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Integral Sliding Mode Output Feedback Control for Discrete-Time Systems with Uncertainty

Tao Liu* and Hengliang Tang
School of Information, Beijing Wuzi University, Beijing, China
*Corresponding author

***Abstract*—A new design method of integral sliding mode control based on fast output feedback is presented for discrete-time systems with uncertainty. A new control scheme is characterized by fast output feedback and new integral sliding mode manifold. It will be shown in this work the stability and dynamic performance could be ensured. An example and its simulation results are presented to illustrate the proposed approach.**

Keywords-discrete-time system; integral sliding mode; fast output feedback; uncertainty

I. INTRODUCTION

With the development of computer technology more attention has been paid to discrete-time system model. Discrete-time system has some unique characteristics and properties [1]-[10].

There are two directions in the study of discrete sliding-mode control. The first direction is to discretize the continuous time systems sliding mode control. This makes the existence of switching discrete-time sliding mode control which result in chattering. The other direction is based on the equivalent control and the disturbance estimator which is based on the equivalent control and the disturbance estimator. In the case of unknown disturbance, a boundary layer order is also generated and the equivalent control needs a large amount of control [11].

As in [12], the sliding mode control strategy is established under the condition of known state. But for most control system, the system state is not measurable. In this case, it is necessary to add a state observer or compensator in the system. This tends to increase the complexity of the system and bring difficulties to the implementation of the control strategy.

In this paper, the integral sliding mode surface is discretized and improved. A new discrete-time integral sliding surface design method is presented. Discrete-time integral sliding mode output feedback control using fast output feedback technique. The poles of the system can be configured arbitrarily. The control strategy is more practical.

II. SYSTEM DESCRIPTION

Consider the following uncertain continuous time system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + f(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

Where is $x \in R^n$ $u \in R^r$ $y \in R^m$ $A \in R^{n\times n}$ $B \in R^{n\times r}$ $C \in R^{m\times n}$ $f \in R^{n\times 1}$. $f(t)$ is bounded smooth. $f_0 \le f(t) \le f_1$.

The system is completely controllable and observable. The discrete-time system corresponding to (1) with sampling period is

$$\begin{aligned} x(k+1) &= \Phi_\tau x(k) + \Gamma_\tau u(k) + d(k) \\ y(k) &= Cx(k) \end{aligned} \tag{2}$$

Where is

$$\Phi_\tau = e^{A\tau} \quad \Gamma_\tau = \int_0^\tau e^{At} dtB \quad d(k) = \int_0^\tau e^{At} f((k+1)\tau - t)dt$$

We assume that system satisfies

(A1). The output sampling period is Δ. $\Delta = \tau / N$. N is greater than or equal to the observation index of the system.

(A2). The control input $u(k)$ and disturbance of the system $d(k)$ are constant in each period.

System (1) is completely controllable so system (2) is also completely controllable.

State equation in system (2) can be expressed as

$$x(k+1) = \Phi_\tau x(k) + \Gamma_\tau u(k) + d(k) \tag{3}$$

As in [13], we can get

$$x(k) = L_y y_k + L_u u(k-1) + L_d d(k-1) \tag{4}$$

Where is

$$L_y = \Phi_\tau (C_0{}^T C_0)^{-1} C_0{}^T \quad L_u = \Gamma_\tau - L_y D_0 \; L_d = I - L_y C_d$$

$$y_k = \begin{bmatrix} y(k\tau-\tau) \\ y(k\tau-\tau+\Delta) \\ \vdots \\ y(k\tau-\Delta) \end{bmatrix} \qquad C_0 = \begin{bmatrix} 0 \\ C\Phi_\Delta \\ \vdots \\ C\Phi_\Delta^{N-1} \end{bmatrix}$$

$$D_0 = \begin{bmatrix} 0 \\ C\Gamma_\Delta \\ \vdots \\ C\sum_{i=0}^{N-2}\Phi_\Delta^i\Gamma_\Delta \end{bmatrix} \qquad C_d = \begin{bmatrix} 0 \\ C \\ \vdots \\ C\sum_{i=0}^{N-2}\Phi_\Delta^i \end{bmatrix}(\sum_{i=0}^{N-1}\Phi_\Delta^i)^{-1}$$

$$\Phi_\tau = \Phi_\Delta^N \quad \Gamma_\tau = \sum_{i=0}^{N-1}\Phi_\Delta^i\Gamma_\Delta \quad d(k) = \sum_{i=0}^{N-1}\Phi_\Delta^i d'(k\tau)$$

$$\Phi_\Delta = e^{A\Delta} \quad \Gamma_\Delta = \int_0^\Delta e^{At}dtB \quad d'(k) = \int_0^\Delta e^{At} f((k+1)\Delta-t)dt$$

From (4) we can see that the state of the moment $k\tau$ can be represented by the output of the moment $k\tau$ to $k\tau-\Delta$ the input of the time $k\tau-\tau$ and the interference of the time $k\tau-\tau$.

III. Integral Sliding Mode Output Feedback Control

A. Integral Sliding Surface Design

Based on the integral sliding mode surface presented as in [12], we present an improved integral sliding mode surface

$$\sigma(k) = Dx(k) - Dx(0) + \varepsilon'(k) = 0 \tag{5}$$

Where is

$$\varepsilon'(k) = \varepsilon(k) - EL_d\sum_{i=0}^{k-1} d(i-1) \tag{6}$$

$$\varepsilon(k) = \varepsilon(k-1) + Ex(k-1) = E\sum_{i=0}^{k-1} x(i) \tag{7}$$

$$d(i) = 0 \;\; i<0 \;\; \sigma \in R^r \;\; \varepsilon' \in R^r$$

$D \in R^{r\times n}$ $E \in R^{r\times n}$ is constant matrix to be designed with $rankD = r$ $rankE = r$.

In (5) $Dx(0)$ is to eliminate the system arrival process so that the state of the system is located on the sliding surface at the beginning. Combine (6) and (7), we can get

$$\varepsilon'(k) = E\sum_{i=0}^{k-1}(L_y y_i + L_u u(i-1))$$

B. Control Strategy

For system (2) we always find $K \in R^{r\times n}$ so that the eigenvalues of $\Phi_\tau - \Gamma_\tau K$ is in the unit circle.

Assume that system satisfies $D\Gamma_\tau$ is reversible and define

$$E = -D(\Phi_\tau - I - \Gamma_\tau K) \tag{8}$$

$$W = -(D\Gamma_\tau)^{-1}(D\Phi_\tau + E) \tag{9}$$

$$d_l \le d(k) \le d_u \quad d_0 = \frac{d_l + d_u}{2} \quad d_1 = \frac{d_u - d_l}{2} \tag{10}$$

$$l(k) = WL_d d(k) \quad l_l \le l(k) \le l_u \quad l_0 = \frac{l_l + l_u}{2} \quad l_1 = \frac{l_u - l_l}{2} \tag{11}$$

$$m(k) = (D\Gamma_\tau)^{-1}Dd(k) \tag{12}$$

$$m_l \le m(k) \le m_u \quad m_0 = \frac{m_l + m_u}{2} \quad m_1 = \frac{m_u - m_l}{2}$$

Theorem1. Exit the control law

$$u(k) = WL_y y_k + WL_u u(k-1) - (D\Gamma_\tau)^{-1}\varepsilon'(k) + (D\Gamma_\tau)^{-1}Dx(0) + l_0 - m_0 \tag{13}$$

Bring closed-loop dynamic of system (2) is

$$x(k+1) = (\Phi_\tau - \Gamma_\tau K)x(k) + \zeta(k) \tag{14}$$

Where is

$$\zeta(k) = d(k) - \Gamma_\tau(D\Gamma_\tau)^{-1}Dd(k-1) - \Gamma_\tau l(k-1) + \Gamma_\tau l(k-2)$$

Proof. Both sides of (4) is multiplied by W

$$Wx(k) = WL_y y_k + WL_u u(k-1) + WL_d d(k-1)$$

$$WL_y y_k + WL_u u(k-1) = Wx(k) - l(k-1) \tag{15}$$

The control law (13) changes to

$$u(k)=Wx(k)-l(k-1)-(D\Gamma_\tau)^{-1}\varepsilon'(k)+ (D\Gamma_\tau)^{-1}Dx(0)+l_0-m_0 \tag{16}$$

From (6) and (7) we get

$$\begin{aligned}\varepsilon'(k+1)&=\varepsilon(k+1)-EL_d\sum_{i=0}^{k}d(i-1)\\ &=\varepsilon(k)+Ex(k)-EL_d\sum_{i=0}^{k}d(i-1)\\ &=\varepsilon'(k)+Ex(k)\end{aligned} \tag{17}$$

Combine (3) (5) (8)-(12) (16) (17), we can draw

$$\begin{aligned}\sigma(k+1)&=Dx(k+1)-Dx(0)+\varepsilon'(k+1)\\ &=D(\Phi_\tau x(k)+\Gamma_\tau u(k)+d(k))-Dx(0)+\varepsilon'(k)+Ex(k)\\ &=(D\Phi_\tau+E)x(k)+D\Gamma_\tau u(k)+Dd(k)+\varepsilon'(k)-Dx(0)\\ &=(D\Phi_\tau+E)x(k)+\\ &\quad D\Gamma_\tau(Wx(k)-l(k-1)-(D\Gamma_\tau)^{-1}\varepsilon'(k)+\\ &\quad (D\Gamma_\tau)^{-1}Dx(0)+l_0-m_0)+Dd(k)+\varepsilon'(k)-Dx(0)\\ &=(D\Phi_\tau+E+D\Gamma_\tau W)x(k)-D\Gamma_\tau l(k-1)+\\ &\quad D\Gamma_\tau(l_0-m_0)+Dd(k)\\ &=D(d(k)-d_0)-D\Gamma_\tau(l(k-1)-l_0)\end{aligned} \tag{18}$$

$$\begin{aligned}\varepsilon'(k)&=\sigma(k)-Dx(k)+Dx(0)\\ &=D\big(d(k-1)-d_0\big)-D\Gamma_\tau\big(l(k-2)-l_0\big)-\\ &\quad Dx(k)+Dx(0)\end{aligned} \tag{19}$$

Control law (16) is substituted into dynamic equation combining with (8) (12) (19). Closed loop equation can be obtained

$$\begin{aligned}x(k+1)&=(\Phi_\tau+\Gamma_\tau W)x(k)-\Gamma_\tau(D\Gamma_\tau)^{-1}\varepsilon'(k)+\\ &\quad \Gamma_\tau(D\Gamma_\tau)^{-1}Dx(0)+\Gamma_\tau(l_0-m_0)+d(k)-\\ &\quad \Gamma_\tau l(k-1)\\ &=(\Phi_\tau-\Gamma_\tau(D\Gamma_\tau)^{-1}(D\Phi_\tau+E-D))x(k)+\\ &\quad \Gamma_\tau l(k-2)-\Gamma_\tau(D\Gamma_\tau)^{-1}Dd(k-1)+\\ &\quad \Gamma_\tau(D\Gamma_\tau)^{-1}Dd_0-\Gamma_\tau m_0+d(k)-\Gamma_\tau l(k-1)\\ &=(\Phi_\tau-\Gamma_\tau K)x(k)+\zeta(k)\end{aligned} \tag{20}$$

Where is

$$\zeta(k)=d(k)-\Gamma_\tau(D\Gamma_\tau)^{-1}Dd(k-1)-\Gamma_\tau l(k-1)+\Gamma_\tau l(k-2)$$

Remark1. Select D as long as $D\Gamma_\tau$ is reversible. And we can get E and W according to (8) and (9).

Remark2. The closed-loop system (14) is stable because the eigenvalues of $\Phi_\tau-\Gamma_\tau K$ is in the unit circle.

C. *Robustness Analysis*

According to (18)

$$\begin{aligned}\|\sigma(k+1)\|&=\|D(d(k)-d_0)-D\Gamma_\tau(l(k-1)-l_0)\|\\ &\le\|D(d(k)-d_0)\|+\|D\Gamma_\tau(l(k-1)-l_0)\|\\ &=\sigma_d+\sigma_l\end{aligned}$$

Where is

$$\sigma_d=D(d(k)-d_0)\qquad \sigma_l=D\Gamma_\tau(l(k)-l_0)$$

The state of the system in the sliding mode region has the maximum deviation $\sigma_d+\sigma_l$ i.e. the integral sliding mode output feedback control strategy is robust.

IV. NUMERICAL SIMULATION

Consider the following uncertain discrete-time system

$$\begin{aligned}x(k+1)&=\Phi_\tau x(k)+\Gamma_\tau u(k)+d(k)\\ y(k)&=Cx(k)\end{aligned}$$

Where is

$$\Phi_\tau=\begin{bmatrix}0&-1\\1&-1\end{bmatrix}\quad \Gamma_\tau=\begin{bmatrix}2\\1\end{bmatrix}\quad d(k)=\begin{bmatrix}\frac{1}{50}\sin\frac{k}{10}\\ \frac{1}{100}\cos\frac{k}{10}\end{bmatrix}\quad x(0)=\begin{bmatrix}5\\5\end{bmatrix}$$

System input sampling period $\tau=0.1\sec$ $N=4$ $\Delta=\tau/N=0.025\sec$.

Select $D=[0 \quad 1]$ that $D\Gamma_{\tau}$ is reversible. $K=[0 \quad -0.3]$ $\rho(\Phi_{\tau}-\Gamma_{\tau}K)<1$.

By calculating available

$$C_0=\begin{bmatrix}1 & 0\\ 0 & 1\\ -1 & 1\\ -1 & 0\end{bmatrix} \quad C_d=\begin{bmatrix}0 & 0\\ 0.3333 & -0.6667\\ 1.000 & -1\\ 1.33 & -0.6667\end{bmatrix} \quad \mathrm{D}_0=\begin{bmatrix}0\\0\\1\\2\end{bmatrix}$$

$$E=\begin{bmatrix}-1\\1.7\end{bmatrix}^T \quad W=\begin{bmatrix}0\\-0.7\end{bmatrix}^T \quad L_y=\begin{bmatrix}-0.2 & -0.6 & -0.4 & 0.2\\ 0.2 & -0.4 & -0.6 & -0.2\end{bmatrix}$$

$$L_u=\begin{bmatrix}2\\2\end{bmatrix} \quad l_0=0 \quad l_1=0.0104 \quad m_0=0 \quad m_1=0.0067$$

Control law is

$$u(k)=WL_y y_k+WL_u u(k-1)-(D\Gamma_{\tau})^{-1}\varepsilon'(k)+(D\Gamma_{\tau})^{-1}Dx(0)+$$
$$l_0-m_0$$
$$=\begin{bmatrix}-0.14 & 0.28 & 0.42 & 0.14\end{bmatrix}y_k-1.4u(k-1)-\varepsilon'(k)+5$$

Where is

$$\varepsilon'(k)=E\sum_{i=0}^{k-1}(L_y y_i+L_u u(i-1))$$

Figure1- Figure 3 shows the dynamics of the system state, the control input and the integral sliding mode.

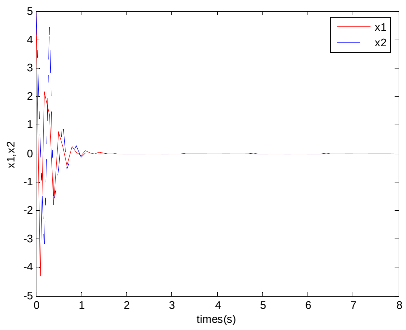

FIGURE I. SYSTEM STATE

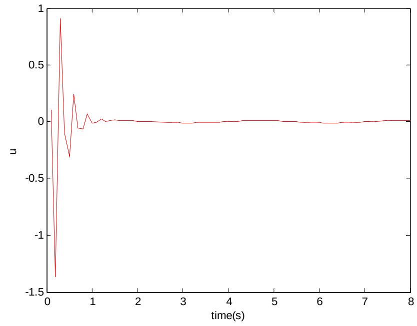

FIGURE II. CONTROL INPUT

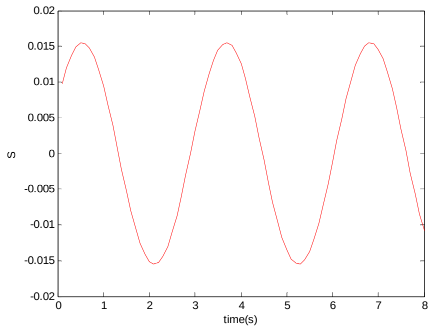

FIGURE III. INTEGRAL SLIDING MODE FUNCTION

The simulation results show that the integral sliding mode output feedback control can make the state of the system converge quickly to the origin. The state of the system remains in a bounded region of the integral sliding mode surface. The system does not produce excessive control input.

V. CONCLUSION

In this paper a discrete-time sliding mode output feedback controller is proposed which can keep the state of the system in a bounded region of the integral sliding mode surface. This method can be used for any pole assignment of the closed-loop system which makes the system can realize the dynamic performance of the system. The controller design and system stability are proved and analyzed in detail. The simulation results show the effectiveness of the control algorithm.

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