate that the evaluation function $J(k) = 0.0045429 > J_{th}$, so the appeared fault can be detected after 6 time steps. Fig.1 shows the generated residual signal $r(k)$ without stepwise fault signal. Fig.2 demonstrates the evolution of residual evaluation function, in which the blue dashed line stands for fault-free case and the red solid line for the fault case. The simulation results are given in Fig. 1 and Fig.2, from which effectiveness of the method is illustrated.

Fig.1 Residual $r(k)$ with Stepwise fault signal

Fig.2 Evolution of residual evaluation $J(k)$

Conclusions

The problem of *RFDF* has been considered in this paper for a class of linear discrete

time-varying systems with multiple measurement packet dropouts. The *RFDF* based on H_∞ filtering has been used as residual generator. Then the design problem of *RFDF* has been reduced to H_∞ filtering problem. A sufficient condition for the existence of the *RFDF* has been established by means of LMI, and the parameters of the *RFDF* are determined directly by solutions of the LMI. The simulation result has showed the effectiveness and applicability of the obtained approach.

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