The Study on Allocation Model of Shared Parking Slots in Multi-parking Lots

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Abstract. To solve the parking slot allocation problem based on shared parking theory, under a comprehensive analysis of the operator's benefit and the factors affecting the user's parking behavior, a 0-1 programming model which aims to maximize system efficiency is proposed. Besides, some evaluation indexes related to operating revenue, customer satisfaction and resource utilization efficiency are put forward. By comparing the results between the proposed model and the first-come-first-serve(FCFS) model, it shows that the result of the proposed model improves the operating revenue by 13.3% and increases parking utilization rate by 8.33% as well as ensuring the customer satisfaction when parking slot cannot meet the demand. The proposed model can solve the allocation problem of multi-parking lots effectively. At last, entropy weight method is introduced to evaluate the results based on different values of penalty factor. It provides a theoretical reference for determining the optimal value of the penalty factor under different request demand.

INTRODUCTION

Traffic congestion is becoming more and more serious. The research shows, in the center of the city, about 30% of the traffic congestion is resulted by the vehicles looking for parking slots.So it is important to find a solution of urban parkingproblem. With the popularity of mobile networks, users can share their information in real-time through applications(app) on mobile phones. The operator manages the demand and supply information of the parking slots by the e-parking platform. Shared parking can be applied easily.

In recent years, many researchers have explored the feasibility and condition of shared parking in downtown areas. Shao(2016) introduced the penalty factor of rejecting users’ requests and built a 0-1 linear programming model aiming to maximize the operating benefit[1]. Although the model was proved to be efficient, it can only solve the shared parking problem in a single parking lot. Chen(2015) and Ayala(2011) considered the travel time and travel distance as the main factors in parking problem and built the model for shared parking allocation which aimed at minimizing both the total social cost and total travel distance[2,3]. Based on the analysis of shared parking problem for commuter groups, Xu(2016) proposed the allocation methods in different conditions and proved their effectiveness through numerical experiments[4]. Chen and Xie(2015) studied the shared parking strategy between university and its surrounding parking areas under different supply and demand conditions[5]. To simulate the process of parking slot allocation, an improved SEM-Logit parking behavior selection model was introduced to a dynamic bi-level programming model.

The existing researches on the shared parking slot allocation problem fail to consider the time and the space factor comprehensively. This paper improves the model proposed by Shao(2016)[1] and proposes the parking slot allocation model for multi-parking lots under different time window information. The model considers both the influence of the interests of operators and users' satisfaction. Furthermore, this paper puts forward three evaluation indexes, uses the entropy weight method to evaluate the solutions obtained under different penalty factors, and provides theoretical reference for determining reasonable value of the penalty factor.

MODEL DESCRIPTION

As for scientific and reasonable allocation of shared parking slot, a theoretic model is built as shown in Fig.1. Assuming an e-parking platform service for a particular area and let I denote the total number of parking lot in this area. Supposed the platform receive M requests and N parking slots before a certain time. The allocation of this model results by binary decision variable x_{mn}, so we get the decision variable matrix $X_{M\times N} = [x_{mn}], m = 1,2,\dots,M; n = 1,2,\dots,N$.

$$x_{mn} = \begin{cases} 1, & \text{if request } m \text{ is allocated to parking slot } n \\ 0, & \text{if request } m \text{ is not allocated to parking slot } n \end{cases} \tag{1}$$

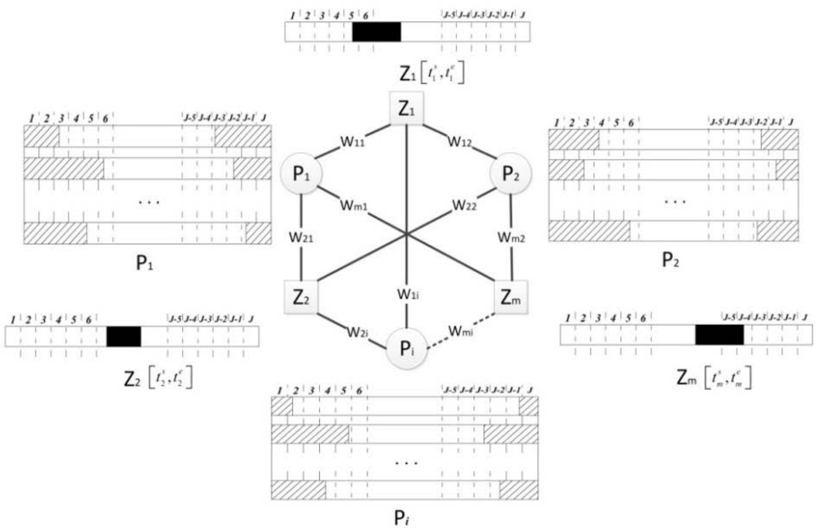


FIGURE 1. Shared parking allocation theory model

The allocation of parking slots is based on the supply and demand information. Among them, the supply information including parking available time windows $[T_n^s, T_n^e]$, the serial number of parking slot in the parking lot $P_n = i, n = 1,2,\dots,N, i = 1,2,\dots,I$.The demand information including parking slot using time windows $[t_{m,}^s t_m^e]$, destination Z_m, maximum acceptable walking distance l_m.

The key problem of shared parking slot allocation is meeting the supply and demand both sides of the time window constraints. To solve the problem, this model mainly refer to Shao's method[1]. The service time is divided into *J* intervals. We introduce a binary variable a_{nj} to describe whether the parking slot is occupied in each interval. So we can get the parking supply matrix $A_{N\times J} = [a_{nj}]$. Also, we defined a binary variable d_{mj} to describe whether request m occupy the interval *j*. So we have the parking demand matrix $D_{M\times J} = [d_{mj}]$.

$$a_{nj} = \begin{cases} 1, & \text{if parking slot } n \text{ is available in interval } j \\ 0, & \text{if parking slot } n \text{ is unavailable in interval } j \end{cases} \tag{2}$$

$$d_{mj} = \begin{cases} 1, & \text{request } m \text{ occupy the interval } j \\ 0, & \text{request } m \text{ not occupy the interval } j \end{cases} \tag{3}$$

As the parking slots are allocated to parking demand users, the state of parking slots in each interval is changing. To describe the state of parking slots, we introduce a binary variable f_{nj} which is defined to denote whether parking slot *n* is occupied in interval *j*. So we get the parking slots occupy matrix $F_{N\times J} = [f_{nj}] = X_{N\times M} * D_{M\times J}$.

$$f_{nj} = \sum_{m=1}^{M} x_{nm} * d_{mj} (n = 1,2,\dots,N;\ j = 1,2,\dots,J) \tag{4}$$

$$f_{nj} = \begin{cases} 1, & \text{if parking slot } n \text{ is occupied in interval } j \\ 0, & \text{if parking slot } n \text{ is not occupied in interval } j \end{cases} \tag{5}$$

We simplify the model by ignoring users' sensitivity to the price and locations of parking slots. Supposing that users do not have specific preference for any parking slot in each parking lot. Parking slots are purchased by operators at the price of P_b \$/h and parking slots are selling at the price of P_s \$/h.

Walking distance is a main index to evaluate the allocation result. We defined factor w_{mi} as the distance between destination Z_m and parking spot P_i. Moreover, we also introduce a binary factor c_{ni} to illustrate whether parking slot n is located in parking lot *j*. So we get the walking distance matrix $W_{M\times I}$ and parking slot position matrix$C_{N\times I} = [c_{ni}]$.

$$c_{ni} = \begin{cases} 1, & \text{if parking slot } n \text{ locates in parking lot } j \\ 0, & \text{if parking slot } n \text{ locates in parking lot } j \end{cases} \tag{6}$$

According to the relationship between matrix $X_{M\times N}$ and $C_{N\times I}$, we can get parking lot distribution matrix $Y_{M\times I} = [y_{mi}]$ by $X_{M\times N} * C_{N\times I}$ .Binary variable y_{mi} indicates whether the request m is allocated to the parking lot *i*.

$$y_{mi} = \sum_{n=1}^{N} x_{mn} * c_{ni} (m = 1,2,\dots,M;\ i = 1,2,\dots,I) \tag{7}$$

$$y_{mi} = \begin{cases} 1, \text{if request } m \text{ is allocated to parking lot } i \\ 0, \text{if request } m \text{ is not allocated to parking lot } i \end{cases} \tag{8}$$

In the shared parking slot allocation problem, operators need to take account of operation revenue and user satisfaction. To improve the satisfaction of parking users, the model considers the influence of walking distance and denial of request. On the one hand, the total walking distance is minimized to reduce the user's time cost; on the other hand, a penalty factor is introduced to reduce the negative impact of rejecting user. In addition, on the basis of ensuring customer's satisfaction, the operators achieve maximum revenue. As the description above, a 0-1 programming model with maximum system benefit is established as follows.

$$\max Z = P_s\sum_{n=1}^{N}\sum_{j=1}^{J}v_{nj} - P_b\sum_{n=1}^{N}\sum_{j=1}^{J}a_{nj} - u(M-\sum_{m=1}^{M}\sum_{n=1}^{N}x_{mn}) - Q\sum_{m=1}^{M}\sum_{i=1}^{I}w_{mi}y_{mi}/v \tag{9}$$

$$s.t.\begin{cases}\sum_{n=1}^{N}x_{mn} \le 1, (m=1,2,...,M; n=1,2,...,N); & ① \\ if\ \ y_{mi}=0\ and\ c_{ni}=1, x_{mn}=0; & ② \\ v_{nj} \le a_{nj}, (n=1,2,...,N; j=1,2,...,J); & ③ \\ x_{mn}、\ y_{mi} \in \{0,1\}; & \end{cases}$$

In the above formula :

Q—The value of unit travel time

v—Average walking speed

In this model, constraint ① means that each user can be allocated to at most one parking slot. Constraint ② means that the parking slot greater than the maximum acceptable walking distance will not allocated to user. Constraint ③ means that the parking slot allocated to user must be available.

EVALUATION INDEX SYSTEM

Operating Income Evaluation Index

To ensure the long-term operation of the system platform, improving operational income is the primary goal. Operating revenue is the total parking revenue paid by users deducting the purchase cost of the parking slot. It can reflect the operation of the system.

$$R = P_s\sum_{n=1}^{N}\sum_{j=1}^{J}v_{nj} - P_b\sum_{n=1}^{N}\sum_{j=1}^{J}a_{nj} \tag{10}$$

Service Quality Evaluation Index

1. Rejection rate

When the supply does not meet the demand, some of the request will be rejected. But rejecting requests can lower customer satisfaction and impose a negative impact on the future operation of the system. Rejection rate represents the ratio of the number of rejected requests to the total number of requests. The smaller the value, the higher the service quality.

$$\alpha = (M - \sum_{m=1}^{M}\sum_{n=1}^{N}x_{mn}) \Big/ M \tag{11}$$

2. Average walking distance

In the parking allocation problem of multiple parking lots, we should take into account the different distances from the parking lot to the destination of the user. Walking distance is directly related to the convenience of user. So we defined average walking distance to evaluate the service quality.

$$d_{avg} = \sum_{i=1}^{I}\sum_{m=1}^{M} w_{mi} \times y_{mi} \Big/ \sum_{m=1}^{M}\sum_{n=1}^{N} x_{mn} \tag{12}$$

Resource Utilization Efficiency Evaluation Index

The purpose of sharing parking slots is to improve the utilization ratio of existing parking slot resources. Parking slot utilization ratio is the ratio of the total occupied time to the total available time of the parking slots. It can directly reflect the utilization of parking slots.

$$\beta = \sum_{n=1}^{N}\sum_{j=1}^{J} v_{nj} \Big/ \sum_{n=1}^{N}\sum_{j=1}^{J} a_{nj} \tag{13}$$

NUMERICAL EXPERIMENTS

Assume that the system service area is shown in Fig.2. In the region, there are three parking lots scattered on different road. The operation company purchase the parking slot at the price of P_b=4 \$/h, then selling it to parking users at the price of P_s=10 \$/h. Set the service time of shared parking system is from 7 a.m. to 6 p.m. and the modeling time interval is 0.5 h. We supposed that operation company buy 90 parking slots from suppliers, and each parking lot has 30 parking slots. The idle interval of each parking slot starts at any one of the first 6 intervals and terminates at any one of the last 6 intervals. Supposed that users arrive according to a possion process and parking duration follows the negative exponential distribution with a mean of 3 h. Among other parameter values, user's maximum acceptable walking distance l_m is 500m. Average walking speed is 6km/h. The value of users' unit travel time is 50\$/h.

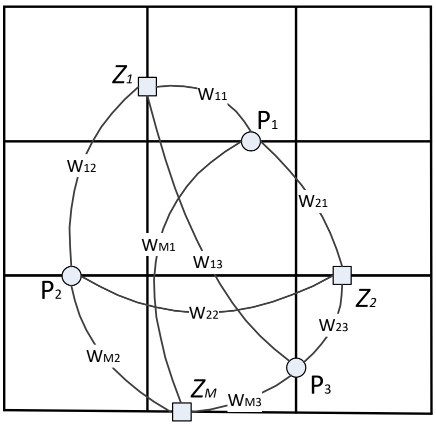


FIGURE 2. Distribution of Parking lot Locations and users' destinations

Shared parking has not formed a complete management mechanism currently. First-come-first-serve (FCFS) is the main model to simulate the allocation of parking slots. We conduct numerical experiments with different number of requests and analyze the optimization effect of the model by comparing the results between two methods.

As shown in Table 1, when *M*=100, the system rejection rate is 0. So we can know that the supply can meet the demand. The results are the same when the penalty factor is 0 and 50. It reflects that the penalty factor is not sensitive to the result when the supply is greater than the demand.

Table 1. The evaluation index value of allocation result under different number of request

Request number	method	Penalty factor	Operating revenue/$	Rejection rate/%	Average walking distance/m	Parking utilization rate/%
$M=100$	OM	0	-551	0	360.74	38.16
		50	-551	0	360.74	38.16
	FCFS	—	-565	0.4	360.82	38.01
$M=200$	OM	0	2714	5.00	345.49	75.75
		50	2674	3.50	348.19	73.33
	FCFS	—	2360	7.80	345.91	65.00
$M=300$	OM	0	4309	24.00	343.36	90.65
		50	4109	15.33	349.49	87.53
	FCFS	—	3644	22.47	344.42	78.60

(OM—the proposed optimization model in this paper)

A small number of requests will be rejected by system when M=200. It indicates that the supply parking slots are slightly larger than parking demand. Compared with the FCFS model, the result of optimization model can increase the operating revenue by more than 13.3% and increase the parking utilization rate by more than 8.33%. When μ=50, the rejection rate decreased by 4.3%, and there is no significant change in average walking distance. The optimization effect of this model is obvious.

When M=300, the parking slot supply cannot meet the demand. Rejection rate is reduced by 8.67% when penalty factor change from 0 to 50. This reflects that when the supply is less than the demand, the results of the model show a significant reduction in rejection rate.

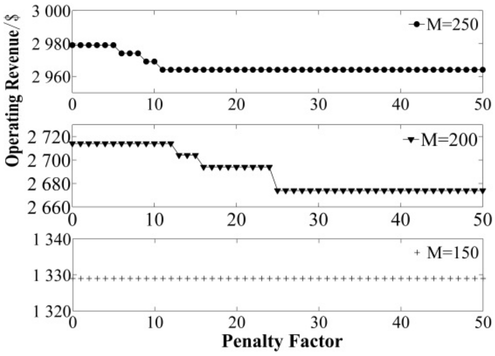
Through the analysis of the above results, we discover that the optimization effect varies with the number of requests. When the supply parking slots are greater than the demand, the result of the model is almost same as the FCFS model. As the parking demand increases, the optimization effect of the model is more obvious and the penalty factor is also more sensitive to the rejection rate.

ALLOCATION SCHEME EVALUATION BASED ON ENTROPY WEIGHT METHOD

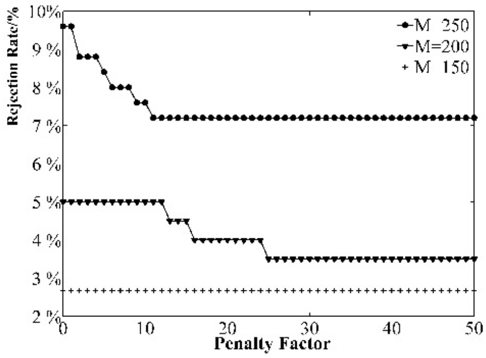
From Fig.3, we can see that, the rejection penalty factor μ has different effects on system benefit R and rejection rate α under different requests number. When M greater than 200, with the increase of penalty factor, the operating revenue and rejection rate are decreasing, so we can get different schemes. To obtain the optimal result, the entropy weight method is applied to evaluate the schemes.

The entropy weight method takes full account of the information provided by each index. It not only reflects the relative importance of each index accurately, but also avoids the subjective influence of the traditional method. Due to advantages above, this paper uses the entropy weight method to evaluate the allocation schemes obtained when M=250 so as to provide the basis for determining reasonable ranged of penalty factor. Among the existing indicators, the operating revenue and parking utilization ratio are linearly related. So we use operating revenue, rejection rate and average walking distance as evaluation indexes to eliminate the influence of the correlation between indexes.

When M=250 and $\mu \in [0,50]$, the allocation schemes obtained are shown in Table.2. The three indexes under each plan constitute the decision matrix $H_{6\times3}$. And the information distribution matrix $R_{6\times3}$ is obtained by standardizing and normalizing the matrix $H_{6\times3}$. According to the principle of entropy weight method and Equation (14)-(16), the calculation results of each index including the entropy E_i, the discrimination degree F_i and the weight are shown in Table 3.



(a)



(b)

FIGURE 3. (a) Change in operating revenue with the value of penalty factor

(b) Change in rejection rate with the value of penalty factor

Table 2. Initial decision matrix of entropy weight method

Scheme number	**Penalty factor**	**Operating revenue /$**	**Rejection rate /%**	**Average walking distance /m**
1	[0,1]	2979	9.60%	346.36
2	[2,4]	2979	8.80%	345.26
3	5	2979	8.40%	346.10
4	[6,8]	2974	8.00%	344.71
5	[9,10]	2969	7.60%	345.23
6	[11,50]	2964	7.20%	346.65

$$E_i = -\frac{1}{\ln m}\sum_{i=1}^{m} r_{ij} \ln r_{ij} \tag{14}$$

$$F_i = 1 - E_i \tag{15}$$

$$w_i = F_j \Big/ \sum_{k=1}^{n} F_k \tag{16}$$

In Equation (14), m represents the total number of index.

Table 3. Computational result of entropy weight method

Index / **Value**	**Operating revenue**	**Rejection rate**	**Average walking distance**
Entropy	0.873	0.695	0.693
Discrimination degree	0.127	0.305	0.307
Weight	0.171	0.413	0.415

Table 4. The evaluation of allocation schemes

Scheme number	Penalty factor	Operating revenue	Rejection rate	Average walking distance	Total score
1	[0,1]	0.154	0.041	0.359	0.554
2	[2,4]	0.154	0.152	0.369	0.675
3	5	0.154	0.207	0.361	0.722
4	[6,8]	0.109	0.262	0.374	0.744
5	[9,10]	0.063	0.317	0.369	0.749
6	[11,50]	0.017	0.372	0.356	0.745

Finally, the total scores of the above 6 schemes are obtained and shown in Table.4. The results indicate that scheme 5 gets the highest score and the best scheme is obtained when $\mu \in [9,10]$.

CONCLUSION

There are many studies focused on shared parking slot allocation. Operational principles and optimization methods are proposed to solve the allocation problem in single parking lot. This paper aims at solving the parking slots allocation problem in multi-parking lots, analyses the main influencing factors of parking slot allocation and builds a 0-1 programming model with the objective of maximizing the total benefits. Numerical experiments are conducted and the results show that the optimization model can both improve the operating income and time utilization ratio significantly as well as reducing the rejection rate, compared with the first-come-first-serve model. The proposed shared parking strategy can increase the utilization ratio of parking slot resource, reduce the investment of constructing a large scale of parking facilities and alleviate the parking pressure in metropolis.

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