Quasi-Sliding Mode Control Based on Reduced Order Mode

Tao Liu* and Hengliang Tang

School of Information, Beijing Wuzi University, Beijing, China

*Corresponding author

Abstract—A new design method of quasi-sliding mode control is presented for discrete-time system. It is show that the quasi-sliding mode controller for high order discrete-time system could be changed into the same problem for low discrete-time system utilizing reaching law and reduced model. The numeration complexity for controller design is cut down in large range. At the same time, the stability and dynamic performance could be ensured by new control decision. An example and its simulation results are presented to illustrate the proposed approach.

Keywords—discrete-time system; quasi-sliding mode; reduced model; reaching law

I. INTRODUCTION

With the rapid development of computer technology, most of the actual control system design is based on discrete-time system. When the sliding-mode control method is used for the discrete-time system, the motion trajectory of the system has quasi-sliding mode and the chattering phenomenon occurs [1]-[3].

There are some achievements in the study of quasi-sliding-mode control for discrete-time systems [4]-[11]. It is generally based on the original high order system. With the increase of the order of the system, the calculation of the system is greatly increased.

We use model reduction method and combine with reaching law method that is proposed to solve the above problems. In this paper, we could see that the quasi-sliding mode control design of high order system is transformed into the design of quasi-sliding mode control strategy for reduced order model. The computational cost of the high order discrete control system is greatly decreased, and satisfactory results are obtained.

II. SYSTEM DESCRIPTION

Consider the following discrete-time system

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

Where is

$$x \in R^n \quad u \in R^m$$

A, B is with appropriate dimensions. (A, B) is controllable.

By nonsingular transformation

$$\hat{x}(k) = Px(k)$$

Discrete-time system (1) can transform into Jordan system

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(k) \quad (2)$$

Where is

$$\hat{x}_1 \in R^r \quad \hat{x}_2 \in R^{n-r} \quad m < r < n$$

Λ_1 and Λ_2 is Jordan block with corresponding dimension. B_1 and B_2 is matrix with corresponding dimension.

We assume

(A1). The Jordan block in the system (2) is stable, that is $\rho(\Lambda_2) < 1$.

III. DISCRETE QUASI-SLIDING MODE CONTROL

A. Design of Sliding Mode Surface

Consider reduced order system

$$z(k+1) = \Lambda_1 z(k) + B_1 u(k) \quad (3)$$

Where is

$$z(k) = C_a \hat{x}(k) = \bar{C}_a x(k) \quad C_a = [I_r \quad 0] \quad \bar{C}_a = C_a P^{-1}$$

As in [8], we can design stable sliding surface for reduced order system (3)

$$s_z(k) = C^T z(k) = 0 \quad C^T \in R^{m \times r} \quad (4)$$

Theorem1. If the ideal quasi-sliding mode of reduced order system (3) on sliding surface

$$s_z(k) = C^T z(k) = 0$$

is asymptotically stable then the ideal quasi-sliding mode of high order system (1) on sliding surface

$$s(k) = C^T \bar{C}_a x(k) = 0$$

is asymptotically stable.

Proof. Ideal quasi-sliding mode of reduced order system (3) satisfies

$$s_z(k+1) - s_z(k) = C^T \Lambda_1 z(k) + C^T B_1(k) - C^T z(k) = 0 \quad (5)$$

Combine system (3) sliding surface (4) and (5), we get equivalent control

$$u(k) = -(C^T B_1)^{-1} C^T \Lambda_1 z(k) \quad (6)$$

Substitute (6) into system (3) the ideal quasi-sliding mode equation is obtained

$$z(k+1) = \left[\Lambda_1 - B_1 (C^T B_1)^{-1} C^T \Lambda_1 \right] z(k) \quad (7)$$

So if the ideal quasi-sliding mode of reduced order system (3) on sliding surface

$$s_z(k) = C^T z(k) = 0$$

is asymptotically stable then

$$\rho(\Lambda_1 - B_1 (C^T B_1)^{-1} C^T \Lambda_1) < 1 \quad (8)$$

The corresponding dynamic for system (2) with control law (6)

$$\begin{aligned} \hat{x}(k+1) &= \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \hat{x}(k) - \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} (C^T B_1)^{-1} C^T \Lambda_1 z(k) \\ &= \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \hat{x}(k) - \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} (C^T B_1)^{-1} C^T [\Lambda_1 \quad 0] \hat{x}(k) \\ &= \begin{bmatrix} \Lambda_1 - B_1 (C^T B_1)^{-1} C^T \Lambda_1 & 0 \\ -B_2 (C^T B_1)^{-1} C^T \Lambda_1 & \Lambda_2 \end{bmatrix} \hat{x}(k) \end{aligned} \quad (9)$$

According to (8) and (A1) the corresponding dynamic for system (2) is asymptotically stable.

So high order system (1) is asymptotically stable with control (6), and do ideal quasi-sliding mode motion on sliding surface

$$s(k) = C^T \bar{C}_a x(k) = 0$$

B. Design of Quasi-Sliding Mode Control

The reduced order model of the system (3) use the reaching law as in [8]

$$s_z(k+1) - s_z(k) = -qTs_z(k) - \varepsilon T \operatorname{sgn}(s_z(k)) \quad (10)$$

Combine (3) with (4) the quasi-sliding mode control based on reaching law (10) can be obtained

$$u(k) = \beta_1 z(k) + \beta_2 \operatorname{sgn}(C^T z(k)) \quad (11)$$

Where is

$$\beta_1 = -(C^T B_1)^{-1} C^T (\Lambda_1 - I + qT) \quad \beta_2 = -(C^T B_1)^{-1} \varepsilon T \quad (12)$$

By

$$z(k) = \bar{C}_a x(k)$$

(11) can be transformed into

$$u(k) = \beta_1 \bar{C}_a x(k) + \beta_2 \operatorname{sgn}(C^T \bar{C}_a x(k)) \quad (13)$$

Theorem2. The control law (13) derived from the low order system is used for high order system (1).It can ensure that the original high order system (1) reaches the sliding surface

$$s(k) = C^T \bar{C}_a x(k) = 0 \quad (14)$$

in finite time and do quasi-sliding mode motion along the sliding surface.

Proof. According to system (1), expression (12), control input (13) and sliding mode function (14), we can get

$$P = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 3 & 2 \\ 7 & 9 & 6 \end{bmatrix}$$

$$\begin{aligned} s(k+1) &= C^T \bar{C}_a x(k+1) \\ &= C^T \bar{C}_a A x(k) + C^T \bar{C}_a B u(k) \\ &= C^T \bar{C}_a A x(k) + C^T \bar{C}_a B (\beta_1 \bar{C}_a x(k) + \\ &\quad \beta_2 \operatorname{sgn}(C^T \bar{C}_a x(k))) \\ &= C^T \bar{C}_a A x(k) + C^T \bar{C}_a B \beta_1 \bar{C}_a x(k) + \\ &\quad C^T \bar{C}_a \beta_2 \operatorname{sgn}(C^T \bar{C}_a x(k)) \\ &= (C^T \bar{C}_a A + C^T \bar{C}_a B \beta_1 \bar{C}_a) x(k) + \\ &\quad C^T \bar{C}_a \beta_2 \operatorname{sgn}(C^T \bar{C}_a x(k)) \\ &= (C^T \bar{C}_a P^{-1} A - C^T \bar{C}_a P^{-1} B (C^T B_1)^{-1} C^T (\Lambda_1 - I + \\ &\quad qT) \bar{C}_a) x(k) - C^T \bar{C}_a (C^T B_1)^{-1} \varepsilon T \operatorname{sgn}(C^T \bar{C}_a x(k)) \\ &= (C^T \Lambda_1 - C^T (\Lambda_1 - I + qT) \bar{C}_a) x(k) - \\ &\quad \varepsilon T \operatorname{sgn}(C^T \bar{C}_a x(k)) \\ &= (1 - qT) C^T \bar{C}_a x(k) - \varepsilon T \operatorname{sgn}(C^T \bar{C}_a x(k)) \\ &= (1 - qT) s(k) - \varepsilon T \operatorname{sgn} s(k) \end{aligned} \quad (15)$$

Combining the above results (15) we can see high order system (1) has the reaching law similar with reduced order system (3). Further, high order system (1) can reach the sliding surface (14) in finite time and do quasi-sliding mode motion along the sliding surface as in [8].

IV. NUMERICAL SIMULATION

Consider the following three order discrete-time system (high order system)

$$x(k+1) = \begin{bmatrix} 4.2072 & -2.7579 & 0.2014 \\ 6.4598 & -4.5658 & 0.4120 \\ 19.3694 & -16.6863 & 2.2339 \end{bmatrix} x(k) + \begin{bmatrix} -0.3431 \\ -0.5546 \\ -1.6633 \end{bmatrix} u(k) \quad (16)$$

Where is

$$x(0) = [0.5 \quad 0.5 \quad 0.5]$$

By nonsingular transformation for system (16)

$$x(k) = \hat{P} \hat{x}(k)$$

Where is

System (16) become

$$\hat{x}(k+1) = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 0.9753 & 0 \\ 0 & 0 & -0.1 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} -0.0001 \\ -0.1314 \\ -0.08 \end{bmatrix} u(k) \quad (17)$$

The reduced order system of system (17) is

$$z(k+1) = \begin{bmatrix} 1 & 0.001 \\ 0 & 0.9753 \end{bmatrix} z(k) + \begin{bmatrix} -0.0001 \\ -0.1314 \end{bmatrix} u(k) \quad (18)$$

The switching surface of the reduced order model system (18) is obtained

$$s_z(k) = C^T z(k) = [5 \quad 1] z(k)$$

In reaching law (10)

$$T = 1ms \quad q = 20 \quad \varepsilon = 0.3$$

The control law of the reduced order model system (18) based on (11) is

$$u(k) = [0.5686 \quad -0.0356] z(k) + 0.0228 \operatorname{sgn}([5 \quad 1] z(k)) \quad (19)$$

Accordingly the control law of high order system (16) can be obtained

$$u(k) = [-0.0712 \quad 0.4195 \quad -0.1280] x(k) + 0.0228 \operatorname{sgn}([2.0000 \quad 0.8000 \quad -0.6000] x(k))$$

Figure 1- Figure 3 shows the system, the control input and the state and sliding mode function of the high order system.

FIGURE I. SYSTEM STATE

FIGURE II. CONTROL INPUT

FIGURE III. SLIDING MODE FUNCTION

V. CONCLUSION

We propose a new design method of quasi-sliding mode control is presented for discrete-time system. The high order discrete-time system is transformed into Jordan standard system by nonsingular transformation. According to the Jordan standard system to obtain the corresponding reduced order system. The control law that is on the basis of reaching law design method for reduced system is applied directly to the high order system. The stability and dynamic performance of

the system are analyzed. The simulation results show that the discrete-time system control based on reduced order model can make the state of the system converge quickly to the origin.

ACKNOWLEDGMENT

This paper is supported by Funding Project for Beijing Key Laboratory of Intelligent Logistics System and Beijing Intelligent Logistics System Collaborative Innovation Center, Beijing Outstanding Talent Fund Key Individual Project (No.2014000020124G091), Beijing Wuzi University High-level Project Cultivate Fund (No. GJB20143005), Beijing Social Science Fund Youth Project (No.16GLC064), General Program of Science and Technology Development Project of Beijing Municipal Education Commission of China (No. KM201610037001, No. KM201710037001), Open Project of State Key Laboratory of Virtual Reality Technology and System of China (No.BUAA-VR-16KF-21).

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