

A Closed form Localization Method using AOA and TOA Measurements based on WSN in NLOS Environments

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Abstract—This paper proposed a closed form localization method based on wireless sensor network (WSN) in the non-line of sight (NLOS) environment. We utilized the angle of arrival (AOA) and time of arrival (TOA) measurements to determine the positions of target and reflectors according to the localization geometry. A two-step weighted least square (WLS) estimator is exhibited, where the first-order error analysis is used. In addition, a derivation of the Cram r–Rao lower bound (CRLB) is presented and the performance will be verified by the simulations.

Keywords—AOA; CRLB; NLOS; WLS; WSN; TOA

I. INTRODUCTION

Target localization based on angle of arrival (AOA) and time of arrival (TOA) in wireless sensor networks (WSN) has been of considerable interest in the field of signal processing for its broad applications in target tracking, surveillance, and navigation [1].

This kind of hybrid localization not only yields better accuracy than using TOA alone, but also overcomes the drawback in AOA localization [2]. Numerous AOA and TOA algorithms have been proposed in [3-5], such as hardware oriented algorithm [3], linear least square (LLS) solution [4] and multidimensional scaling (MDS) [5].

These methods can achieve satisfactory accuracy in light of sight (LOS) environment, but the accuracy may be largely deteriorated when the direct wave between the target and the sensor is blocked by obstacles. Therefore, another error generated from non-line of sight (NLOS) environment except measurement noises should be taken into account.

The exiting algorithms to mitigate the NLOS error have been exhibited in [7,8], which can be categorized into two techniques. One is the parametric method, which utilizes the transmission path of the indirect wave with the known reflectors. The other is the statistical approach, eliminating the influence of the NLOS error utilizing its priori knowledge which is usually unavailable. Moreover, the accuracy is lower than parametric method.

This work will present a parametric method in a localization scenario consisting of multi-reflector and a target in WSN. The nonlinear equations derived from AOA and TOA will be transformed into a set of pseudo linear equations by introducing a nuisance variable. We apply the weight least square (WLS) estimator [8] to obtain the reflector positions and the corresponding nuisance

variables. Sequentially, we use the inner relationship of the previous estimation results and WLS method is introduced again to determine the target position. The study proceeds with deriving the CRLB in this NLOS environment and finally testes the localization performance in simulations.

II. MEASUREMENT MODEL AND PROBLEM FORMULATION

Consider a two-dimensional (2D) scenario composed of M known sensors and N reflectors, whose position are denoted by $\mathbf{s}_i = [x_i, y_i]^T$ and $\mathbf{p}_j = [u_j, v_j]^T$ respectively, to determine the target located at $\mathbf{x} = [x, y]^T$. Fig. 1 shows the localization geometry. The AOA and TOA received by sensor i via reflector j in the NLOS environment are denoting by θ_{ij}^o and τ_{ij}^o , whose relationship with the target can be described as

$$\theta_{ij}^o = \arctan \frac{v_j - y_i}{u_j - x_i}, \quad (1)$$

$$r_{ij}^o = c\tau_{ij}^o = \|\mathbf{x} - \mathbf{p}_j\| + \|\mathbf{p}_j - \mathbf{s}_i\|, i=1,2,\dots,M, j=1,2,\dots,N. \quad (2)$$

where c is the signal propagation speed in meter per second.

Denoting the vectors as $\boldsymbol{\theta}_j^o = [\theta_{1j}^o, \theta_{2j}^o, \dots, \theta_{Mj}^o]^T$, $\boldsymbol{\theta}^o = [\boldsymbol{\theta}_1^{oT}, \boldsymbol{\theta}_2^{oT}, \dots, \boldsymbol{\theta}_N^{oT}]^T$, $\mathbf{r}_j^o = [r_{1j}^o, r_{2j}^o, \dots, r_{Mj}^o]^T$, $\mathbf{r}^o = [\mathbf{r}_1^{oT}, \mathbf{r}_2^{oT}, \dots, \mathbf{r}_N^{oT}]^T$ and $\mathbf{p} = [\mathbf{p}_1^T, \dots, \mathbf{p}_N^T]^T$.

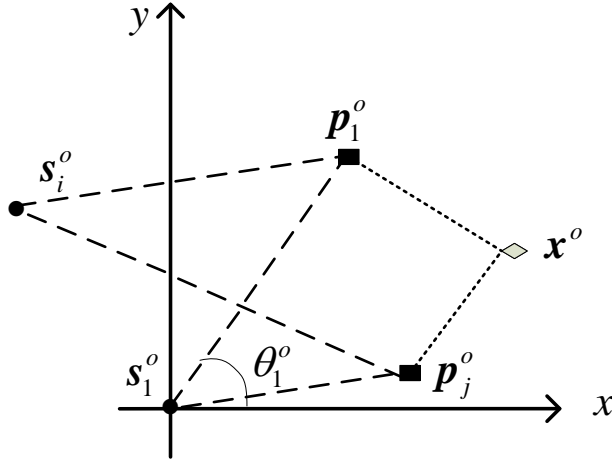


Figure 1. 2D localization scenario with M sensors and N reflectors to determine the target position.

III. PROPOSED METHOD

In this section, we will present a closed form method to obtain the reflector and target positions, where the WLS method is introduced in two steps.

In the first step, with the aid of trigonometric geometrical identities in Fig. 1, AOA equations can be formatted as the pseudo linear equations:

$$(u_j - x_i^o) \sin(\theta_{ij}^o) = (v_j - y_i^o) \cos(\theta_{ij}^o), i = 1, 2, \dots, M. \quad (3)$$

Further, rewrite (3) into vector format:

$$\omega_1^T(\theta_{ij}^o) \mathbf{p}_j = \omega_1^T(\theta_{ij}^o) \mathbf{s}_i^o, i = 1, 2, \dots, M, \quad (4)$$

$$\text{where } \omega_1(\theta_{ij}^o) = [\sin(\theta_{ij}^o), -\cos(\theta_{ij}^o)]^T.$$

Then, TOA equations can be transformed into pseudo linear equations by introducing a nuisance variable $d_j = \|\mathbf{p}_j - \mathbf{x}\|$ in coordination with the geometry relationship:

$$\|\mathbf{s}_i^o - \mathbf{p}_j\| = \omega_2^T(\theta_{ij}^o) (\mathbf{p}_j - \mathbf{s}_i^o) \quad (5)$$

$$\text{where } \omega_2(\theta_{ij}^o) = [\cos(\theta_{ij}^o), \sin(\theta_{ij}^o)]^T.$$

Substitute (5) into (2) and obtain the TOA equations:

$$r_{ij}^o = \|\mathbf{x} - \mathbf{p}_j\| + \omega_2^T(\theta_{ij}^o) (\mathbf{p}_j - \mathbf{s}_i^o), i = 2, \dots, M. \quad (6)$$

We define the vector $\boldsymbol{\mu}_j = [\mathbf{p}_j^T, d_j]^T$ and the joint of (4) and (6) gives the localization equation set based on AOA and TOA measurements as follow

$$\mathbf{A}(\boldsymbol{\theta}_j^o) \boldsymbol{\mu}_j = \mathbf{b}(\boldsymbol{\theta}_j^o, \mathbf{r}_j^o), \quad (7)$$

where

$$\mathbf{A}(\boldsymbol{\theta}_j^o) = \begin{bmatrix} \omega_1^T(\theta_{1j}^o) & 0 \\ \vdots & \vdots \\ \omega_1^T(\theta_{Mj}^o) & 0 \\ \omega_2^T(\theta_{1j}^o) & 1 \\ \vdots & \vdots \\ \omega_2^T(\theta_{Mj}^o) & 1 \end{bmatrix}, \mathbf{b}(\boldsymbol{\theta}_j^o, \mathbf{r}_j^o) = \begin{bmatrix} \omega_1^T(\theta_{1j}^o) \mathbf{s}_1^o \\ \vdots \\ \omega_1^T(\theta_{Mj}^o) \mathbf{s}_M^o \\ r_{1j}^o + \omega_2^T(\theta_{1j}^o) \mathbf{s}_1^o \\ \vdots \\ r_{Mj}^o + \omega_2^T(\theta_{Mj}^o) \mathbf{s}_M^o \end{bmatrix}. \quad (8)$$

In practice, $\boldsymbol{\theta}_j^o$ and \mathbf{r}_j^o interfered by the noises inevitably are described as $\boldsymbol{\theta}_j = \boldsymbol{\theta}_j^o + \mathbf{n}_j$ and $\mathbf{r}_j = \mathbf{r}_j^o + \mathbf{m}_j$. $\mathbf{n}_j = [n_{1j}, n_{2j}, \dots, n_{Mj}]^T$ and $\mathbf{m}_j = [m_{1j}, m_{2j}, \dots, m_{Mj}]^T$ are AOA and TOA measurement noises submitted to zero mean Gaussian distribution with covariance matrix $\mathbf{Q}_{\text{AOA},j} = \mathbf{E}[\mathbf{n}_j \mathbf{n}_j^T]$ and $\mathbf{Q}_{\text{TOA},j} = \mathbf{E}[\mathbf{m}_j \mathbf{m}_j^T]$, respectively.

When the AOA measurements is small, there exist $\sin(n_{ij}) \approx n_{ij}$ and $\cos(n_{ij}) \approx 1$, and we obtain the approximation

$$\begin{cases} \omega_1(\theta_{ij}^o) \approx \omega_1(\theta_{ij}) - n_{ij} \omega_2(\theta_{ij}) \\ \omega_2(\theta_{ij}^o) \approx \omega_2(\theta_{ij}) + n_{ij} \omega_1(\theta_{ij}) \end{cases} \quad (9)$$

Subtract (9) and $\mathbf{r}_{ij}^o = r_{ij} - m_{ij}$ into (7), and obtain the residual term

$$\boldsymbol{\varepsilon}_j = \mathbf{G}_{1,j} \mathbf{n}_j + \mathbf{D} \mathbf{m}_j \approx \mathbf{A}(\boldsymbol{\theta}_j) \boldsymbol{\mu} - \mathbf{b}(\boldsymbol{\theta}_j, \mathbf{r}_j) \quad (10)$$

where $\mathbf{O}_{M \times M}$ denotes the $N \times n$ matrix with all zero entries, $\text{diag}[\cdot]$ represents a diagonal matrix equipped with the elements of a vector and

$$\mathbf{D} = \begin{bmatrix} \mathbf{O}_{M \times M} \\ -\mathbf{I}_M \end{bmatrix}, \quad \mathbf{G}_{1,j} = \begin{bmatrix} \text{diag}(\omega_2^T(\theta_{1j})(\mathbf{p}_j - \mathbf{s}_1^o), \dots, \omega_2^T(\theta_{Mj})(\mathbf{p}_j - \mathbf{s}_M^o)) \\ -\text{diag}(\omega_1^T(\theta_{1j})(\mathbf{p}_j - \mathbf{s}_1^o), \dots, \omega_1^T(\theta_{Mj})(\mathbf{p}_j - \mathbf{s}_M^o)) \end{bmatrix} \quad (11)$$

Applying WLS optimization to (10) yields the solution and covariance matrix of $\boldsymbol{\mu}_j$

$$\mu_j = \left(A^T(\theta_j) W_{1j} A(\theta_j) \right)^{-1} A^T(\theta_j) W_{1j} b(\theta_j, r_j) \quad (12)$$

$$C_j = \text{cov}(\mu_j) = \left(A^T(\theta_j) W_{1j} A(\theta_j) \right)^{-1} \quad (13)$$

where \mathbf{p}_j can be roughly obtain by (4) and the weight matrix is defined as

$$W_{1j} = \left[\mathbf{E}(\varepsilon_j \varepsilon_j^T) \right]^{-1} = \left(\mathbf{G}_{1,j} \mathbf{Q}_{\text{AOA},j} \mathbf{G}_{1,j}^T + \mathbf{D} \mathbf{Q}_{\text{TOA},j} \mathbf{D}^T \right)^{-1} \quad (14)$$

It is obvious that \mathbf{p}_j and d_j are two independent unknowns in the first-step WLS estimator. Therefore, we gather all nuisance variables as $\mathbf{d} = [d_1, \dots, d_N]$ and its covariance matrix is a diagonal matrix which can be constructed as

$$\mathbf{C}_d = \text{cov}(\mathbf{d}) = \text{diag}(C_1(3,3), \dots, C_N(3,3)) \quad (15)$$

However, internal relation exists in the position of reflector and the nuisance variable actually, which can be exploited to estimate the target position for the further step.

The relationship of the true value d_j^o and estimate value d_j of nuisance variable is

$$d_j = d_j^o + \Delta d_j = \|\mathbf{p}_j^o - \mathbf{x}\| + \Delta d_j \quad (16)$$

where Δd_j is the estimate error of d_j . And transpose Δd_j in (16) to the left and square both sides. Give the second-order error term and we have the approximation

$$d_j^2 - 2d_j \Delta d_j = \|\mathbf{p}_j^o\|^2 - 2\mathbf{x}^T \mathbf{p}_j^o + \|\mathbf{x}\|^2 \quad (17)$$

Make a difference of (17) decided by reflector j and reflector 1 in order to realize a linear equation of \mathbf{x} as

$$(\mathbf{p}_j^o - \mathbf{p}_1^o)^T \mathbf{x} - \frac{1}{2} (\|\mathbf{p}_j^o\|^2 - \|\mathbf{p}_1^o\|^2 - d_j^2 + d_1^2) = d_j \Delta d_j - d_1 \Delta d_1. \quad (18)$$

Rewrite (18) as the vector form as

$$\mathbf{e} = \mathbf{G}_2 \mathbf{D} \Delta \mathbf{d} = \mathbf{H} \mathbf{x} - \mathbf{h}, \quad (19)$$

where

$$\mathbf{H} = \begin{bmatrix} (\mathbf{p}_2^o - \mathbf{p}_1^o)^T \\ \vdots \\ (\mathbf{p}_N^o - \mathbf{p}_1^o)^T \end{bmatrix}, \quad \mathbf{h} = \frac{1}{2} \begin{bmatrix} \|\mathbf{p}_1^o\|^2 - d_1^2 \\ \vdots \\ \|\mathbf{p}_N^o\|^2 - d_N^2 \end{bmatrix}, \quad \mathbf{D} = \text{diag}(\mathbf{d}) \quad (20)$$

We introduce the the second-step WLS method to obtain the solution

$$\mathbf{x} = (\mathbf{H}^T \mathbf{W}_2 \mathbf{H})^{-1} \mathbf{H}^T \mathbf{W}_2 \mathbf{h} \quad (21)$$

where the weight matrix is

$$\mathbf{W}_2 = (\mathbf{G}_2 \mathbf{D} \text{cov}(\mathbf{d}) \mathbf{D}^T \mathbf{G}_2^T)^{-1} = (\mathbf{G}_2 \mathbf{D} \mathbf{C}_d \mathbf{D}^T \mathbf{G}_2^T)^{-1} \quad (22)$$

IV. CRLB

Given that AOA and TOA measurement noises \mathbf{n} and \mathbf{m} are Gaussian distributed with different covariance matrix and independent of each other, the logarithm of the joint probability density function (pdf) of θ and \mathbf{r}

parameterized on $\boldsymbol{\rho} = [\mathbf{p}^T, \mathbf{x}^T]^T$ is

$$\ln f(\theta, \mathbf{r}; \boldsymbol{\rho}) = k_1 - \frac{1}{2} (\theta - \theta^o)^T \mathbf{Q}_{\text{AOA}}^{-1} (\theta - \theta^o) + k_2 - \frac{1}{2} (\mathbf{r} - \mathbf{r}^o)^T \mathbf{Q}_{\text{TOA}}^{-1} (\mathbf{r} - \mathbf{r}^o) \quad (23)$$

where $k_1 = -1/2 \ln((2\pi)^{2M} |\mathbf{Q}_{\text{AOA}}|)$ and $k_2 = -1/2 \ln((2\pi)^{2M} |\mathbf{Q}_{\text{TOA}}|)$ are constants.

The CRLB of $\boldsymbol{\rho}$ is equal to

$$\text{CRLB}(\boldsymbol{\rho}) = -\mathbf{E} \left[\frac{\partial^2 \ln f(\xi, \mathbf{s}; \boldsymbol{\rho})}{\partial \boldsymbol{\rho} \partial \boldsymbol{\rho}^T} \right]^{-1} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} \\ \mathbf{X}_{12}^T & \mathbf{X}_{22} \end{bmatrix}^{-1}, \quad (24)$$

$$\begin{cases} \mathbf{X}_{11} = \mathbf{Q}_{\theta^o, p}^T \mathbf{Q}_{\text{AOA}}^{-1} \mathbf{Q}_{\theta^o, p} + \mathbf{Q}_{\mathbf{r}^o, p}^T \mathbf{Q}_{\text{TOA}}^{-1} \mathbf{Q}_{\mathbf{r}^o, p} \\ \mathbf{X}_{12} = \mathbf{Q}_{\mathbf{r}^o, p}^T \mathbf{Q}_{\text{TOA}}^{-1} \mathbf{Q}_{\mathbf{r}^o, x} \\ \mathbf{X}_{22} = \mathbf{Q}_{\mathbf{r}^o, x}^T \mathbf{Q}_{\text{TOA}}^{-1} \mathbf{Q}_{\mathbf{r}^o, x} \end{cases} \quad (25)$$

where $\mathbf{Q}_{\text{AOA}} = \text{blkdiag}(\mathbf{Q}_{\text{AOA},1}, \dots, \mathbf{Q}_{\text{AOA},N})$, $\mathbf{Q}_{\text{TOA}} = \text{blkdiag}(\mathbf{Q}_{\text{TOA},1}, \dots, \mathbf{Q}_{\text{TOA},N})$, $\text{blkdiag}[\cdot]$ represents a block diagonal matrix consisting of matrices.

In summary, the CRLB of \mathbf{p} and \mathbf{x} are given as

$$\text{CRLB}(\mathbf{p}) = (\mathbf{X}_{11} - \mathbf{X}_{12} \mathbf{X}_{22}^{-1} \mathbf{X}_{12}^T)^{-1}, \quad (26)$$

$$\text{CRLB}(\mathbf{x}) = (\mathbf{X}_{22} - \mathbf{X}_{12}^T \mathbf{X}_{11}^{-1} \mathbf{X}_{12})^{-1} \quad (27)$$

V. SIMULATIONS

In these simulations, we consider a two-dimensional scenario where consists of 3 reflectors and 3 sensors and their locations are listed in Table 1. The true position of

target is $\mathbf{x}^o = [550, 500]^T$. We claim $\sigma_{\text{AOA}} = 10^{-4} \sigma_m (\text{rad})$ and $\sigma_{\text{TDOA}} = \sigma_m (\text{m})$, where

σ_m represents the standard deviation of measurement noises varying from 1 to 10. The estimation accuracy is evaluated in terms of $\text{RMSE}(\mathbf{x}) = \left(\sum_{k=1}^K \|\mathbf{x} - \mathbf{x}^0\|^2 / K \right)^{1/2}$ with $K=500$ Monte Carlo experiments.

The RMSEs of reflectors positions and nuisance variables in the first-step WLS are shown in Fig.2, where the estimate accuracy is consistent perfectly with the CRLB. It demonstrates the feasibility that use the estimation results of the first-step WLS to the second step WLS estimator and the localization accuracy can be guaranteed.

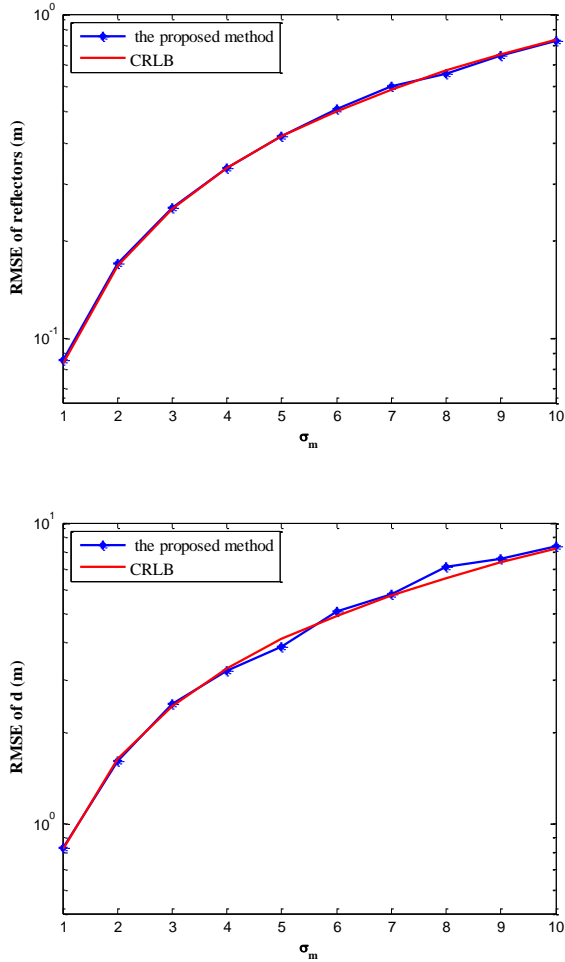


Figure 2. RMSEs of the first-step WLS estimator compared with the CRLB

Next, we examine the estimate accuracy of source and the simulation results verify that target position can be estimated successfully by the second-step WLS estimation. Fig.3 shows that the RMSE coincides well with theory value, both of which is much closed to CRLB but cannot attain it. It can be interpreted that we only consider the estimate errors of nuisance variables derived from the first-step WLS estimation and ignore the errors in estimation of reflectors positions. Hence the second-step WLS estimator is not the minimum variance unbiased (MVU) estimator.

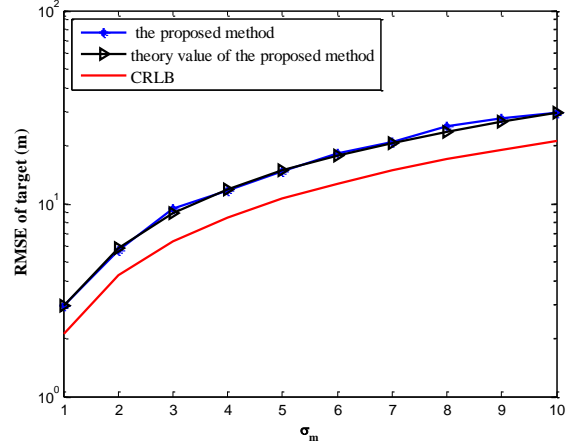


Figure 3. Localization RMSEs of the reflectors and target compared with the CRLB

TABLE I. POSITION OF THE SENSORS AND REFLECTORS (UNIT: M)

| No. | 1 | 2 | 3 |
|-----------|-----------|------------|------------|
| sensor | (0,0) | (-500,500) | (-200,400) |
| reflector | (200,300) | (300,800) | (100,600) |

VI. CONCLUSIONS

A closed form method based on AOA and TOA measurements in NOLS environment has been exhibited in this paper. We construct a set of pseudo linear equations and minimize the sum of residual errors squares. Two-step WLS estimator is employed to obtain the positons of both the reflectors and target. Simulation results illustrate that the first-step WLS estimator with respect to the reflectors positions can achieve the CRLB even though in high noises. And the second-step WLS estimator could determine the target and its localization performance gradually achieves CRLB.

ACKNOWLEDGEMENT

The author acknowledges support from National Natural Science Foundation of China (Grant No. 61201381), China Postdoctoral Science Foundation of (Grant No. 2016M592989), the Self-Topic Foundation of Information Engineering University (Grant No. 2016600701), and the Outstanding Youth Foundation of Information Engineering University (Grant No. 2016603201).

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