License Plate Auction of Bidder with Different Risk Appetite

Yi-Tian WU
East China University of Science and Technology
ECUST
Shanghai, China
E-mail: tkong941009@163.com

Jia-Hua SUN
East China University of Science and Technology
ECUST
Shanghai, China
E-mail: jimmysjh@163.com

***Abstract*—In recent year, auction is used to solve the problem that the requirement and supply of license plate is not equal. The common rule of auction is that the bidders win the license plate by price order after submitting the price at the same time. The discriminate auction model of license plate is built in this article to find the best bid strategy. Then classify the bidders by the different risk appetite. The winning probability of different risk appetite is analyzed. The best valuation expression can be got according the different probability.**

Key words*：*License plate auction, different rick appetite

I. THE DISCRIMINATE AUCTION MODEL

Model hypothesis:(1)The number of the same License plate is k (2)The number of the bidders is n($n \ge k$) (3)Each auctioneer can only get a license plate (4)The auction is a non-cooperative game (5)The valuation of bidders obey the same distribution function and won't change according to other information.

Every bidder have the valuation v_i of the auction, the valuation of bidders is independent and obey the same distribution function $F(v_i)$. $F(v_i)$ Is strictly increasing and differentiable in the Domain $[0,\theta]$ .And the probability density function if $f(v_i)$. Every bidder will submit the price g_i according to the valuation v_i . $g_i = g(v_i)$ Is monotonically increasing and differentiable. According to the auction $g_i \le v_i$.

Let's investigate the bidder X. His valuation is v_x .His bid function is $G = g_x = g(v_x)$. Other bidders' valuation and bid function is v_i , $g_i = g(v_i)$ (i=1,2…n and $i \ne x$). $P_x(win)$ Represents the probability that the bidder X win. We definite the income function of the bidder X

$$R_x = (v_x - G) \times P_x(win)$$

We introduce the inverse function $h(g_i)$, $h(g_i) = v_i$. The function h is monotonous, so

$$\begin{aligned} P_x(win) = {} & P(G > \{g_i\}_1) + P(\{g_i\}_1 > G \\ & > \{g_i\}_2) + \cdots + P(\{g_i\}_{k-1} > G > \{g_i\}_k) \end{aligned}$$

$$\begin{aligned} = {} & C_{n-1}^0 F[h(G)]^{n-1} + C_{n-1}^1 F[h(G)]^{n-2}[1 - F(h(G))] \\ & + \cdots + C_{n-1}^{k-1} F[h(G)]^{n-k}[1 - F(h(G))]^{k-1} \end{aligned}$$

$$= \sum_{i=1}^{k} C_{n-1}^{i-1}[1 - F(h(G))]^{i-1} F[h(G)]^{n-i}$$

So,

$$\begin{aligned} R_x = {} & (v_x - G) \times \\ & \sum_{i=1}^{k} C_{n-1}^{i-1}[1 - F(h(G))]^{i-1} F[h(G)]^{n-i} \end{aligned}$$

To solve the problem conveniently. We introduce the $L_i(x) = [1 - F(x)]^{i-1} F(x)^{n-i}$, so

$$R_x = (v_x - G) \times \sum_{i=1}^{k} C_{n-1}^{i-1} L_i[h(G)]$$

For the bidder X, His best bid G is the bid that can make the biggest R_x . So

$$\begin{aligned} \frac{\partial R_x}{\partial G} & = (v_x - G) \times \sum_{i=1}^{k} C_{n-1}^{i-1} L_i'[h(G)] \cdot h'(G) \\ & \quad - \sum_{i=1}^{k} C_{n-1}^{i-1} L_i[h(G)] \\ & = 0 \end{aligned}$$

For the bidder X: $G = g_x = g(v_x)$, $h(G) = v_x$, $h'(G) = 1/g'(v_x)$,then integral

$$G = v_x - \frac{\sum_{i=1}^{k} C_{n-1}^{i-1} \int_0^{v_x} L_i(y)dy}{\sum_{i=1}^{k} C_{n-1}^{i-1} L_i(v_x)}$$

Let put $L_i(x)=[1-F(x)]^{i-1}F(x)^{n-i}$ into the expression. We get the best bid strategy.

II. Bidder's best strategy with different risk appetite

In reality, everybody's risk appetite is different. For the bidders has different risk appetite, their bids are reason reasonably different. We classify the bidders has different risk appetite for three categories. The three categories are risk bidders, neutral bidders and conservative bidders. Their corresponding bidding function is g_1, g_2, g_3. The number of three kinds of bidders are n_1, n_2, n_3 $\left(n_1+n_2+n_3=n\right)$.The number of three kinds of bidders winning are k_1, k_2, k_3,The remaining assumptions are the same as the above model.

Considering the bidder X,His valuation and bid is v_x and G. We definite the income function $R_x=(v_x-G)\times P_x(win)$. Bidder X is the bidders of three categories. We definite the three bids G_1 , G_2 G_3 according to three categories. So the income function is

$$\begin{aligned} R_x = \quad & (v_x-G_1)\cdot P_x^{risky}(win) \\ & +(v_x-G_2)\cdot P_x^{neutral}(win) \\ & +(v_x-G_3)\cdot P_x^{conservative}(win) \end{aligned}$$

We introduce the symbol $\{g_i\}_k$. The symbol $\{g_i\}_k$ represents the kth bid after arranging descending bid g_i .As shown on the right, $\{g_i\}_k$ intersects function g_1 , g_2 , g_3 ,and the abscissa of intersection point are v_1, v_2, v_3.(It mean the lowest valuation of the three kinds of bidders if they win).You can see this suitation in the Figure 1.

Considering the risk bidders: the number of the bidders are n_1 . v_1 Is the lowest valuation if they win $F(v_1)$ represent the probability that valuation is less than v_1. So the number of the risk bid winning is $k_1=[1-\mathrm{F}(v_1)]\cdot n_1$.

$$P_x^{risky}(win)=\sum_{i=1}^{k_1}C_{n_1-1}^{i-1}[1-F(v_x)]^{i-1}F[v_x]^{n_1-i},$$
$$\text{and}\, k_1=[1-F(v_1)]\cdot n_1$$

As the same inference, we can get the winning probability of the neutral bidders and conservative bidders. So

$$\begin{aligned} R_x=&[v_x-g_1(v_x)]\sum_{i=1}^{k_1}C_{n_1-1}^{i-1}[1-F(v_x)]^{i-1}F[v_x]^{n_1-i}\cdot\frac{n_1}{n}+ \\ &[v_x-g_2(v_x)]\sum_{i=1}^{k_2}C_{n_2-1}^{i-1}[1-F(v_x)]^{i-1}F[v_x]^{n_2-i}\cdot\frac{n_2}{n}+ \\ &[v_x-g_3(v_x)]\sum_{i=1}^{k_3}C_{n_3-1}^{i-1}[1-F(v_x)]^{i-1}F[v_x]^{n_3-i}\cdot\frac{n_3}{n} \end{aligned}$$

$$k_1=[1-\mathrm{F}(v_1)]\cdot n_1 \quad k_2=[1-\mathrm{F}(v_2)]\cdot n_2$$

$$k_3=[1-\mathrm{F}(v_3)]\cdot n_3$$

Because the bidding strategy is different according to the different risk appetite, we want to find the best valuation. The best valuation v_x is what make the R_x biggest.

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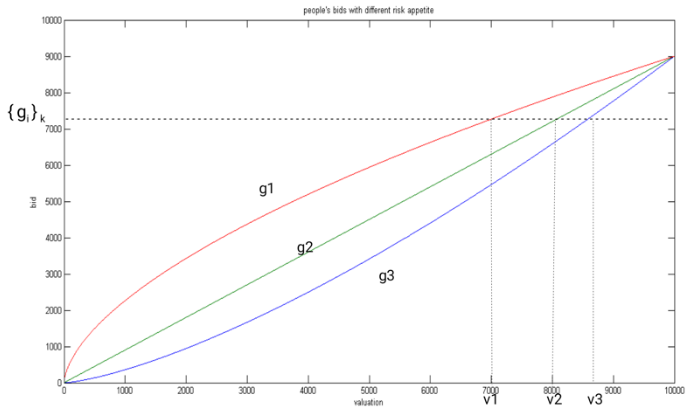

Figure 1. People’s bids with different risk appetite