

# Kinematic Errors Modeling Method of Five-axis Machine Tool with a Universal Spindle Head

Ya Zhang

Zhejiang University of Science and technology, Hangzhou, 310027, China

yazhang1982@126.com

**Keywords:** Kinematic Errors ; Machine Tool ; Modeling

**Abstract.** A simple mathematical modelling method of five-axis machine tool with a universal spindle head was proposed according to the structural characteristics of the machine tool. The mathematical formula of the kinematic errors of the machine tool, which has two rotary axes superimposed together, is derived. The mathematical formula of this model is simple, the physical meaning of the modelling process is clear. It is a theoretical foundation for the kinematic error recognition of five-axis machine tool with a universal spindle head.

## Introduction

Five-axis machine tools are widely applied in the high-precision machining area[1,2]. Motion accuracy is the most basic indicators of high-precision five-axis machine tools. In order to improve the motion accuracy, the kinematic errors should be estimated accurately. Some methods have been proposed to identify the kinematic errors of the machine tool with a universal spindle head recently, such as the estimating method by telescoping double ball bar (DBB)[3-6] and the “R-Test”[7], and so on. The structural characteristics of five-axis machine tool with a universal spindle head is shown in Fig.1.

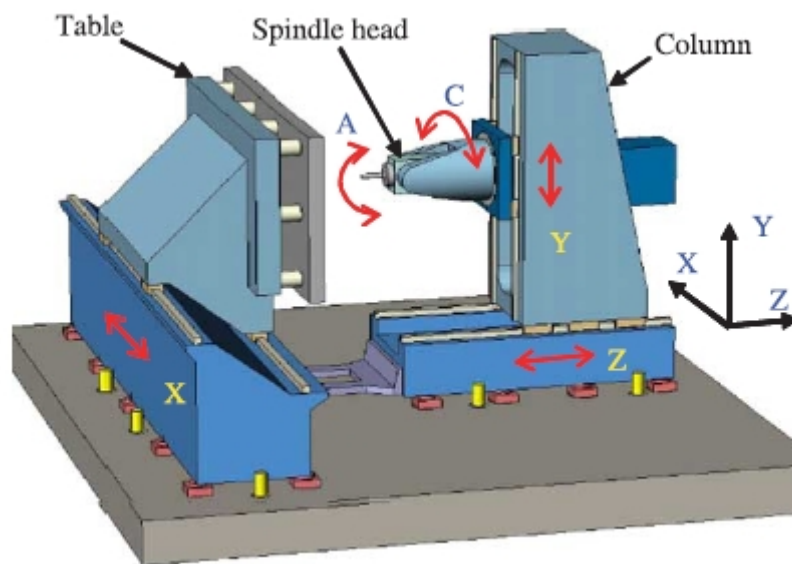


Fig.1 Five-axis machine tool with a universal spindle head[8]

## Coordinate system settings of the tool ring of five-axis Machine tool

According to the structural characteristics of the spindle head of the machine tool, the coordinate systems of the spindle head were set up with a simple method in this paper, as can be seen in Fig.2.

The coordinate systems of the machine tool with a universal spindle head was established, as can be seen in Figure 2. Firstly, establish the reference coordinate system {F} in the centerline intersection point of A-axis and C-axis (point O) when A-axis and C-axis remain stationary. Obviously, nominal

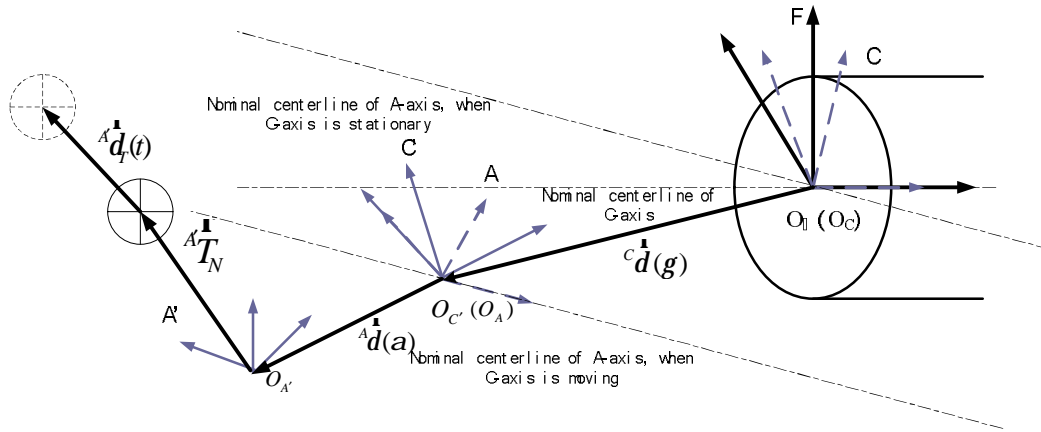


Fig.2 Kinematic error modeling of the universal spindle head of five-axis machine tool

coordinate system of C-axis {C} can be established in  $O_F$ . Hence, there is only a rotation transformation for coordinate system {C} relative to coordinate system {F}. Secondly, when C-axis has an actual displacement, the position of  $O_C$  changes to  $O_{C'}$ , establish the coordinate system {A} (nominal coordinate system of A-axis) in  $O_{C'}$ .

### The modeling process of the kinematic errors of five-axis machine tool with a universal spindle head

(1) The modeling process of the kinematic errors of tool ring

The modeling process of the kinematic errors is described as follows:

1) Description of tool center point (TCP)

Suppose the initial position of the tool center point is in +Z direction and has a distance of L from the reference point of the machine tool. The position of tool center point in Coordinate system can be represented as

$${}^{A'}\mathbf{r}_{T_N} = \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix} \quad (1)$$

2) The differential transformation

There is a differential transformation from coordinate system {F} to coordinate system {A}. The differential transformation is completed by the differential movement transformation and the differential rotation transformation.

$${}^{A'}\mathbf{T} = {}^{A'}\mathbf{T}_N + {}^{A'}\mathbf{d}_{T,N} \quad (2)$$

Where

$${}^{A'}\mathbf{d}_T = \begin{bmatrix} {}^{A'}\mathbf{d}_{T,x} \\ {}^{A'}\mathbf{d}_{T,y} \\ {}^{A'}\mathbf{d}_{T,z} \end{bmatrix} \quad (3)$$

Based on the assumptions of small angle [9], the operator of the differential rotation transformation from coordinate system to coordinate system C can be represented as

$${}^C_R = \begin{bmatrix} 1 & -x_z(g) & x_y(g) \\ x_z(g) & 1 & -x_x(g) \\ -x_y(g) & x_x(g) & 1 \end{bmatrix} \quad (4)$$

${}^{A'}R$  can be represented as

$${}^A_R = \begin{bmatrix} 1 & -x_z(a) & x_y(a) \\ x_z(a) & 1 & -x_x(a) \\ -x_y(a) & x_x(a) & 1 \end{bmatrix} \quad (5)$$

Similarly, there is a differential transformation from coordinate system to coordinate system C. The differential transformation is completed by the differential movement transformation and the differential rotation transformation.

$${}^C\dot{T} = {}^C\dot{d}(g) + {}^C_R {}^C\dot{T} \quad (6)$$

Where

$${}^C\dot{d}(g) = \begin{bmatrix} {}^C d_x(g) \\ {}^C d_y(g) \\ {}^C d_z(g) \end{bmatrix} \quad (7)$$

${}^C_R$  can be represented as

$${}^C_R = \begin{bmatrix} 1 & -x_z(g) & x_y(g) \\ x_z(g) & 1 & -x_x(g) \\ -x_y(g) & x_x(g) & 1 \end{bmatrix} \quad (8)$$

There are two rotation transformation operators  ${}^{C'}_A R$  and  ${}^F_C R$

$${}^{C'}_A R = \begin{bmatrix} Ca & -Sa & 0 \\ Sa & Ca & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

$${}^F_C R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & Cg & -Sg \\ 0 & Sg & Cg \end{bmatrix} \quad (10)$$

From the above analysis, the position vector of the tool center point in coordinate system F can be obtained.

$${}^F\dot{T} = {}^F_R [{}^C\dot{d}(g) + {}^C_R [{}^{C'}_A R [{}^A\dot{d}(a) + {}^A_R [{}^A\dot{T}_N + {}^A\dot{d}_{T,N}(t)]]]] \quad (11)$$

According to the principle of matrix transformation, we can obtain

$${}^F\dot{T} = \begin{bmatrix} {}^C\dot{d}_x(g) + \cos a * ({}^A\dot{d}_{T,x}(t) + {}^A d_x(a)) - \sin a * ({}^A\dot{d}_{T,y}(t) + {}^A d_y(a)) + \\ {}^C x_y(g) * ({}^A d_z(a) - L + {}^A\dot{d}_{T,z}(t)); \\ \cos g * ({}^C d_y(g) + \cos a * ({}^A\dot{d}_{T,y}(t) + {}^A d_y(a)) + \sin a * ({}^A\dot{d}_{T,x}(t) + {}^A d_x(a)) - \\ {}^C x_x(g) * ({}^A d_z(a) - L + {}^A\dot{d}_{T,z}(t))) - \sin g * ({}^A d_z(a) - L + {}^A\dot{d}_{T,z}(t) + \\ {}^C x_x(g) * (\cos a * ({}^A\dot{d}_{T,y}(t) + {}^A d_y(a)) + \sin a * ({}^A\dot{d}_{T,x}(t) + {}^A d_x(a)) - \\ {}^C x_y(g) * (\cos a * ({}^A\dot{d}_{T,x}(t) + {}^A d_x(a)) - \sin a * ({}^A\dot{d}_{T,y}(t) + {}^A d_y(a))); \\ \sin g * ({}^C d_y(g) + \cos a * ({}^A\dot{d}_{T,y}(t) + {}^A d_y(a)) + \sin a * ({}^A\dot{d}_{T,x}(t) + {}^A d_x(a)) - \\ {}^C x_x(g) * ({}^A d_z(a) - L + {}^A\dot{d}_{T,z}(t))) + \cos g * ({}^A d_z(a) - L + {}^A\dot{d}_{T,z}(t) + \\ {}^C x_x(g) * (\cos a * ({}^A\dot{d}_{T,y}(t) + {}^A d_y(a)) + \sin a * ({}^A\dot{d}_{T,x}(t) + {}^A d_x(a)) - \\ {}^C x_y(g) * (\cos a * ({}^A\dot{d}_{T,x}(t) + {}^A d_x(a)) - \sin a * ({}^A\dot{d}_{T,y}(t) + {}^A d_y(a))) \end{bmatrix} \quad (12)$$

(2) The modeling process of the kinematic errors of workpiece ring

The position of the workpiece is in the end coordinate system of the workpiece ring of five-axis machine tool. It can be described directly with a position vector.

$${}^F\dot{W} = {}^F P_{OX} + {}^F R^X \dot{W} \quad (13)$$

(3) The position description of the tool center point in the coordinate system of workpiece

The volumetric error of the tool center point can be obtained. It can be represented as a position vector:

$$\dot{R} = {}^F \dot{T} - {}^F \dot{W} \quad (14)$$

## Conclusions

A five-axis machine tool with a universal spindle head has two rotary axes superimposed together. It is difficult to separate the kinematic errors of the spindle head. A simple mathematic model of the kinematic errors of the machine tool with a spindle head is established according to the structural characteristics of the machine tool. The mathematical expression of the tool center position in the coordinate system of the workpiece is deduced. It provides a mathematical basis for developing the machining tests to identify and compensate the kinematic errors.

## Acknowledgements

This work was financially supported by Startup Foundation of Zhejiang University of Science and Technology (F701102H02).

## References

- [1] K. Inasaki, H. Kishinami, N. Sakamoto, Y. Sugimura, F. Takeuchi, Shape generation theory for machine tools, Yokendo, (1997)95–100.
- [2] E.L.J. Bohez, Five-axis machine tool kinematic chain design and analysis, International Journal of Machine Tools & Manufacture 42 (2002) 505–520.
- [3] J.B. Bryan, A simple method for testing measuring machines and machine tools, Part I: Principle and applications, Precision Engineering 4 (2) (1982) 61–69.
- [4] S. Sakamoto, I. Inasaki, Identification of alignment errors in fiveaxis machining centers, Transactions of Japan Society of Mechanical Engineers 60 (C-575) (1994) 2475–2483.
- [5] S. Sakamoto, I. Inasaki, Identification of alignment errors in fiveaxis machining centers using telescoping ball bar, Transactions of Japan Society of Mechanical Engineers 63 (C-605) (1997)262–267.
- [6] Sang Peng , The DBB-Based Errors Modeling and Measurement Method for 5-axis High Speed CNC Machine Tools [D] , dissertation of Tian Jin University , 2010.
- [7] Weikert, S. (2004). “R-test, a new device for accuracy measurements on five axis machine tools.” Cirp Annals-Manufacturing Technology 53(1): 429-432.
- [8] K. Dassanayake, M. Tsutsumi, and A. Saito, A strategy for identifying static deviations in universal spindle head type multi-axis machining centers. Int. J. of Machine Tools and Manufacture, 2006. 46(10): p.1097-1106.
- [9] Youlun X (1993) Robotics. China Machine, Beijing.