The Forward Displacement Analysis of 3-RPS Parallel Mechanism Based on Matlab Symbolic Operation

Xiao-Dong LIANG^{1,a}, Feng-Long YIN^{1,b}, Er-Kang LI^{1,c}, Hong-Bo LIAO^{1,d}

¹Northwest Institute of Nuclear Technology, xi'an 710024, China

^axiaodoudong@126.com, ^byinfenglongwade@163.com, ^clektt@qq.com, ^d13975890826@163.com

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Abstract. Through analyzing the structural features of the 3-RPS parallel mechanism, establish the position constraint equations of the parallel mechanism, input the initial value, calculate the value of positive solution to location by symbolic operation in the Matlab and get the corresponding institutions posture figure. It shows that the equations for the forward displacement analysis of 3-RPS can get all the value positive solutions, which is accurate and reliable.

Introduction

A parallel mechanism is a closed loop mechanism in which the end effector is connected by at least two independent kinematic chains and the basic platform, with two or more two degrees of freedom, and driven in parallel. The mechanism has the advantages of high speed, high strength, high bearing capacity and error characteristics accumulated by domestic and foreign scholars, including Australia Hunt proposed a 3-RPS parallel mechanism to with two dimensional rotation and one dimensional movement, which is a research hotspot^[1,2].

Description and establishment of coordinate system for 3-RPS

The 3-RPS parallel mechanism consists of the lower platform (fixed platform), the upper platform (moving platform) and the 3 identical RPS (revolute pair-moving pair-spherical bearing pair) connecting the upper and lower platforms. With the extension of the moving pair (P), change the position and posture of the upper platform, and realize the two dimensional rotation and one dimensional movement of the upper platform. Establish a fixed coordinate system in $O-XYZ$ platform in which O is the center of triangle $A_1A_2A_3$ circle, Y axis through the point A_1 , Z axis perpendicular to the OXY plane, and the X axis is determined by the right-hand rule, On the basis of the characteristics of institutions. Similarly, on the upper platform to establish dynamic coordinate system $O'-X'Y'Z'$, as shown in figure 1.

Fig.1 3-RPS parallel mechanism

Forward displacement analysis of 3-RPS

As shown in Figure 2, the three branched chain vectors are $\vec{W}_i$ ($i=1,2,3$), the angle between the branch and the OXY plane is θ_i ($i=1,2,3$), $\phi=30^\circ$, the length of the electric cylinder is L_i ($i=1,2,3$), and the length of side of the equilateral triangle in the lower platform is c .

Fig.2 Structure sketch of 3-RPS parallel mechanism

Since the three branches of the mechanism are constrained by the rotational pair, the branch A_iB_i ($i=1, 2, 3$) can only rotate in the plane OA_iB_i . The three branched unit vectors $\vec{W}_i$ and the position vectors $\vec{OA}_i$ ($i=1, 2, 3$) are as follows.

$$\vec{W}_1 = \cos \theta_1 \vec{j} + \sin \theta_1 \vec{k} \quad (1)$$

$$\vec{W}_2 = -\cos \theta_2 \cos \phi \vec{i} - \cos \theta_2 \sin \phi \vec{j} + \sin \theta_2 \vec{k} \quad (2)$$

$$\vec{W}_3 = \cos \theta_3 \cos \phi \vec{i} - \cos \theta_3 \sin \phi \vec{j} + \sin \theta_3 \vec{k} \quad (3)$$

Position vector $\vec{OA}_i$ ($i=1,2,3$):

$$\vec{OA}_1 = -R \vec{j} \quad (4)$$

$$\vec{OA}_2 = \frac{\sqrt{3}}{2} R \vec{i} + \frac{R}{2} \vec{j} \quad (5)$$

$$\vec{OA}_3 = -\frac{\sqrt{3}}{2} R \vec{i} + \frac{R}{2} \vec{j} \quad (6)$$

There among, $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the unit vectors for X , Y and Z axis, R is the radius of circumcircle in lower platform. According to the laws of vectors $\vec{OB}_i = \vec{OA}_i + L_i \vec{W}_i$, then

$$\left| \vec{OB}_1 - \vec{OB}_2 \right|^2 = c^2 \quad (7)$$

$$\left| \vec{OB}_2 - \vec{OB}_3 \right|^2 = c^2 \quad (8)$$

$$\left| \vec{OB}_3 - \vec{OB}_1 \right|^2 = c^2 \quad (9)$$

Obtained from the above formula that:

$$\begin{cases} A_1 \cos \theta_1 + B_1 \cos \theta_2 + D_1 \cos \theta_1 \cos \theta_2 + E_1 \sin \theta_1 \sin \theta_2 + F_1 = 0 \\ A_2 \cos \theta_2 + B_2 \cos \theta_3 + D_2 \cos \theta_2 \cos \theta_3 + E_2 \sin \theta_2 \sin \theta_3 + F_2 = 0 \\ A_3 \cos \theta_3 + B_3 \cos \theta_1 + D_3 \cos \theta_1 \cos \theta_3 + E_3 \sin \theta_1 \sin \theta_3 + F_3 = 0 \end{cases} \quad (10)$$

There among,

$$\begin{aligned} A_1 &= -3RL_1 & B_1 &= -3RL_2 & D_1 &= L_1L_2 & E_1 &= -2L_1L_2 & F_1 &= 3R^2 + L_1^2 + L_2^2 - c^2 \\ A_2 &= -3RL_2 & B_2 &= -3RL_3 & D_2 &= L_2L_3 & E_2 &= -2L_2L_3 & F_2 &= 3R^2 + L_2^2 + L_3^2 - c^2 \\ A_3 &= -3RL_3 & B_3 &= -3RL_1 & D_3 &= L_1L_3 & E_3 &= -2L_1L_3 & F_3 &= 3R^2 + L_1^2 + L_3^2 - c^2 \end{aligned}$$

Assign

$$x_i = \tan \frac{\theta_i}{2} \quad (11)$$

Then

$$\sin \theta_i = \frac{2x_i}{1+x_i^2}; \cos \theta_i = \frac{1-x_i^2}{1+x_i^2} \quad (i = 1, 2, 3), \text{ the Equation (10) can be obtained}$$

as:

$$a_1x_2^2 + b_1x_2 + c_1 = 0 \quad (12)$$

$$a_2x_3^2 + b_2x_3 + c_2 = 0 \quad (13)$$

$$a_3x_3^2 + b_3x_3 + c_3 = 0 \quad (14)$$

There among

$$\begin{aligned} a_1 &= (A_1 - B_1 - D_1 + F_1) + (-A_1 - B_1 + D_1 + F_1)x_1^2 \\ b_1 &= 4E_1x_1 \\ c_1 &= (A_1 + B_1 + D_1 + F_1) + (-A_1 + B_1 - D_1 + F_1)x_1^2 \\ a_2 &= (A_2 - B_2 - D_2 + F_2) + (-A_2 - B_2 + D_2 + F_2)x_2^2 \\ b_2 &= 4E_2x_2 \\ c_2 &= (A_2 + B_2 + D_2 + F_2) + (-A_2 + B_2 - D_2 + F_2)x_2^2 \\ a_3 &= (-A_3 + B_3 - D_3 + F_3) + (-A_3 - B_3 + D_3 + F_3)x_1^2 \\ b_3 &= 4E_3x_1 \\ c_3 &= (A_3 + B_3 + D_3 + F_3) + (A_3 - B_3 - D_3 + F_3)x_1^2 \end{aligned}$$

Eliminate an x_3 from formula (13), (14), then:

$$(a_2c_3 - a_3c_2)x_3 + (b_2c_3 - b_3c_2) = 0 \quad (15)$$

Eliminate an x_3^2 from formula (13), (14), then

$$(a_2b_3 - a_3b_2)x_3 + (a_2c_3 - a_3c_2) = 0 \quad (16)$$

Eliminate x_3 from formula (15) and (16), then obtain an algebraic equation with x_1 and x_2 which is no higher than about 4 times.

$$(a_2c_3 - a_3c_2)^2 - (a_2b_3 - a_3b_2)(b_2c_3 - b_3c_2) = 0 \quad (17)$$

Write formula (17) as an explicit expression of x_2 :

$$k_1x_2^4 + k_2x_2^3 + k_3x_2^2 + k_4x_2 + k_5 = 0 \quad (18)$$

There among, each coefficient k_i is a function of x_1 .
 x_2 multiplies formula (18)

$$k_1x_2^5 + k_2x_2^4 + k_3x_2^3 + k_4x_2^2 + k_5x_2 = 0 \quad (19)$$

x_2 , x_2^2 , x_2^3 multiplies formula (18), respectively

$$a_1x_2^3 + b_1x_2^2 + c_1x_2 = 0 \quad (20)$$

$$a_1x_2^4 + b_1x_2^3 + c_1x_2^2 = 0 \quad (21)$$

$$a_1x_2^5 + b_1x_2^4 + c_1x_2^3 = 0 \quad (22)$$

Write the expressions (12), (18), (19), (20), (21) and (22) in matrix form

$$\begin{bmatrix} 0 & k_1 & k_2 & k_3 & k_4 & k_5 \\ k_1 & k_2 & k_3 & k_4 & k_5 & 0 \\ a_1 & b_1 & c_1 & 0 & 0 & 0 \\ 0 & a_1 & b_1 & c_1 & 0 & 0 \\ 0 & 0 & a_1 & b_1 & c_1 & 0 \\ 0 & 0 & 0 & a_1 & b_1 & c_1 \end{bmatrix} \begin{bmatrix} x_2^5 \\ x_2^4 \\ x_2^3 \\ x_2^2 \\ x_2 \\ 1 \end{bmatrix} = 0 \quad (23)$$

As $[x_2^5 \ x_2^4 \ x_2^3 \ x_2^2 \ x_2 \ 1]^T \neq 0$, according to the necessary and sufficient conditions, which is the system of homogeneous equations exist nonzero solutions, the following equations can be obtained.

$$\begin{vmatrix} 0 & k_1 & k_2 & k_3 & k_4 & k_5 \\ k_1 & k_2 & k_3 & k_4 & k_5 & 0 \\ a_1 & b_1 & c_1 & 0 & 0 & 0 \\ 0 & a_1 & b_1 & c_1 & 0 & 0 \\ 0 & 0 & a_1 & b_1 & c_1 & 0 \\ 0 & 0 & 0 & a_1 & b_1 & c_1 \end{vmatrix} = 0 \quad (24)$$

It is a 16 algebraic equation of x_1 , the entire solution of x_1 can be obtained by solving this equation, and the corresponding x_2 and x_3 equations can be obtained by formula (23) and formula (16). By substituting the result in formula (11), θ_i can be obtained, and the coordinates of the endpoints of the upper platform in the fixed coordinate system can be calculated with $\vec{OB}_i$.

Numerical example

Assign the initial values of structure $L_1=916mm$, $L_2=958mm$, $L_3=1089mm$, $R=800mm$, $c=799mm$, the 16 order equation of x_1 can be obtained with Matlab symbolic operation.

$$0.2330 \times 10^5 x_1^{16} + 0.4085 \times 10^5 x_1^{14} - 0.6589 \times 10^5 x_1^{12} + 0.1344 \times 10^5 x_1^{10} + 0.6512 \times 10^4 x_1^8 - 0.9611 \times 10^3 x_1^6 - 0.3289 \times 10^3 x_1^4 - 0.1331 \times 10^2 x_1^2 + 0.6987 = 0$$

The root is

$$x_1 = \begin{Bmatrix} \pm 0.1718 & \pm 0.6630 & \pm 0.6907 & \pm 0.7456 \\ \pm 0.3620i & \pm 0.3860i & \pm 0.3982i & \pm 1.6783i \end{Bmatrix}$$

The 16 roots can be obtained by solving the 16 order equation. Considering the actual situation, 8 real roots are taken as x_1 solution, then the corresponding coordinates of endpoint B_i in upper platform can be obtained, as shown in table 1.

Table 1 Endpoints B_i coordinates on upper platform in mechanism

Class	(x_{B1}, y_{B1}, z_{B1})	(x_{B2}, y_{B2}, z_{B2})	(x_{B3}, y_{B3}, z_{B3})
1	(0,63.92,305.89)	(445.96,257.48,915.01)	(-325.28,187.80,1003.20)
	(0,63.92,-305.89)	(445.96,257.48,-915.01)	(-325.28,187.80,-1003.20)
2	(0,-443.19,844.17)	(401.34,234.48,901.59)	(-400.18,210.19,1026.10)
	(0,-443.19,-844.17)	(401.34,234.48,-901.59)	(-400.18,210.19,-1026.10)
3	(0,-475.52,857.12)	(371.75,214.63,883.77)	(232.16,-134.04,214.06)
	(0,-475.52,-857.12)	(371.75,214.63,-883.77)	(232.16,-134.04,-214.06)
4	(0,-538.46,878.37)	(-102.98,-59.46,272.22)	(-299.04,172.65,989.88)
	(0,-538.46,-878.37)	(-102.98,-59.46,-272.22)	(-299.04,172.65,-989.88)

According to the platform endpoint coordinates shown in Table 1, the position of the upper platform is plotted, as shown in Figure 3.

Fig.3 position and posture of platform on platform

Conclusion

In this article, according to the geometrical relations of 3-RPS parallel mechanism, establish the equation of position constraint with analytic method. Work out the 16 sets of solutions for position constraint equations with Matlab symbol operation, and then obtain the corresponding mechanism posture figures. The results show that the method is accurate and reliable.

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