Blind Signal Separation of Unknown Source Number Under the Constraint Based on Uniform Linear Array

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Abstract. Regarding blind signal separation based on eigenvalue decomposition, when the spatial dimension of signal is wrong in case of unknown source quantity, it will result in significant separation errors. By making use of kurtosis as the cost function, this paper attempts to construct a blind signal separation algorithm taking “space-kurtosis” spectrum as the basis, thus avoiding eigenvalue decomposition. Meanwhile, in the operation process, the source signal statistic independence and spatial distribution independence are fully utilized. It is proved by simulation experiments that the algorithm is with characteristics of high accuracy, fast operation and strong robustness.

Introduction

Blind signal separation(BSS) is to separate or extract the source signal from hybrid signal received by the sensor array in case the transmission channel characteristic and real source signal are unknown[1][2]. It is one of the hottest new disciplines of signal processing and is widely applied to fields like communications, remote sensing, radar, sonar, and noise control. Previous blind signal separation algorithms can be divided into two categories, one needs to know the source signal number in advance, and the other does not. A main representative of the former is principal component analysis (PCA)[3][4]. With these methods, the covariance matrix is calculated first, and then the eigenvector corresponding to the eigenvalue with the same number as source signals is found, and finally, the principal component vector obtained by transforming Karhunen-Loeveb, which is also known as the separated source signal[5][6][7]. However, in many application fields, the source signal number is unknown, so the rational threshold is set via the eigenvalue of the covariance matrix, and the number of eigenvalues larger than this threshold will be the number of source signals. Nonetheless, as a rational threshold is hard to be set, especially under the circumstance of low signal to noise ratio, the principal separation signal error is large. A typical algorithm that does not need to know the source signal number in advance is the blind signal separation algorithm brought forward by Amari [8]. Via this algorithm, the Kullback-Leibler divergence is taken as the cost function, and the natural gradient is adopted as the learning rule, so as to realize positive separation effect [9]. It can be applied to any group of blind source extraction and is applicable to blind signal separation occasions of unknown source number. However, the probability density distribution function of source signals needs to be known by Kullback-Leibler divergence. In actual applications, the probability density distribution function of source signals is unknown. Thus, a series of nonlinear functions have to be used for replacement in actual computing. In some special cases, these nonlinear functions may fail to match with the actual probability density distribution function of original source signals rigorously, thereby leading to wrong values of the algorithm convergence.

To obtain more information by making use of source signals and to improve the accuracy of blind signal algorithm of the unknown source signal number, the sensor array is designed as a uniform one in this paper, which is likely to be realized in some cases, for instance, the antenna array in communications, underwater sonar, and radar sensor array can all be structuralized as uniform linear

array with high precision. In this case, not only the statistic independence of signals can be used for blind signal separation, but also the spatial independence of signals can be utilized. Kurtosis is taken as the cost function for signal independence, and the “space-kurtosis” spectrum is constructed. By making use of the general direction of arrival corresponding to the peak of the spectrum, the separated weight vector as well as the source signal can be calculated. Assuming source signals are independent mutually, the probability density distribution function of source signals does not need to be known with the method of this paper, neither does the source signal number. In addition, the natural gradient algorithm is not utilized, so the learning step does not need to be selected. The time consumption of the algorithm is fixed and is steadily converged to various separation weight vectors. As the spatial information of source signals is applied by this algorithm compared to previous blind signal separation algorithms, it has been verified by simulation experiments that it has characteristics like high accuracy, fast operation and strong robustness.

Description of the blind signal separation issue

Blind signal separation is to separate the hybrid signal received by the sensor array, as shown in Figure 1. Sensor arrays are isotropy uniform linear arrays.

Fig.1 A model of blind signal separation based on uniform linear array

The hybrid system in Figure 1 can be described as:

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k) \quad (1)$$

Where $\mathbf{x}(k) = [x_1(k), x_2(k), \dots, x_M(k)]^T$ is the M -dimension observed signal vector;

$\mathbf{s}(k) = [s_1(k), s_2(k), \dots, s_N(k)]^T$ is the N -inention source signal vector;

A is the unknown hybrid matrix $M \times N$ dimension under the constraint of the array structure, which is also known as the direction matrix.

$$\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_N] \quad (2)$$

By taking No.1 array element for reference, the i -th column of vector will be

$$\mathbf{a}_i = [1 \quad \exp(j2\pi f_c \sin(\theta_i)/c) \quad \dots \quad \exp(j2\pi f_c \sin(\theta_M)/c)]^T \quad (3)$$

N is the incident source signal number; M is the array element number; $\mathbf{n}(k)$ is the superimposed noise of array elements; $d=c/2f_c$ is the distance between array elements; c is the spread speed of signals in the medium; and f_c is the central frequency of the narrow-band signal.

Although columns of the direction matrix A in formula (1) are the same as those in formula (3), θ_i is unknown. Therefore, the signal separation model of Figure 1 still belongs to blind signal separation.

The observed signal $\mathbf{x}(k)$ is used to determine the separation matrix $\mathbf{W}$ in the blind separation process, making

$$\mathbf{y}(k) = \mathbf{W}\mathbf{x}(k) \quad (4)$$

Is the estimation of $\mathbf{s}(k)$ source signals.

Separated weight vectors based on the direction of arrival

When the direction of arrival θ_i is known, the minimum variance distortion (MVDR) can be used to calculate the i -th row vector of the separation matrix $\mathbf{W}$ according to formula (4), that is

$$\mathbf{w}_i = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_i)}{\mathbf{a}^H(\theta_i)\mathbf{R}^{-1}\mathbf{a}(\theta_i)} \quad (5)$$

$\mathbf{R}$ is the covariance matrix receiving signal $\mathbf{x}(k)$.

Direction of arrival estimation of unknown source number based on kurtosis

In actual application, the direction of arrival θ_i in formula (5) is unknown, and it has to be estimated first. The most classic DOA estimation way is the MUSIC algorithm based on subspace decomposition. By decomposing the covariance matrix $\mathbf{R}$ into N -dimension signal subspace and $(M-N)$ -dimension noise subspace, the orthogonal projection on the noise subspace will be

$$\Pi^\perp = \mathbf{U}_n \mathbf{U}_n^H \quad (6)$$

Thus, the MUSIC “spatial spectrum” can be defined:

$$\mathbf{P}_{MUSIC}(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\Pi^\perp\mathbf{a}(\theta)} \quad (7)$$

θ corresponding to the “spectrum peak” is the DOA of source signal. Apparently, $\mathbf{P}_{MUSIC}(\theta)$ is not a real spectrum with any sense; Strictly speaking, it is merely the “distance” between the signal direction vector and the noise subspace, but it manages to show the “spectrum peak” near the actual DOA, thus presenting the DOA of different signals accurately with super-resolution.

However, when the unknown source signal number N is unknown, the source number has to be estimated before taking MUSIC algorithm. In case of low signal to noise ratio, underestimation may occur, leading to the classification of signal vectors into the noise subspace. Thus, the finally estimated DOA error will be large, and the signal separation accuracy will ultimately be affected.

To resolve the DOA estimation issue in case the source number is unknown, a “space-kurtosis” spectrum similar to $\mathbf{P}_{MUSIC}(\theta)$ by taking kurtosis as the cost function is constructed in this paper. It is assumed the unknown DOA is θ , the corresponding separated weight vector obtained via MVDR will be

$$\mathbf{w}(\theta) = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta)} \quad (8)$$

At this time, the signal corresponding to DOA separated from the hybrid signal can be

$$\mathbf{y}(k) = \mathbf{w}(\theta)\mathbf{x}(k) \quad (9)$$

The kurtosis of the signal will be

$$P_{\kappa}(\theta) = \frac{E[|y(k)|^4]}{E^2[|y(k)|^2]} - 3 \quad (10)$$

By putting formula (8) and (9) into (10), the “space-kurtosis” spectrum can be defined as

$$P_{\kappa}(\theta) = \frac{E[|\frac{\mathbf{R}^{-1}\mathbf{a}(\theta)\mathbf{x}(k)}{\mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta)}|^4]}{E^2[|\frac{\mathbf{R}^{-1}\mathbf{a}(\theta)\mathbf{x}(k)}{\mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta)}|^2]} - 3 \quad (11)$$

By changing the value of θ and scanning all direction, the entire “space-kurtosis” spectrum can be obtained which can show the “spectrum peak” in the real DOA, that is, what the spectrum peak corresponds to is the DOA of the source signal $(\theta_1, \theta_2, \dots)$. There’s no need to decompose the signal space and noise space for the covariance matrix $\mathbf{R}$ in the whole computing process, so the number of source signals does not have to be estimated.

The BSS of “space- kurtosis” spectrum

When the DOA $(\theta_1, \theta_2, \dots)$ is estimated, the MVDR is utilized again to obtain the corresponding separated weight vector

$$\mathbf{w}(\theta_i) = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_i)}{\mathbf{a}^H(\theta_i)\mathbf{R}^{-1}\mathbf{a}(\theta_i)} \quad (12)$$

The corresponding separation signal is

$$y_i(k) = \mathbf{w}(\theta_i)\mathbf{x}(k) \quad (13)$$

According to the above analysis, the process frame based on the “space- kurtosis” spectrum algorithm can be drawn, as shown in Figure 2.

Fig.2 BSS frame based on the “space-kurtosis” spectrum (the virtual frame is the “space-kurtosis” spectrum generation process)

In Figure 2, the receiving sensor array is the uniform linear array, and θ changes from 0° to 90° by taking $step$ as the step size. Via steps in the circulation virtual frame, the “space-kurtosis” spectrum is obtained, and the angle corresponding to the spectrum peak is the DOA. Later, formula (12) and (13) are used to separated signal corresponding to the DOA. It is especially worth noting that some fake spectrum peaks may exist in the “space-kurtosis” spectrum, and source signals do not exist

in the corresponding DOA. Therefore, it is the noise signal that is separated from these fake spectrum peaks, but their existence does not affect the precision of separating source signals by making use of real spectrum peaks.

Simulation experiments

To examine the effectiveness of the algorithm, simulation experiments of blind signal separation are carried out. In the experiment, the hybrid and separation of signals are as shown in Figure 1. The receiving sensor array is the uniform array, and the gap between array elements is half the size of the sound signal wave. Noise interference is available to all array elements, and relevant parameters of the algorithm are. Three voice signals (s1, s2 and s3) provided by Cichocki advanced brain signal processing laboratory (ABSP) is adopted as incident signals, and their waveforms are shown below[10]:

Fig.3 Waveforms of source signals

To carry out quantitative analysis of the blind separation effect on hybrid signals by the algorithm, the performance index (PI) of formula (14) is used for measurement.

$$PI = \sum_{i=1}^N \max(\rho(y_i, y_j)) \quad (14)$$

Where

$$\rho(y_i, y_j) = \frac{|E(y_i y_j) - E(y_i)E(y_j)|}{\sqrt{E^2(y_i - E(y_i)) * E^2(y_j - E(y_j))}} \quad , (j=1, \dots, N, \text{but } j \neq i) \quad (15)$$

The smaller the PI value is, the better the separation effect will be.

Experiment 1: The influence of noise on signal separation

Experiment conditions are: there are 10 array elements, and the incidence of source signal s1, s2 and s3 is from 10°, 30° and 70°. The SNR of figure 4 (a), (b) and (c) is -10 dB, 0 dB, and 10 dB respectively. The left of the graph shows the waveforms of three components of hybrid signals, and waveforms and PI value of separation signals are shown on the right.

Fig.4 Influence of noise on the signal separation effect

It can be seen from Figure 4(a), when SNR=-10 dB, the separated signal component y_1 , y_2 and y_3 are corresponding to the source signal s_1 , s_2 and s_3 . Although waveforms are severely covered by the noise, source signals can still be identified by hearing, and the corresponding PI value is very low (0.0434). When SNR=0 dB and 10 dB, the separation effect is positive. Thus, it is verified that the algorithm of this paper can result in favorable robustness towards noise.

Experiment 2: The influence of the number of array elements on signal separation

Experiment conditions: SNR=5dB, and the incidence of source signal s_1 , s_2 and s_3 is from 10° , 30° and 70° . Figure 5(a) shows waveforms of three components of hybrid signals, and waveforms of separation signals are shown when the array element number of (b), (c) and (d) is 3, 10 and 20.

Fig.5 The influence of the number of array elements on signal separation

As can be seen from Figure 5, when the source number is smaller than or equal to the number of array elements, source signals can be separated effectively with this algorithm, which is because signal subspace decomposition does not have to be conducted by taking the kurtosis as the cost function. Meanwhile, as the number of array elements increases, the separation effect can be better.

Experiment 3: The influence of the number of array elements on the calculation speed

Experiment conditions are: SNR=10dB, and the incidence of source signal s_1 , s_2 and s_3 is from 10° , 30° and 70° . changes from 0° to 90° by taking step=0.1° as the step size. The Monte Carlo experiment is conducted for 300 times on the condition of different numbers of array elements, and the mean time consumption is obtained, as shown in Table 1.

Table 1 The influence of the number of array elements on the calculation speed

NO.	3	6	10	20	40
TIME (s)	0.563744	0.570071	0.587945	0.618130	0.835219

Seen from Table 1, the number of array elements changes from 3 to 40, but the time for calculation does not change much, which is conducive to the hardware solidification and the application to large-scale arrays. As the number of array elements increases, the direction matrix and each snapshot will be lengthened, but the operation amount has not been affected significantly.

Experiment 4: The influence of the number of array elements on resolution

Experiment conditions are: SNR=10dB, and the incidence of the source signal s_1 is from 10° , that of the s_2 is from $10^\circ + \Delta$. θ changes from 0° to 90° by taking step=0.1° as the step size. In case the number of array elements is different, the separation effect PI is shown in Figure 6.

Fig. 6. The influence of the number of array elements on resolution

As shown in the Figure, when $\Delta = 0.5^\circ$ and there are more than 15 array elements, source signals can be well separated; when $\Delta = 1^\circ, 2^\circ$ and there are more than 10 array elements, source signals can also be well separated. The experiment suggests that this algorithm can help to separate source signals in similar spatial direction as long as the number of array elements is sufficient.

Conclusions

By taking kurtosis as the cost function, this paper takes “space-kurtosis” spectrum as the basis for blind signal separation. Under the constraint of uniform linear array, the kurtosis corresponding to separation signals of the entire array flow pattern is calculated first, thus obtaining the “space-kurtosis” spectrum peak. By searching the spectrum peak, the DOA of source signals can be got, and then the separated weight vector can be computed by making use of the MVDR. Finally, signals can be separated via the separation of weighted vectors. Steepest descent iteration for the cost function does not need to be carried out in the whole process, thus avoiding the algorithm divergence caused by the setting of irrational iteration step. The statistic independence and spatial direction independence of signals can be fully utilized as well. Through simulation experiments, it is proved that the algorithm proposed by this paper has characteristics like non-estimation of the source signal number, high reliability, and fast operation.

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