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Design of Analytical Model for Predicting the Remaining Battery Discharge Time

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Abstract. Based on the data of voltage, current intensity and remaining battery discharge time, a mathematical model of lead-acid battery discharge curve is established. The model was used to fit the sampling data of the curve about lead battery discharge with different current intensity. The nlinfit function of the optimization toolbox of MATLAB is applied along with calculation to obtain the model coefficient, results of which indicate the mean relative error (MRE) of the model fitting the curve about battery discharge time with different current intensity is 2.393%, 1.538%, 1.185%, 0.560%, 0.460%, 0.697, 0.745%, 0.642%, and 0.673% respectively. They are all lower than 3%, suggesting the model is with high accuracy.

Introduction

It has been over 150 years of history since the emergence of lead-acid batteries in 1859. With advantages like low costs, easy to be manufactured, multiple models, easy to be maintained or maintenance-free, long floating charge life, safe in use, and high voltage of a single battery, they have widely been applied to industries, military, and people's daily life. China is a country with the largest production and maintenance of electric bicycles in the world, and lead-acid batteries are taken as the power source for 95% of its electric bicycles. In the communications industry, lead-acid batteries are used by two-thirds of the standby power supply. In the automotive industry, lead-acid batteries are used for starting, lighting, and ignition by over 90 % of vehicles [1]. If lead-acid batteries are frequently discharged thoroughly, their service life will be greatly reduced. Therefore, it is of great significance to predict the remaining discharge time of lead-acid batteries under the current load, which will facilitate the rational management of batteries, avoid damage to batteries caused by over-discharge, and give full play to the performance of batteries. The study on discharge models of lead-acid batteries mainly includes the equivalent circuit model, neural network model, and least square model [2,3,4,5,6]. However, the discharge model of classical lead-acid batteries is usually very complicated with a complex resolving process, so it is difficult to meet actual needs. Therefore, based on the discharge monitoring data of lead-acid batteries with different current intensity, this paper attempts to establish the initial mathematical model of arbitrary current intensity discharge curves and predict the remaining discharge time of batteries under arbitrary current intensity by making use of the model.

Data and methods

Data

The data in this paper refers to the sampling data of the discharge curve about the same batch of lead-acid batteries with different current intensity. Partial data is given in Table 1, and the complete data is shown in question C of China Undergraduate Mathematic Contest in Modeling in 2016. As for the data, some explanations are provided below:

(1) During the discharge of lead-acid batteries with a constant current intensity, the voltage decreases monotonously with the discharge time until it reaches the rated minimum protection voltage (Um, 9V in this paper).

(2) Heat can be produced in the battery discharge process, leading to changes in the experimental temperature which will affect the data of the discharge curve. However, it is assumed in this paper external environmental conditions are always unchanged during the sampling experiment.

Table 1 Data of Question C of China Undergraduate Mathematical Contest in Modeling in 2016

	Discharge current(A)/ Discharge Voltage(V)								
Discharge Time(min)	20	30	40	50	60	70	80	90	100
0	11.1781	11.0514	11.0650	11.0821	11.1043	11.1243	11.1536	11.1864	11.2179
2	10.8913	10.7179	10.6421	10.5650	10.5086	10.4257	10.3736	10.3250	10.2750
4	10.7419	10.6171	10.5521	10.4793	10.4221	10.3450	10.2979	10.2529	10.2150
6	10.6288	10.5886	10.5321	10.4657	10.4157	10.3507	10.3143	10.2843	10.2636
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Mathematical Model

According to the given data, 9 corresponding discharge curves are drawn, as shown in Fig. 1. Each discharge curve in the figure is obtained in condition of some constant discharge current intensity. To establish the discharge curve with arbitrary current intensity, it is necessary to establish a mathematical model about voltage and discharge time and current, that is

$$U = f(t, I), \tag{1}$$

Where, U is the voltage, t is the discharge time (min), and I is the discharge current intensity (A).

As the battery voltage is correlated with the remaining capacity, it is pointed out by Dong Jing et al. that the battery voltage and remaining capacity relation curve can be obtained via the cubic spline margin calculation and fitting [6]. Thus, it can be assumed there is power function relationship between the voltage and remaining capacity. In the discharge process with constant current intensity, the remaining capacity can be expressed as

$$Q = It, \tag{2}$$

Where Q is the remaining capacity, t is the remaining discharge time (min), and I is the discharge current intensity (A). Therefore, it can be assumed there is power function relationship between the voltage and the remaining discharge time. According to the given data, the discharge curve between the voltage and remaining time with constant current can be drawn, as shown in Fig. 2.

According to Peukerts equation [7]

$$t_1 I^n = K, \tag{3}$$

Where t_1 is the discharge time (min), I is the discharge current intensity (A), and K, n are constants. It can be assumed there is power function relationship between the discharge time t_1 and the discharge current I, thus, the power function relationship between the voltage and discharge current intensity I can be demonstrated.

In the lead-acid battery discharge process with a constant current intensity, the voltage decreases monotonically with the discharge time until it reaches the rated minimum protection voltage (Um, 9V in this paper), so the mathematical model of the voltage and discharge time and current is established.

$$U = a_1 t^{a_2} \left(I + a_3\right)^{a_4} + 9,$$
(4)

Where U is the voltage, t is the remaining discharge time (min), I is the discharge current intensity (A), and a_1, a_2, a_3, a_4 are undetermined coefficients.

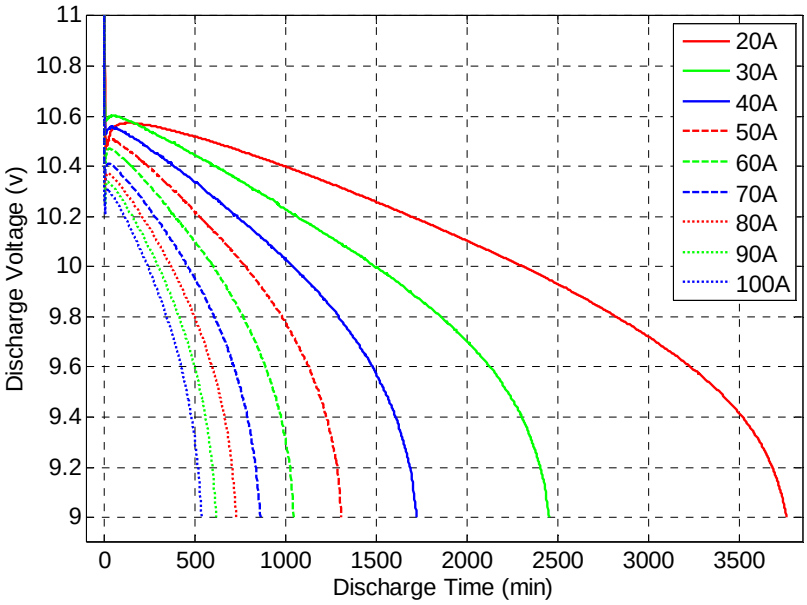

Fig .1 The discharged curve

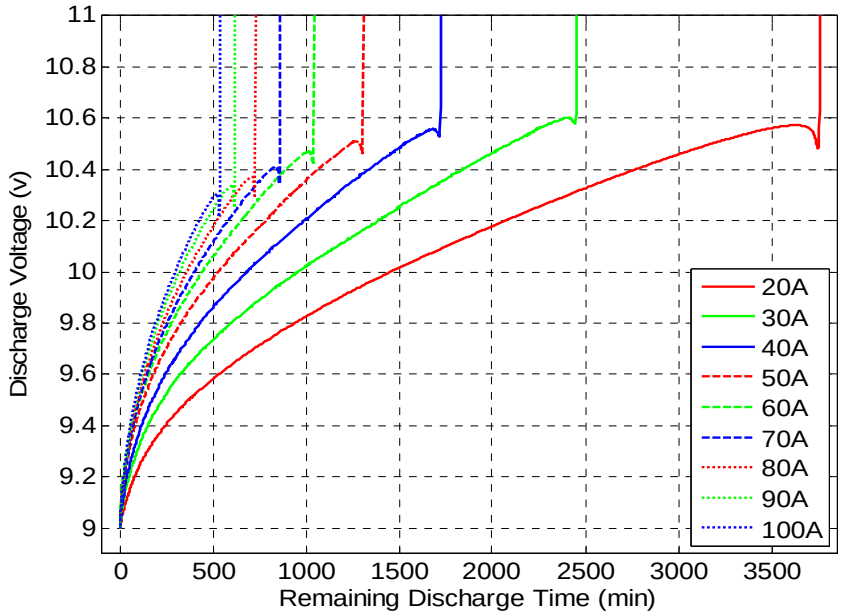

Fig .2 The remaining discharged curve

Results and verification

The nlinfit function in the optimization toolbox of MATLAB (The Mathworks, Inc.) is used for calculation and obtaining of coefficients of the model (4). Values of these coefficients are shown in Table 2. According to results of the coefficient fitting, the curve fit by the model (4) and the actually measured discharge curve are compared, as shown in Fig. 3. It can be seen from the figure that the model is well-fit for the discharge curve.

Table 2 Parameters of the model(4)

Parameter	a_1	a_2	a_3	a_3
Value	0.00811	0.50430	5.62511	0.42943

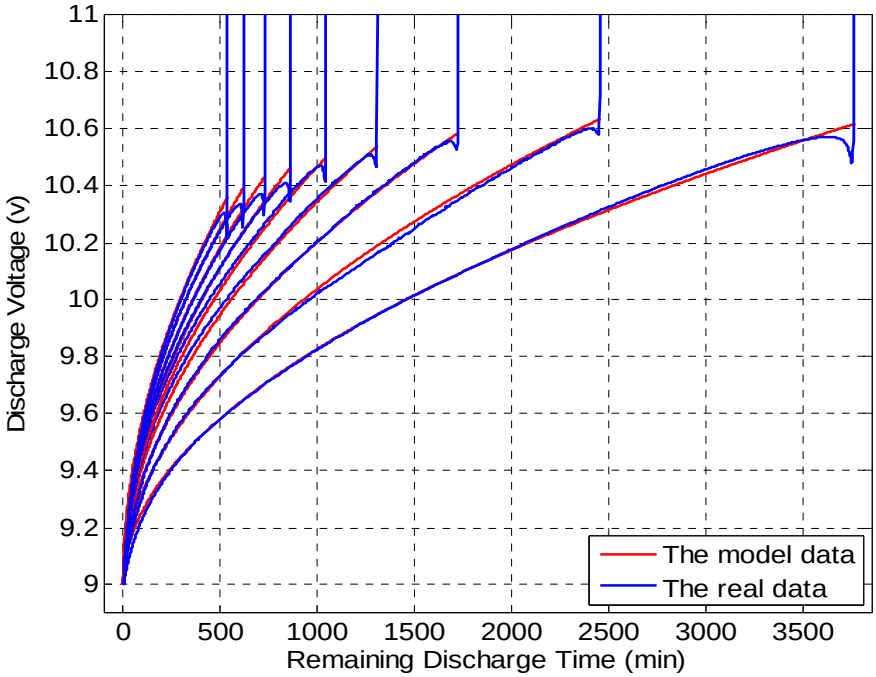

Fig. 3 Comparison between the curve of model (4) and discharge curve

The accuracy of predicting the battery remaining discharge time (or remaining capacity) by making use of the discharge curve is dependent on the quality of the discharge curve on the low-voltage section. Therefore, starting from Um(9v), we take 200 voltage sample points specific to the discharge curve with different current intensity. The mean relative error between the discharge time of the model corresponding to these voltage values and that of samples is MRE whose mathematic expression is

$$MRE = \frac{1}{n}\sum_{i=1}^{200}\left|\frac{t_i - \hat{t}_i}{t_i}\right|, \tag{5}$$

Where $\hat{t}$ is the discharge time of the model corresponding to voltage samples, t_i is the actual discharge time corresponding to voltage samples, and n is the number of samples. According to model (4), the remaining discharge time corresponding to voltage samples can be obtained, thus getting the corresponding discharge time of the model. By making use of formula (5), MRE with different current intensity can be obtained, as shown in Table 3.

Table 3 The mean relative error of discharge curve with different current intensity

Discharge Current(A)	20	30	40	50	60
MRE	0.02393	0.01538	0.01185	0.00560	0.00460
Discharge current(A)	70	80	90	100	
MRE	0.00697	0.00745	0.00642	0.00673	

Seen from Table 3, MRE values are all lower than 3%, the maximum of which is 0.02393 (the discharge curve whose current intensity is 20A) and the minimum is 0.00460 (the discharge curve whose current intensity is 60A). Therefore, the data fitting result of model (4) is very positive.

Conclusions

As the internal chemical reaction of lead-acid batteries in actual working state is very complicated, it is difficult to be described precisely by mathematical models. Based on vast experimental data, the relationship of voltage, current intensity and remaining discharge time is explored in this paper. A lead-acid battery discharge curve model is set up, and the model is used to fit the sampling data of the discharge curve about lead-acid batteries with different current intensity. The nlinfit function in the optimization toolbox of MATLAB (The Mathworks, Inc.) is adopted for calculation and obtaining of coefficients of model (4). Seen from comparative analysis of results, the MRE of predicting the battery discharge time with different current intensity by the model is 2.393%, 1.538%, 1.185%, 0.560%, 0.460%, 0.697%, 0.745%, 0.642%, and 0.673% respectively. They are all lower than 3%, suggesting the model is with high accuracy. In addition, what we establish is an initial mathematical model which has a concise form and is easy to be resolved. Thus, it is with certain application value.

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