Order Properties of Conditional Warm Standby System at Random Time

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Abstract—This article discusses the residual life and inactivity time of warm standby system at random time. The reliability function of the residual life(inactivity time) of the warm standby system at random time is obtained. And by using stochastic order, we compared the stochastic properties of residual life at different times.

Keywords—Stochastic order; warm standby system; Residual life

I. INTRODUCTION

In general, three types of standby redundancy are usually discussed in literature, namely, cold standby, hot standby, and warm standby. In a cold standby redundancy, the redundant component is inactive, that is, it has zero failure rate and hence never fails in standby state; in a system with hot standby redundancy, and the redundant component also works in the usual environment and may fail when it is in standby state. That is, the active component and the redundant one form a parallel system. The warm standby means that the components work in a warm environment when they are in standby status. Both the residual life and inactivity time of systems have been investigated in literature. For example, Asadi[1], Khaledi[2], Hashemi[3], Zhang[4].

Let X be the lifetime of the redundancy in the usual environment with distribution function $F(t)$ and survival function $\bar{F}(t) = 1 - F(t)$. Since the redundant component in standby statu operates in a milder environment, the lifetime in standby statu should be stochastically larger than that in the usual environment. Light of the accelerated life model (see Bagdonavicius and Nikulin [5]) and the decelerated life model (see Singpurwalla [6]), the lifetime of the redundancy in standby status, say X^* , $F^*(t)$ be the distribution function of X^* . Since the standby unit is operated under milder environment than the usual level of environment during warm standby state, it is assumed that

$$X^* \leq_{st} X \quad (1)$$

For any $t \geq 0$, (1) implies that

$$F^*(t) = F(\gamma(t)).$$

Where continuous non-decreasing function $\gamma(t)$ depends on the accelerated environment, $t \geq \gamma(t) \geq \gamma(0) \equiv 0$ for all $t \geq 0$.

According to the virtual age model (see Kijima[7], Finkelstein[8]), it is reasonable to assume that a standby redundancy switching to the usual environment at its age $t > 0$ has the virtual age $\omega(t)$ satisfying (i) $t \geq \omega(t) \geq \omega(0) \equiv 0$ for all $t \geq 0$, (ii) $\omega(t)$ is continuous and increases in $t \geq 0$.

Let Y be the lifetime of the active component with distribution function $G(t)$. Assume that X and Y are independent. According to Li et al [9], the total lifetime of the general standby system composed of the active component Y and the standby one X is

$$T_{Y|X} = \begin{cases} Y, & \text{if } Y \geq X^*, \\ Y + X_{\omega(Y)}, & \text{if } Y < X^*. \end{cases}$$

Here $\omega(t) = h(t, \gamma(t))$ for some bivariate function h_1 , and

$$X_{\omega(Y)} = X - \omega(Y) \mid X > \omega(Y)$$

Is the residual life of the lifetime X at random time $\omega(Y)$. The reliability function of the lifetime $T_{Y|X}$ of the standby system as below. For any $t \geq 0$,

$$\bar{H}(t) = \bar{G}(t) + \int_0^t \frac{\bar{F}(t - \delta(u))}{\bar{F}(\omega(u))} \bar{F}(\gamma(u)) dG(u),$$

Where $\delta(u) \equiv u - \omega(u)$ for all $u \geq 0$.

Definition 1[10] the lifetime of the component is assumed to be M , the residual life and inactivity time of the component at $t \geq 0$ are

$$M_t = (M - t \mid M > t), M_{(t)} = (t - M \mid M \leq t).$$

The survival function is

$$\bar{H}_t(x) = \bar{H}(t+x) / \bar{H}(t), \bar{H}_{(t)}(x) = H(t-x) / H(t).$$

Definition 2[11] suppose X and Y have their distribution functions F and G , denote $\bar{F} = 1 - F$ and $\bar{G} = 1 - G$ as their survival functions, respectively. For any $x \geq 0$

a) If $\bar{F}(x) \leq \bar{G}(x)$, then X is said to be smaller than Y in the usual stochastic order (denoted by $X \leq_{st} Y$);

b) If $\bar{F}(x) / \bar{G}(x)$ is decreasing in $x \geq 0$, then X is said to be smaller than Y in the hazard rate order (denoted by $X \leq_{hr} Y$);

c) if $F(x) / G(x)$ is decreasing in $x \geq 0$, then X is said to be smaller than Y in the reversed hazard rate order (denoted by $X \leq_{rh} Y$).

For ease of reference, relations among these orderings are presented as follows(see, e.g., Shaked M, Shanthikumar [12]):

$$X \leq_{st} Y \Rightarrow X \leq_{hr} Y, X \leq_{st} Y \Rightarrow X \leq_{rh} Y.$$

This article discusses the residual life and inactivity time of the warm standby system, we consider the reliability function of the residual life (inactivity time) of the warm standby system at random time Θ . The influence of time on the residual life of the system is studied.

II. MAIN RESULTS

Think of t as a nonnegative random variable, as Θ . The distribution function and survival function of Θ are $H, \bar{H}$. In what follows, we consider the residual life (inactivity time) of the warm standby system at random time $\Theta \geq 0$. That is

$$(T(X, Y) - \Theta | \min(X, Y) \geq \Theta).$$

Theorem 1. Let $R^\Theta(x)$ be the reliability function of the residual life $(T(X, Y) - \Theta | \min(X, Y) \geq \Theta)$. Then, for any $x \geq 0, \theta \geq 0$,

$$R^\Theta(x) = \int_0^\infty \int_\theta^\infty \frac{\bar{F}(x+\theta-u+\omega(u))\bar{F}(\gamma(u))}{\bar{F}(\gamma(\theta))\bar{F}(\omega(u))\bar{G}(\theta)} dG(u)dH(\theta) + \int_0^\infty \frac{\bar{G}(\theta+x)}{\bar{G}(\theta)} dH(\theta).$$

Proof. For any $x \geq 0, \theta \geq 0$, we have

$$\begin{aligned} R^\Theta(x) &= P(T(X, Y) - \Theta \geq x | \min(X, Y) \geq \Theta) \\ &= \frac{P(T(X, Y) - \Theta \geq x, \min(X, Y) \geq \Theta)}{P(\min(X, Y) \geq \Theta)} \\ &= \int_0^\infty \frac{P(T(X, Y) - \theta \geq x, \min(X, Y) \geq \theta)}{P(\min(X, Y) \geq \theta)} dH(\theta). \end{aligned}$$

Among,

$$P(\min(X, Y) \geq \Theta) = \bar{F}(\gamma(\theta))\bar{G}(\theta),$$

and,

$$\begin{aligned} &P(T(X, Y) - \theta > x, \min(X, Y) \geq \theta) \\ &= P(T - \theta > x, X > Y, Y \geq \theta) \\ &\quad + P(T - \theta > x, X < Y, X \geq \theta) \\ &= P(Y + X^* > x + \theta, X > Y, Y \geq \theta) \\ &\quad + P(Y > x + \theta, X^* \geq \theta) \\ &= \int_0^\infty P(Y + X^* > x + \theta, X > Y, Y \geq \theta | Y = u) dG(u) \\ &\quad + \bar{G}(x + \theta)\bar{F}(\gamma(\theta)) \\ &= \int_\theta^\infty [P(X - \omega(u) > x + \theta - u | X > u, Y = u) \\ &\quad P(X > u | Y = u)] dG(u) + \bar{G}(x + \theta)\bar{F}(\gamma(\theta)) \\ &= \int_\theta^\infty \frac{\bar{F}(x + \theta - u + \omega(u))}{\bar{F}(\omega(u))} \bar{F}(\gamma(u)) dG(u) \\ &\quad + \bar{G}(x + \theta)\bar{F}(\gamma(\theta)). \end{aligned}$$

Hence,

$$\begin{aligned} R^\Theta(x) &= \int_0^\infty \int_\theta^\infty \frac{\bar{F}(x + \theta - u + \omega(u))\bar{F}(\gamma(u))}{\bar{F}(\gamma(\theta))\bar{F}(\omega(u))\bar{G}(\theta)} \\ &\quad dG(u)dH(\theta) + \int_0^\infty \frac{\bar{G}(\theta+x)}{\bar{G}(\theta)} dH(\theta). \end{aligned}$$

The following theorem is illustrated the influence of time on the residual life of the system is studied.

Theorem 2. If $\Theta_1 \leq_{st} \Theta_2$, then

$$\begin{aligned} &(T(X, Y) - \Theta_1 | \min(X, Y) \geq \Theta_1) \\ &\geq_{st} (T(X, Y) - \Theta_2 | \min(X, Y) \geq \Theta_2). \end{aligned}$$

Proof. Due to Theorem 1, for any $x \geq 0, \Theta \geq 0$, we

have

$$R^{\Theta_{1,2}}(x) = \int_0^\infty \int_{\Theta_{1,2}}^\infty \frac{\bar{F}(x + \theta_{1,2} - u + \omega(u)) \bar{F}(\gamma(u))}{\bar{F}(\gamma(\theta_{1,2})) \bar{F}(\omega(u)) \bar{G}(\theta_{1,2})} dG(u) dH(\theta_{1,2}) + \int_0^\infty \frac{\bar{G}(\theta_{1,2} + x)}{\bar{G}(\theta_{1,2})} dH(\theta_{1,2}),$$

So,

$$\begin{aligned} R^{\Theta_1}(x) - R^{\Theta_2}(x) &= \int_0^\infty \int_{\Theta_1}^\infty \frac{\bar{F}(x + \theta_1 - u + \omega(u)) \bar{F}(\gamma(u))}{\bar{F}(\gamma(\theta_1)) \bar{F}(\omega(u)) \bar{G}(\theta_1)} dG(u) dH(\theta_1) \\ &\quad + \int_0^\infty \frac{\bar{G}(\theta_1 + x)}{\bar{G}(\theta_1)} dH(\theta_1) - \int_0^\infty \frac{\bar{G}(\theta_2 + x)}{\bar{G}(\theta_2)} dH(\theta_2) \\ &\quad - \int_0^\infty \int_{\Theta_2}^\infty \frac{\bar{F}(x + \theta_2 - u + \omega(u)) \bar{F}(\gamma(u))}{\bar{F}(\gamma(\theta_2)) \bar{F}(\omega(u)) \bar{G}(\theta_2)} dG(u) dH(\theta_2) \\ &\geq 0, \end{aligned}$$

Hence,

$$\begin{aligned} (T(X, Y) - \Theta_1 | \min(X, Y) \geq \Theta_1) \\ \geq_{st} (T(X, Y) - \Theta_2 | \min(X, Y) \geq \Theta_2). \end{aligned}$$

By Theorem 2, we can easily obtain the following theorem.

Theorem 3. If $\Theta_1 \leq_{hr} \Theta_2$, then

$$\begin{aligned} (T(X, Y) - \Theta_1 | \min(X, Y) \geq \Theta_1) &\geq_{st} \\ (T(X, Y) - \Theta_2 | \min(X, Y) \geq \Theta_2). \end{aligned}$$

Theorem 4. If $\Theta_1 \leq_{rh} \Theta_2$, then

$$\begin{aligned} (T(X, Y) - \Theta_1 | \min(X, Y) \geq \Theta_1) \\ \geq_{st} (T(X, Y) - \Theta_2 | \min(X, Y) \geq \Theta_2). \end{aligned}$$

Theorem 3 and Theorem 4 show that in the case of hazard rate order and reversed hazard rate order, the same conclusion as Theorem 2 can be obtained, i.e., the residual life of the system decreases as time increases.

Then, we consider the inactivity time of the warm standby system at random time $\Theta \geq 0$. That is

$$(\Theta - T(X, Y) | \max(X, Y) \leq \Theta).$$

Theorem 5. Let $R^{(\Theta)}(x)$ be the reliability function of the inactivity time $(\Theta - T(X, Y) | \max(X, Y) \leq \Theta)$. Then, for any $x \geq 0$, $\theta \geq 0$,

$$\begin{aligned} R^{(\Theta)}(x) &= \int_0^\infty \int_0^\theta \frac{F(\theta - x - u + \omega(u))}{F(\theta)G(\theta)\bar{F}(\omega(u))} \bar{F}(\gamma(u)) \\ &\quad dG(u) dH(\theta) + \int_0^\infty \frac{F(\gamma(\theta))G(\theta - x)}{F(\theta)G(\theta)} dH(\theta). \end{aligned}$$

Proof. For any $x \geq 0$, $\theta \geq 0$, we have

$$\begin{aligned} R^{(\Theta)}(x) &= P(\Theta - T(X, Y) \geq x | \max(X, Y) \leq \Theta) \\ &= \int_0^\infty \frac{P(\theta - T(X, Y) \geq x, \max(X, Y) \leq \theta)}{P(\max(X, Y) \leq \theta)} dH(\theta) \end{aligned}$$

Among,

$$P(\max(X, Y) \leq \theta) = F(\theta)G(\theta),$$

and,

$$\begin{aligned} &P(T(X, Y) \leq \theta - x, \max(X, Y) \leq \theta) \\ &= P(T(X, Y) \leq \theta - x, X < Y \leq \theta) \\ &\quad + P(T(X, Y) \leq \theta - x, Y < X \leq \theta) \\ &= P(Y \leq \theta - x, X^* \leq \theta) + P(Y + X^* \leq \theta - x, Y < X \leq \theta) \\ &= G(\theta - x)F(\gamma(\theta)) \\ &\quad + \int_0^\infty P(Y + X^* \leq \theta - x | X > u, Y = u) dG(u) \\ &= G(\theta - x)F(\gamma(\theta)) + \\ &\quad \int_0^\theta P(X - \omega(u) \leq \theta - x - u | X > \omega(u)) \\ &\quad P(X > u | Y = u) dG(u) \\ &= G(\theta - x)F(\gamma(\theta)) \\ &\quad + \int_0^\theta \frac{F(\theta - x - u + \omega(u))}{\bar{F}(\omega(u))} \bar{F}(\gamma(u)) dG(u). \end{aligned}$$

hence,

$$\begin{aligned} R^{(\Theta)}(x) &= \int_0^\infty \int_0^\theta \frac{F(\theta - x - u + \omega(u))}{F(\theta)G(\theta)\bar{F}(\omega(u))} \bar{F}(\gamma(u)) dG(u) dH(\theta) \\ &\quad + \int_0^\infty \frac{F(\gamma(\theta))G(\theta - x)}{F(\theta)G(\theta)} dH(\theta). \end{aligned}$$

III. CONCLUSIONS

In this work, we investigate the reliability function of the residual life (inactivity time) of the warm standby system at random time Θ . And compare the stochastic properties of Residual life at different times, the results show that with the increase of time, the residual life of the system becomes

smaller and smaller.

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