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Improved SPH Boundary Treatment Method and Numerical Simulation of Water-Soil Two-Phase Flow

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Abstract. The underwater landslide often appears in the construction of underwater facilities, causing serious personal property damage. It is of great significance for engineering design and disaster prevention to simulate the large deformation and two-phase flow problems. Smoothed Particle Hydrodynamics (SPH) method is a Lagrange meshless method, with inherent advantage for simulating the large deformation and two-phase flow problem. In the SPH method, appropriate boundary treatment method can guarantee higher accuracy. Boundary particle massless scheme is proposed, and underwater landslide is simulated by SPH method and split phase model. The law of motion and the dynamic response of underwater landslide are studied. The influence of physical parameters such as hydraulic conductivity and dilatancy angle on the process of underwater landslide is analyzed. The accuracy of massless boundary particle method is proved.

Introduction

The underwater landslide usually occurs in the underwater constructions, such as pave pavement and submersed tube construction. It will generate internal wave and cause tsunamis if seriously. It is important but difficult to simulate underwater landslide, since the methods based on mesh are difficult to handle the large deformation problem. Besides, two-phase flow problem is always a hard ball for simulation, since interaction between mixture is hard to simulate. Smoothed Particle Hydrodynamics (SPH), a Lagrange meshless method, has inherent advantage for simulating the large deformation and two-phase flow problem and holds remarkable achievement in mixture problems [1].

Two models can be selected to solve mixture problems in SPH. One of models considers mixture medium as a single-phase mixture. Miao et al. [2] assumed that the viscous debris flow could be described by compressible Navier-Stokes equations, and established a SPH model based on Bingham movement law. Although the model can reveal the global phenomenon of two-phase flow, the interaction between mixture cannot be investigated. The other is split phase model, in which each phase of the mixture satisfies its own governing equation, and the interaction between mixture model will satisfies other equation, which is determined by model. Bui et al. [3] proposed a spilt phase model, in which the soil is regarded as an elastic-perfectly plastic material and the water as a Newtonian fluid. The interaction between two water and soil was modeled by the Darcy's law and pore water pressure. Wang et al. [4,5] developed the spilt flow model by introducing the influence of fraction of volume.

Appropriate boundary treatment method is a key to get high accuracy in SPH [6]. Traditional SPH researchers always arrange many particles which have mass property on the boundary. But boundary capture would be difficult and the precision would be decreased near boundary in this kind of treatment method.

In this paper, massless boundary particle method is introduced into spilt phase model, and the reliability of the new method is proved by the simulation of underwater landslide. At the same time, the process of underwater landslide is studied. The evolutions of velocity and pressure are investigated, and the impacts of hydraulic conductivity and dilation angle are discussed.

Mathematical Formulation

Spilt Phase Model. In spilt phase model, Water-soil two-phase flow is based on the mixture theory [7], in which every point in space is occupied simultaneously by one particle of each constituent. In this theory, every constituent has to satisfy individual balance laws for the conservation of mass and momentum

$$\frac{D\rho_\delta}{Dt} = -\rho_\delta \boldsymbol{\nabla} \boldsymbol{u}_\delta, \tag{1}$$

$$\rho_\delta \frac{d\tilde{\boldsymbol{u}}_\delta}{dt} = \boldsymbol{\nabla}\boldsymbol{\sigma}_\delta + \rho_\delta \boldsymbol{g} + \boldsymbol{f}_\delta, \tag{2}$$

where $\delta = s, w$ for soil and water, and ϕ the fraction of volume, and $\boldsymbol{g}$ the gravitational acceleration. $\boldsymbol{f}_\delta$ is interaction force and obeys Newton's third law i.e. $\boldsymbol{f}_s + \boldsymbol{f}_w = 1$. $\rho, \boldsymbol{u}, \boldsymbol{\sigma}$ is partial density, partial velocity, partial stress. In mixture theory, the partial and intrinsic fields can be contacted by

$$\rho_\delta = \tilde{\rho}_\delta \phi_\delta, \tag{3}$$

$$\boldsymbol{u}_\delta = \tilde{\boldsymbol{u}}_\delta, \tag{4}$$

where $\tilde{\rho}, \tilde{\boldsymbol{u}}$ is intrinsic density and intrinsic velocity. True intrinsic $\tilde{\rho}$ is the density of the particles that make up the only one constituent, not including the pore spaces in between the grains, different from the partial density, so as $\tilde{\boldsymbol{u}}$. For the stress, we assume that

$$\boldsymbol{\sigma}_s = \phi_s \tilde{\boldsymbol{\sigma}}_s, \tag{5}$$

$$\boldsymbol{\sigma}_w = -p\boldsymbol{I} + \phi_w \tilde{\boldsymbol{\tau}}_w, \tag{6}$$

where $\boldsymbol{I}$ is unit tensor, p the pore water pressure, $\tilde{\boldsymbol{\tau}}_w$ the intrinsic deviatoric stress tensor of the water. The interaction force $\boldsymbol{f}_s$ can be solved by pore water pressure and Darcy seepage force

$$\boldsymbol{f}_s = -\phi_s \boldsymbol{\nabla} p_w + \boldsymbol{f}_d, \tag{7}$$

where the pore water pressure p_w is related to density by equation of state, which regards the water as a weakly compressible fluid,

$$p_w = \frac{\rho_0 c_0^2}{\xi}\left[\left(\frac{\rho}{\rho_0}\right)^{\xi} - 1\right], \tag{8}$$

where ρ_0 is initial density of water, a constant 1000kg/m^3; $\xi = \text{constant}$ normally set to seven; c_0 artificial sound speed. The Darcy seepage force $\boldsymbol{f}_d$ is related to the velocity difference between water and soil

$$\boldsymbol{f}_d = C_d(\boldsymbol{u}_w - \boldsymbol{u}_s), \tag{9}$$

where C_d is the drag coefficient, given by Darcy's law

$$C_d = \rho_w \phi_w \boldsymbol{g}/k, \tag{10}$$

where k is hydraulic conductivity.

Constitutive Model. In the paper, the water phase is considered as a Newtonian fluid, so we can get deviatoric stress tensor

$$\tilde{\tau}_w^{\alpha\beta} = \mu \tilde{\varepsilon}_w^{\alpha\beta}, \tag{11}$$

where μ is dynamic viscosity of the water, $\tilde{\varepsilon}_w^{\alpha\beta}$ strain of water. And α, β are free indices.

The soil phase is regarded as elastic-perfectly plastic material, obeying Drucker-Prager yield criterion and non-associated flow rate. The constitutive equation can be written as

$$\dot{\sigma}_s^{\alpha\beta} - \sigma_s^{\alpha\gamma}\dot{\omega}_s^{\beta\gamma} - \sigma_s^{\gamma\beta}\dot{\omega}_s^{\alpha\gamma} = 2G\dot{e}_s^{\alpha\beta} + K\dot{\varepsilon}_s^{\gamma\gamma}\delta^{\alpha\beta} - \dot{\lambda}\left[3\alpha_\theta K\delta^{\alpha\beta} + G/\sqrt{J_2}\tau_s^{\alpha\beta}\right], \tag{12}$$

where $\dot{e}_s^{\alpha\beta}$ is deviatoric strain rate tensor of soil, G shear modulus, K bulk modulus, and γ a dummy index. In this paper, the plastic potential function g has the form

$$g = \sqrt{J_2} + 3I_1 sin\psi, \tag{13}$$

where I_1 is the first invariant of the total stress tensor $\tilde{\boldsymbol{\sigma}}_s$, and we have $p= -I_1/3$ for soil. J_2 is the second invariant of the deviatoric stress tensor $\tilde{\tau}_s^{\alpha\beta}$. ψ is dilatancy, a important parameter for two-phase flow. For the plane strain problem, α_θ is related to internal friction angle θ

$$\alpha_\theta = \frac{tan\theta}{\sqrt{9+12tan^2\theta}}, \tag{14}$$

The left of the equation (12) actually defines the Jaumann rate, with $\dot{\omega}_s^{\alpha\beta}$ torsion tensor. The rate of change of plastic multiplier $\dot{\lambda}$ is calculated by

$$\dot{\lambda} = \frac{3\alpha_\theta K\dot{\varepsilon}_s^{\gamma\gamma} + (G/\sqrt{J_2})\tau_s^{\alpha\beta}\dot{\varepsilon}_s^{\alpha\beta}}{27\alpha_\theta K sin\psi + G}. \tag{15}$$

SPH implementation. A brief introduction to SPH method is presented in this section and detail can be found in [4]. In the SPH method, the problem domain consists of particles with volume, mass, and field function information. A field function of particle have a relationship with the neighboring particles

$$f_a = \sum_{b\in P} V_b f_b w_{ab}, \tag{16}$$

so as the derivative of field function

$$\boldsymbol{\nabla} f_a = \sum_{b\in P} V_b (f_a + f_b)\cdot \boldsymbol{\nabla} w_{ab}, \tag{17}$$

where f_a and f_b is the function of particle a and particle b, such as velocity and density, V_b volumn of particle b, and P assemblage of all particle. The equation (17) is very classic in SPH. It satisfies the symmetric form of particle pairs. w_{ab} is called smoothed kernel function with symmetry and normalization. The value of smoothed kernel function decreases when the distance between the two particles increases, given by

$$w(r,h) = \begin{cases} \alpha_D\left(1-\frac{q}{2}\right)^4(2q+1), 0 \le q \le 2 \\ 0, \qquad q > 2 \end{cases}, \tag{18}$$

where $q = |\boldsymbol{r}_{ab}|/h$; $\boldsymbol{r}_{ab} = \boldsymbol{r}_a - \boldsymbol{r}_b$. $\boldsymbol{r}_a$ and $\boldsymbol{r}_b$ is position vectors of particles a and b, and h is smoothing length. α_D is a parameter to ensure the normalization of the smoothed kernel function, equal to $7/(4\pi h^2)$ in two-dimension model.

With the SPH particle approximations, the momentum equations of soil and water are

$$\frac{d^s u_a^\alpha}{dt} = \sum_{b=1}^M m_b\left(\frac{\sigma_a^{\alpha\beta}\phi_a}{\rho_a^2} + \frac{\sigma_b^{\alpha\beta}\phi_b}{\rho_b^2}\right)\frac{\partial w_{ab}}{\partial x_a^\beta} - \phi_a\sum_{j=1}^N m_j\frac{p_{aj}}{\rho_a\rho_j}\frac{\partial w_{aj}}{\partial x_a^\alpha} + \sum_{j=1}^N m_j\frac{f_{aj}^\alpha}{\rho_a\rho_j}w_{aj} + g_a^\alpha, \tag{19}$$

$$\frac{d^w u_i^\alpha}{dt} = \sum_{j=1}^{N} m_j \left(\frac{\tau_i^{\alpha\beta}\phi_i}{\rho_i^2} + \frac{\tau_j^{\alpha\beta}\phi_j}{\rho_j^2}\right)\frac{\partial w_{ij}}{\partial x_i^\beta} - \phi_i \sum_{j=1}^{N} m_j \frac{p_{ij}}{\rho_i\rho_j}\frac{\partial w_{ij}}{\partial x_i^\alpha} + \sum_{b=1}^{M} m_b \frac{f_{ib}^\alpha}{\rho_i\rho_j} w_{ib} + g_i^\alpha, \tag{20}$$

where subscripts a, b are soil phase particles, and subscripts i, j are water phase particles. M,N are total number of soil particles and water particles. The fraction of volume is solved by

$$\frac{d\phi_a}{dt} = \phi_a \sum_{b=1}^{M} \frac{m_b}{\rho_b} u_{ab}^\beta \cdot \frac{\partial W_{ab}}{\partial x_a^\beta}, \tag{21}$$

$$\frac{d\phi_i}{dt} = -\frac{d\phi_a}{dt} = -\phi_b \sum_{b=1}^{M} \frac{m_b}{\rho_b} u_{ib}^\beta \cdot \frac{\partial W_{ib}}{\partial x_i^\beta}, \tag{22}$$

Numerical techniques, such as artificial viscosity and artificial stress [4], are also added in the model to make sure a great precision. Besides, repulsive force method is applied to the boundary [8].

Massless boundary particle method

In the SPH method, the problem domain is discretized into a finite number of particles. Every particle has its own volume. The sum of volume of all particles is equal to the volume of problem domain. In the physics problem, the boundary of model is usually a wall, which is represent by a line in two-dimensional model. As is known to all, there is no concept of volume for a line. However, in traditional SPH method, the line is located between inter particles and boundary particles with mass. It is so complicated that complex boundaries are difficult to simulate, and it can not simulate fraction of volume of particles precisely near boundary. In this paper, massless boundary particle method (Fig. 1) are introduced into two-phase flow. In Fig. 1, hollow particles are boundary particles, and the black line is boundary. The solid particles are internal particles.

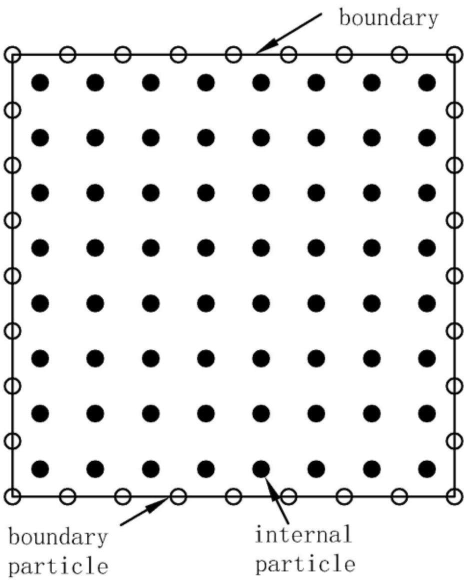


Fig.1 Particle distribution in the treatment of massless boundary particle

Simulations and Results Analysis

In this section, the SPH method and split model are validated in the simulation of underwater landslide. The rules of water-soil mixture is studied. The reliability of massless boundary particle method is proved. The model is seen in Fig.2, the rectangular tank contains a column of water-soil mixture which is delimited by a removable gate.

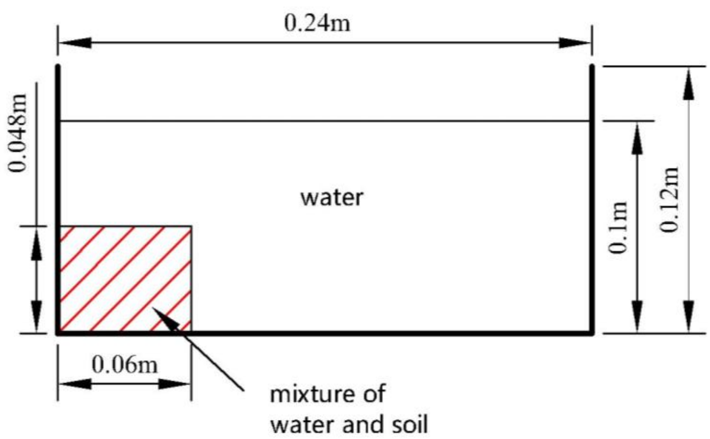
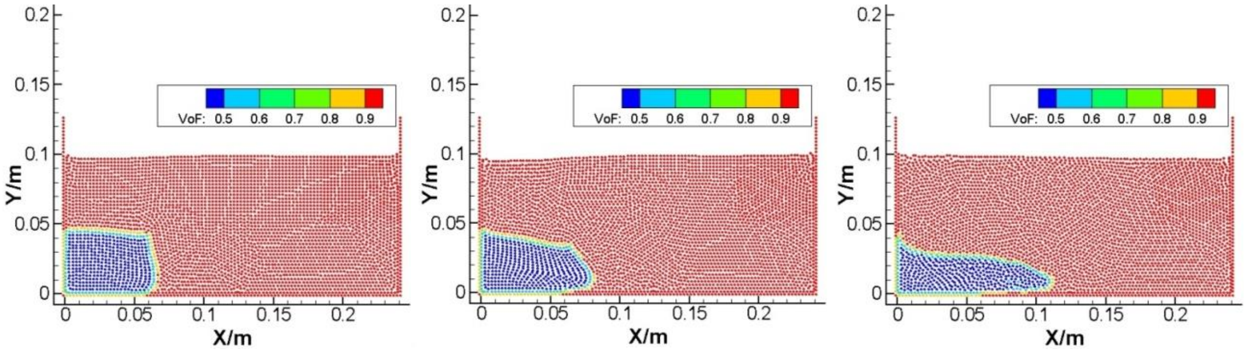


Fig.2 Underwater landslide model

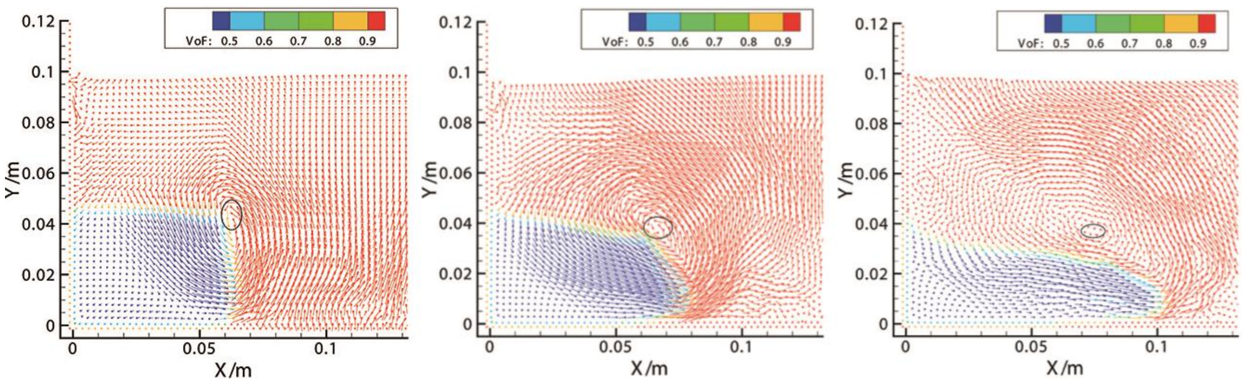
In this paper, landslide unter water is simulated and the position of soil and water mixture is captured by the water fraction of volume. In the model, there are 3840 water phase internal particles, 200 water phase boundary particles, 456 soil phase internal particles, 200 soil phase boundary particles. The artificial soundspeed of water is 10 m/s, the Young's modulus 0.86MPa, the bulk modulus 0.7MPa, the Poisson's ratio 0.3, the internal friction angle take 20^{o}. The volume fraction of soil is 0.55, which is a loose sand based on the research of Pailha et al. [9]. The dilatancy angle should be negative to reveal the compaction of soil. The dilatancy angle is -1.3^{o}, hydraulic conductivity 0.001m/s, initial spacing 0.0025m.

Profile of underwater landslide at the preliminary stage is shown in Fig.3. The distribution of water volume fraction at three representative times can be seen in the figure. When the volume fraction of a water particle is less than 1, we can assume that there are soil particles at the same position or nearly. The direct result of underwater landslide is displayed, and the wave caused by landslide can be seen.



(a)t=0.1s **(b)t=0.2s** **(c)t=0.5s**

Fig.3 Profile of underwater landslide at the preliminary stage



(a)t=0.1s **(b)t=0.2s** **(c)t=0.5s**

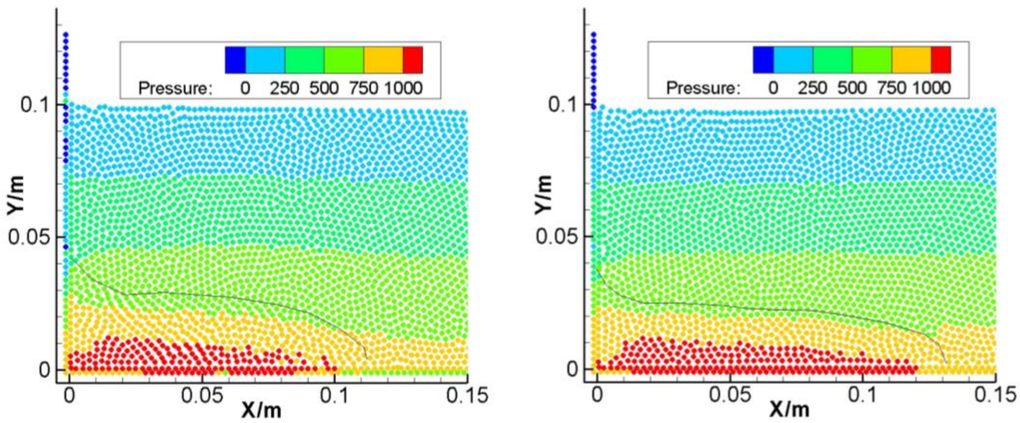
Fig.4 Local profile of velocity vector of underwater landslide

Fig.4 shows local profile of velocity vector of underwater landslide. The direction of the arrow indicates movement direction of the particle, and the length of the arrow indicates the magnitude of the velocity. As shown in the figure, the particle velocity increase at first but decrease then. The impact of landslide on the water is generally decrease from near to far. Landslide from left to right, affects the water particle anticlockwise movement. At first, a vortex appear near the cusp of the upper right mixture. As time goes on, the surface of the mixture tends to be smooth, and the cusp disappears. The center of vortex moves away, as shown in the ellipse.

Fig.5 shows the distribution of pressure of underwater landslide. According to the theory of critical state, loose soil exhibits compaction instead of dilatancy. As a result of the compaction, the water have to provide a high pressure to compress the soil. So the pressure is higher than the pressure of the surrounding fluid at the same depth.

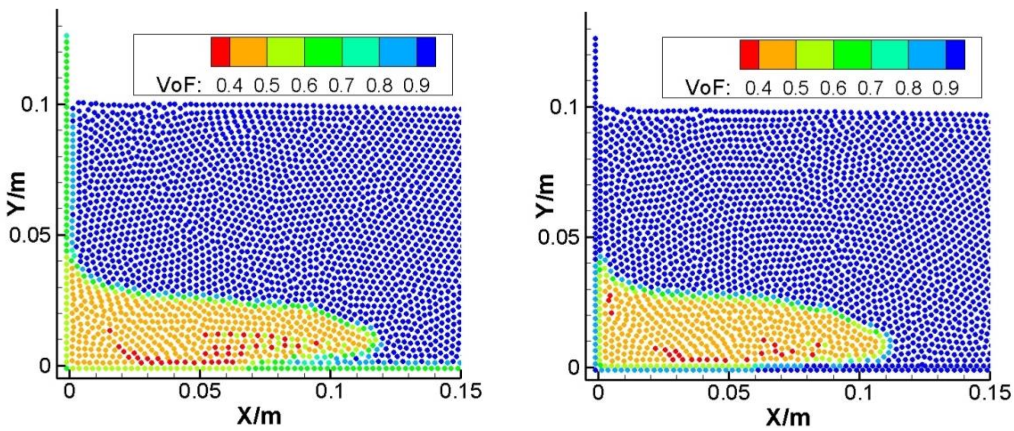
Fig.6 shows the volume fraction of particles near boundary for two treatments at 0.5s. In the traditional treatment, boundary particle with mass, the volume fraction of water phase boundary particles is not equal to 1, because both water particle and soil particle share its own volume on the boundary. In the new treatment, boundary particle without mass, the volume fraction of water phase boundary particles is equal to 1, becausse both water particle and soil particle hold no volumn on the boundary. The volume fraction of particles is a very important parameter in water-soil two-phase flow. The new treatment can simulate better and solve more accurate near boundary.

The simulation result is compared with the experimental result of Rondon et al. [10] and level set method simulating result of Savage et al. [11] in fig.7. The simulation result is closer to the experimental result than level set method simulating result, and the reliability of the method is proved.



(a)t=0.5s **(b)t=1s**

Fig.5 Distribution of pressure of underwater landslide



(a) Boundary particle with mass **(b) Boundary particle without mass**

Fig.6 Volume fraction of particles near boundary for two treatments

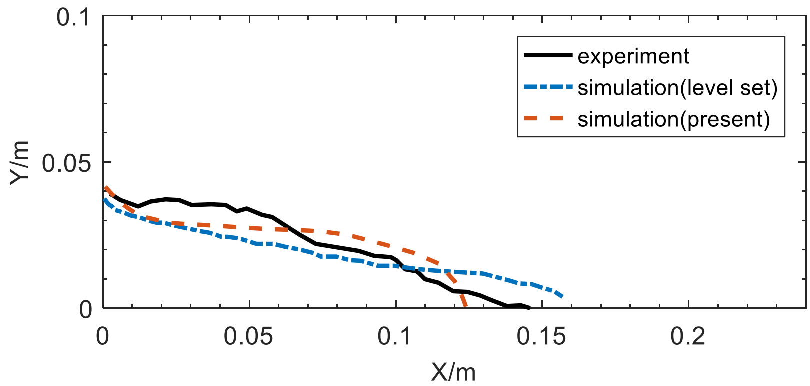
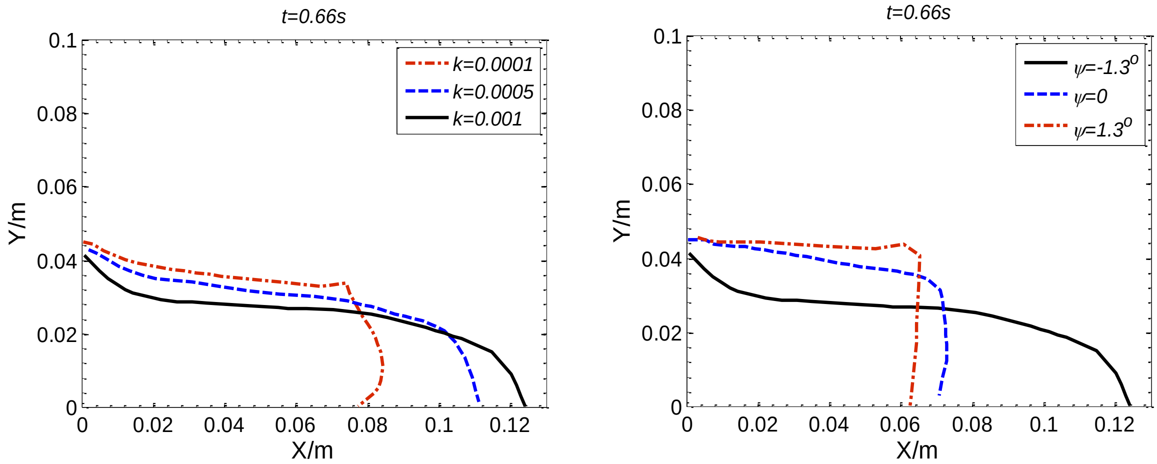


Fig.7 Comparison of simulation results



(a) Different hydraulics conductivity **(b) Different dilation angle**

Fig.8 Influence of important parameters on results

Hydraulic conductivity and dilatancy angle have an great influence on the water-soil two-phase flow simulation, as shown in fig.8. When hydraulic conductivity *k* is too small, the seepage force will be too large so that the soil just can move slowm, as shown in fig.8(a). Enlarging the value of *k*, the soil will move far away for the smaller seepage force. Fig.8(b) shows the dilatancy phenomenon and compaction phenomenon of soil. When dilatancy angle is greater than zero, the dilatancy of soil would appear and slow flow. When dilatancy angle is less than zero, the compaction of soil would appear and accelerate flow.

Summary

In this paper, underwater landslide is simulated by split model and SPH method. In the model, every phase of the mixture satisfies its own conservation equations of mass and momentum, and the interaction between water and soil are simulated by the drag force according to Darcy's law. Because the volume fraction of particles is a very important parameter in water-soil two-phase flow, massless boundary particle is introduced into the model, which make it more accurate near boundary. Underwater landslide is simulated successfully by the model, which verify the reliability of the method.

Furthermore, the evolution of velocity vector and pressure distribution in the process of underwater landslide are analyzed. The evolution law of vortex center in flow field is obtained, and the high-pressure zone phenomenon of soil water mixture is found. The reliability of the improved boundary treatment is proved by the simulation of underwater landslide. Effects of different hydraulic conductivity and dilatancy angle are compared, which too small hydraulic conductivity and positive dilatancy angle will slow the flow.

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