Advances in Computer Science Research, volume 62
2nd Joint International Information Technology, Mechanical and Electronic Engineering Conference (JIMEC 2017)

Study On Inventory Model Considering Fairness Concerns Under Permissible Delay In Payments

Wang Dao-ping, Zhang Bo-qing

Donlinks School of Economics and Management, University of Science and Technology Beijing, Beijing, 100083

Keywords: fairness concerns; Stackelberg games; the delay of payment

Abstract

Optimal retailer's pricing and replenishment policies and supplier's delay-in-payments policy considering fairness concerns under permissible delay in payments is studied in this paper. A model of Stackelberg game between the fairness neutral supplier and the fairness concerned retailer is developed. A numerical example is given to verify the validity of the proposed model and conduct sensitivity analysis on parameters. The results show that, with the increase of the retailer's fairness concerns, the retailer should lower the price, increase the order quantity, and the supplier's optimal delay-in-payments should be shortened; when the fairness concerns coefficient satisfies certain conditions, the utility of the retailer and supplier is higher than without fairness concerns.

1 Introduction

As an important means of promotion, deferred payment is widely used in economic activities. In general merchandise trade, the buyer will pay for the goods immediately after they receive it when they don't have capital constraint. While some enterprises that don't have enough cash flow have strong willing to be provided a deferred payment period by sellers during which the buyer can earn some money to afford the debt. On the other hand, sellers often take measures such as deferred payment strategy to stimulate order quantity. Therefore, deferred payment has been widely recognized used. But it should be noted that this promotion strategy also bring credit risk to sellers which is more likely to incur fairness concern of sellers. In real life, enterprises do not always behave to maximize their profit. Sometimes, people express strongly concern of fairness to their cooperators. When enterprises pay attention to fairness, channel members in supply chain are easy to achieve a win-win situation and the interests of the members can be long-term guarantee. The fairness concern will lead members to behave to maximize their utility rather than profit. So, Therefore, it is of great theoretical value and practical significance to study the influence of the fair concern on the optimal strategy of the decision maker under the condition of delay payment.

At present, many domestic and foreign scholars have studied the inventory model considering deferred payment. Goyal introduced delay payment mechanism into economic order quantity model firstly and then a lot of scholars have made a lot of extensions to the model[1].Hwang proposed the method of determining the optimal extension payment term in the case of the supplier's point of view, considering the price and order quantity of the product are retailer's decision variable[2]. Huang studied the minimum inventory quantity and optimal order quantity of deteriorating products under the condition of fuzzy demand[3]. Based on delay payment, Liang-Yuh and Kai-Wayne take quantity discount contract and allowance of inventory shortage into account and solve the optimal order quantity and shortage period[4]. Ghosh established EOQ model of perishable products considering delay payment[5]. Differing from literature [4] and [5], literature [6] proposed an inventory model which include 3 situations divided according to relationship among shortage period, order period and period of delay payments. Aiming at a supply chain include one manufacturer and multiple distributors Yang shu established a Stackelberg game model to make optimal delay payment decision for manufacture and optimal order quantity decisions for distributors[7].

Literatures above assume that enterprises in supply chain make decisions to maximize their profit. But in reality, many enterprise pay attention to fairness. Cui introduced fairness concern to research of channel coordination first and found that wholesale price contract can coordinate supply chain when members all concern fairness and demand function is linear[8]. Considering a two-stage supply chain , Du studied the influence of retailer's fairness concern on effectiveness of wholesale price contract, revenue sharing contract and buy back contract respectively[9]. Liu studied the influence of retailer's fairness concern and manufacture's fairness concern on price and inventory decision respectively[10]. Under the research background of VMI, Zhao and Lv Reveals the boundary conditions of whether fairness concern influence performance of overall supply chain or not[11].

In conclude, many scholars have introduced the theory of fairness concern to the channels cooperation, supply chain coordination, supplier inventory management, and has achieved fruitful research results. But seldom literature consider the strategy of delay payment. So this paper introduces fairness concern and delay payment into EOQ model simultaneously. Thus, establishes a kind of Stackelberg game based on a two-stage supply chain to solve supplier's optimal decision of delay payment and retailer's optimal price and order decision.

2 Symbolic Description And Model Hypothesis

Under the research background of a two-stage supply chain that contains a fairness neutral supplier and a fairness concern retailer, the strategy of delay payment is implemented by supplier to stimulate retailer's order quantity. And supplier loss interest during the delay period. In the course of the game, supplier the dominant role and decide the delay period while retailer make decision on price and order quantity to maximize it's utility which can be calculated by equation(1).

$$U(\pi_r)=\pi_r+\lambda(\pi_s-k\pi_r) \tag{1}$$

Among equation(1), π_r and π_s represent retailer and supplier's profit respectively. λ means fairness concern ratio and satisfy the inequation $0<\lambda<1$. k is positive real number and means relative contribution rate. λ and k satisfy the inequation $(1-\lambda k)>0$. Other symbols and corresponding definitions used in the model are as follows:

D: demand rate. That is product demand for retailers in unit time,

A: fixed cost got order,

c: order cost per unit,

c_0: cost of production per unit,

h: *hc* is retailer's inventory holding cost per unit for unit time,

p: retail price,

T: retailer's ordering cycle,

B: shortage quantity at the end of ordering cycle,

r: retailer's back order cost per unit for unit time,

s: order loss cost per unit,

η: delay satisfaction rate of shortage quantity,

Q: retailer's order quantity,

M: deferred payment period given by supplier to retailer,

I_e: annual interest rate of income,

I_k: annual interest rate of payment,

$I(t)$: retailer's inventory stock at time t, where $0\le t\le T$.

Hypothesis of this paper are as follows:

(1) the delivery of the goods is instantaneous.

(2)Before the deadline of deferred payment, retailer saves it's revenue to bank for interest with rate revenue rate I_e. After deferred payment period, retailer need to pay interest for inventory hold cost with payment rate I_k.

(3) the retailer has to pay the loan in t=M.

3 Model building

The demand rate that retailer faces is $D=a-bp$. a represent potential demand rate in the market. b is demand price elasticity. So, retailer's inventory satisfy equation $I(t)=(FT-t)D$, $0\le t\le FT$.

FT satisfy equation $I(FT)=0$. If products are all sold before the next replenishment, situation of shortage is happened, and a part of consumer accept to wait for replenishment. The proportion of consumer in those who face shortage is η. Figure 1 gives a brief exhibition of inventory quantity at every time. So order quantity of every cycle Q is $[DFT+\eta D(1-F)T]$.

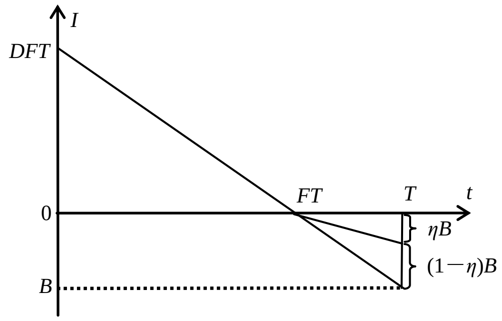

Figure 1： Inventory quantity at every time

3.1 Supplier's profit per time

Supplier's income is revenue come from products wholesaled to retailer. It's cost is lost of revenue interest caused by delay payment strategy. Supplier's profit per time can be calculated by equation (2).

$$\pi_s=(c-c_0-cI_eM)\big(F+\eta(1-F)\big)D \tag{2}$$

3.2 Retailer's utility per time

According to equation (1), we need to calculate retailer's and supplier's profit to obtain retailer's utility. Thus we need to analyze retailer's cost. The component of it's cost include order cost A/T, wholesale cost cQ/T, Inventory holding cost $hcDF^2T/2$, shortage cost SC and interest cost IC which equals interest payment for inventory holding cost subtract interest revenue for sale income. Shortage cost SC consists defer delivery cost and order lost cost. SC can be calculated by equation (3).

$$\begin{aligned}SC&=\frac{1}{T}\left(\frac{r\eta D(1-F)^2T^2}{2}+sDT(1-\eta)(1-F)\right)\\&=\frac{r\eta DT(1-F)^2}{2}+sD(1-\eta)(1-F)\end{aligned} \tag{3}$$

When it comes to IC, we need to discussion on two cases. One case is the situation when deferred payment period satisfy inequation $M\le FT$, while the other case is the situation when M satisfy inequation $M>FT$. IC can be calculated by equation (4).

$$IC=\begin{cases}-\dfrac{pI(t)}{T}\displaystyle\int_0^M Dt\,dt+\dfrac{cI_k}{T}\displaystyle\int_M^{FT}I(t)\,dt & ,M\le FT\\ -\dfrac{pI_e}{T}\left(\displaystyle\int_0^{FT}Dt\,dt+DFT(M-FT)\right) & ,M<FT\end{cases} \tag{4}$$

So, retailer's profit per time π_r is:

$$\begin{aligned}\pi_r=(p-c)D-\frac{A}{T}-\frac{hcDF^2T}{2}-\frac{r\eta D(1-F)^2T}{2}\\-(s+p-c)(1-\eta)(1-F)\,D+IC\end{aligned} \tag{5}$$

Then, we substitution π_r and π_s in equation (1) through equation (3) and (5) to obtain retailer's utility per time $U(\pi_r)$.

3.3 Stackelberg game model

According to retailer's and supplier's decision objectives, the Stackelberg game model can be describe as following:

$$\max\ \pi_s = (c - c_0 - cI_e M)\left(F + \eta(1-F)\right)\left(a - bp\right)$$

$$s.t.\ \{p, T\} = \arg\max U(\pi_r)$$

4 Model Solution

4.1 Retailer's decision

Retailer's decisions which include price p and ordering cycle T on two cases consisting of situation when $M \le FT$ and $M > FT$ hold respectively.

(1) $M \le FT$

According to equation (1), (2) and (5), first-order partial derivatives $\partial U(\pi_r)/\partial p$ and $\partial \Delta U(\pi_r)/\partial T$ are as following:

$$\frac{\partial U(\pi_r)}{\partial p} = (1-k\lambda)\left(\gamma_{11}(D - bp) + \gamma_{12}\right) + \lambda\gamma_{13} \quad (6)$$

$$\frac{\partial U(\pi_r)}{\partial T} = (1-k\lambda)\left(\begin{matrix} \dfrac{A}{T^2} - \dfrac{hcDF^2 + r\eta D(1-F)^2}{2} \\ -\dfrac{pI_e DM^2 + cI_k D(F^2T^2 - M^2)}{2T^2} \end{matrix}\right) \quad (7)$$

γ_{11}, γ_{12} and γ_{13} are as following:

$$\gamma_{11} = 1 - (1-\eta)(1-F) + \frac{I_e M^2}{2T}$$

$$\gamma_{12} = b\left(\begin{matrix} c + (s-c)(1-\eta)(1-F) \\ + \dfrac{hcF^2T^2 + r\eta(1-F)^2T^2 + cI_k(FT-M)^2}{2T} \end{matrix}\right)$$

$$\gamma_{13} = -b(c - c_0 - cI_e M)\left(F + \eta(1-F)\right)$$

Let $\partial U(\pi_r)/\partial p$ and $\partial \Delta U(\pi_r)/\partial T$ equal 0, and we can obtain optimal price p_1^* and ordering cycle T_1^* when $M \le FT$ holds:

$$p_1^* = \frac{(1-k\lambda)(a\gamma_{11} + \gamma_{12}) + \lambda\gamma_{13}}{2b\gamma_{11}(1+k\lambda)} \quad (8)$$

$$T_1^* = \sqrt{\frac{2A + (cI_k - pI_e)(a - bp_1^*)M^2}{\left(hcF^2 + r\eta(1-F)^2 + cI_k F^2\right)(a - bp_1^*)}} \quad (9)$$

If $FT_1^* < M$ or equation (8) and (9) are unsolvable, the optimal ordering cycle is M/F. Substitute T with M/F in equation (8) and the optimal price is get.

(2) $M > FT$

Similar to situation when $M \le FT$ is satisfied, optimal price p_2^* and ordering cycle T_2^* when $M > FT$ holds are as following:

$$p_2^* = \frac{(1-k\lambda)(a\gamma_{21} + \gamma_{22}) + \lambda\gamma_{23}}{2b\gamma_{21}(1+k\lambda)} \quad (10)$$

$$T_2^*(p) = \sqrt{\frac{2A}{\left((hc + pI_e)F^2 + r\eta(1-F)^2\right)(a - bp)}} \quad (11)$$

γ_{11}, γ_{12} and γ_{13} are as following:

$$\gamma_{21} = 1 - (1-\eta)(1-F) + FI_e(M - \frac{1}{2}FT)$$

$$\gamma_{22} = b\left(c + (s-c)(1-\eta)(1-F) + \frac{hcF^2T + r\eta(1-F)^2T}{2}\right)$$

$$\gamma_{23} = -b(c - c_0 - cI_e M)\left(F + \eta(1-F)\right)$$

If $FT_1^* \ge M$ or equation (10) and (11) are unsolvable, the optimal ordering cycle is M/F. Substitute T with M/F in equation (9) and the optimal price is get.

4.2 Supplier's decision

Supplier's profit per time can be get by substituting p in equation (2) with p_1^* and p_2^*:

$$\pi_s = (c - c_0 - cI_e M) \times \left(F + \eta(1-F)\right) \times \left(\frac{(1-k\lambda)(a\gamma_{i1} - \gamma_{i2}) + \lambda\gamma_{i3}}{2\gamma_{i1}(1-k\lambda)}\right) \quad (12)$$

i=1 and i=2 represent the situations when $M \le FT$ and $M > FT$ holds respectively. Supplier make decision on delay payment period M, we construct it's Lagrange function as equation (13), considering M has constrain $M \le FT$ or $M > FT$.

$$\max L(M, \mu_{i1}, \mu_{i2}) = (c - c_0 - cI_e M) \times \left(F + \eta(1-F)\right) \times \left(\frac{(1-k\lambda)(a\gamma_{i1} - \gamma_{i2}) + \lambda\gamma_{i3}}{2\gamma_{i1}(1-k\lambda)}\right) + \mu_{i1}g_{i1}(M) + \mu_{i2}g_{i2}(M)$$

(13)

In equation (13), g_{i1} ,g_{i2},μ_{i1} and μ_{i2} satisfy equation or inequation as following:

$g_{11} = M$

$g_{12} = FT - M$

$g_{21} = M - FT$

$g_{22} = T - M$

$\mu_{i1} \ge 0$

$\mu_{i2} \ge 0$

Then, Karush-Kuhn-Tucker conditions for Lagrange function (13) are as following:

$$\begin{cases} \dfrac{\partial L(M, \mu_{i1}, \mu_{i2})}{\partial M} = 0 \\ \mu_{i1}g_{i1}(M) = 0 \\ \mu_{i2}g_{i2}(M) = 0 \\ \mu_{i1}, \mu_{i2} \ge 0, \\ i = 1, 2 \end{cases} \quad (14)$$

Optimal delay payment period M^* can be obtained by solving equation set (14). Substitute M with M^* in equation (8), (9), (10) ,(11) and we can get retailer's optimal decisions. Analytic solutions for equation set (14) are hard to achieve, so we solve equation set (14) by software matlab in numerical example and make sensitive analysis based on it.

5 Numerical Example

A numerical example is given to analyze the effect of λ and k on optimal decisions. Values of main parameters in the model are given in table 1.

parameter	value	parameter	value
a	100000	r	100
b	40	s	200
A	20000	η	40%
c_0	1600	I_e	0.05
c	2000	I_k	0.10
h	20%	λ	0.25
F	90%	k	2

Table 1: The values of parameters in model.

Consulting literature [12], this paper set k to 2 which means retailer treats the situation as fairness when it's profit is half of supplier's profit. The effect of λ on optimal decisions is shown in table 2. (supplier's utility equals to it's profit) It can be found that:

(1) The retailer's profit is higher than $\pi_s/2$ when it doesn't have fairness concern. Under this circumstances, Fairness concern will incur retailer's utility less than it's profit. As the extent of fairness concern increasing, retailer's utility presents a decreasing trend. Under this trend, retailer act to reduce price and increase ordering quantity. Because, these decisions increase supplier's profit and shrink the difference between supplier's profit and retailer's profit which will improve retailer's utility. Supplier act to shrink delay payment period under the trend of increasing λ which will reduce the loss of interest. Together retailer's and supplier's actions, it is found that supplier's profit shows a increasing trend. Retailer's utility increases as well despite the sacrifice of it's own profit.

(2) While the overall utility of the supply chain is increased, retailer's utility is not always greater than that when λ equals to 0. This shows that when λ is relatively small the effect of increasing supplier's profit can sufficient the effect caused by the decrease of retailer's profit on retailer's utility.

λ	M^*	p^*	T^*	Q^*	π_r	U_r	π_s	$U_s + U_r$
0	0.091	2263	0.101	891	1917889	1917889	3485610	5403499
0.05	0.089	2252	0.098	914	1869063	1864602	3648908	5513510
0.10	0.086	2238	0.096	946	1847326	1863174	3853127	5716301
0.15	0.084	2220	0.093	979	1797780	1875822	4115842	5991664
0.20	0.080	2197	0.089	1014	1699282	1912842	4466363	6379205
0.25	0.076	2164	0.085	1074	1490062	1984404	4957493	6941897
0.30	0.072	2114	0.080	1161	1018946	2116059	5694935	7810994

Table 2: The effect of λ on optimal decisions

k	M^*	p^*	T^*	Q^*	π_r	U_r	π_s	$U_s + U_r$
1.0	0.080	2197	0.089	1162	1699282	1966232	4466363	6432595
1.4	0.079	2187	0.088	1163	1644570	1976643	4617433	6594076
1.8	0.077	2173	0.086	1164	1555119	1983436	4823505	6806941
2.2	0.075	2153	0.084	1165	1402478	1981563	5121296	7102859
2.6	0.072	2122	0.080	1166	1106349	1950559	5589537	7540096
3.0	0.068	2065	0.075	1167	376029	1796297	6433130	8229427

Table 3: The effect of k on optimal decisions

The effect of k on optimal decisions is shown in table 3. As k increasing, retailer's utility increases firstly and then decreases. With the increase of k, retailer's profit is gradually reduced, and supplier's profit is gradually increased. When the profit margin of the supplier increases, the utility of the retailer also increases. when the k is large enough, the supplier's profit margin is not enough to balance the decrease of the retailer's profit. Retailers should take actions at this time to increase it's relative contribution in the channel which means increase it's profit, such as reducing the marginal cost, increasing the demand for products and so on.

5 Conclusion

This paper takes fairness concern and delay payment into consideration and establishes a kind of stacklberg game model between supplier and retailer which can solve supplier's optimal delay payment period and retailer's optimal price and ordering quantity. These are found that:

(1) As the extent of fairness concern increasing, retailer act to reduce price and increase ordering quantity, while supplier act to shrink delay payment period.

(2) As relative contribution rate increasing, retailer's utility increases firstly and then decreases. When supplier's profit is very low, retailer could even sacrifice of it's own profit to improve it's utility.

(3) when the extent of fairness concerns are satisfied with certain conditions, retailer's utility and supplier's profit are greater than these under situation of no fairness concerns.

Future research will extend the model of this paper to competition situations under the background of multiple suppliers or retailers.

References

[1] S. K. Goyal. Economic Order Quantity under conditions of permissible delay in payments[J]. Journal of the Operational Research Society, 1985, 36(4): 35-38.

[2] J.S. Kim, H. Hwang, S.W. Shinn. An optimal credit policy to increase supplier's profits with price dependent demand functions[J]. Planning and Control, 1995, 6(1): 45-50.

[3] Shien-Ping Huang. Retailer's EOQ model in fuzzy annual demand inventory with limited storage space under partially permissible delay in payments[J]. Journal of Statistics and Management Systems, 2010, 13(3): 651-673.

[4] Liang-Yuh Ouyang, Kai-Wayne Chuang. Economic order quantity with allowable shortage and quantity discounts under permissible delay in payments[J]. Journal of Information and Optimization Sciences, 2004, 25(1): 53-61.

[5] S.K. Ghosh, K.S. Chaudhuri. An EOQ model with a quadratic demand, time-proportional deterioration and shortages in all cycles[J]. International Journal of Systems Science, 2006, 37(10): 663-672.

[6] S. Khanra, Buddhadev Mandal, Biswajit Sarkar. An inventory model with time dependent demand and shortages under trade credit policy[J]. Economic Modeling, 2013, 35: 349-355.

[7] Yang S, Liang L, Dong J F, An inventory model for single manufacture and multiple retailers under delay in payments[J]. System Engineering, 2007, 25(4): 9-14.

[8] Tony Haitao Cui, Jagmohan S. Raju, Z. John Zhang. Fairness and channel coordination[J]. Management Science, 2007, 53(8): 1302-1314.

[9] Du S F, Du C, Liang L, Liu T Z, Supply chain coordination considering fairness concerns[J]. Journal of Management Sciences in China, 2010, 13(11): 41-48.

[10] Liu S R, Huang Y N, Chen L L. Inventory and pricing decisions in supply chain under fairness concerns[J]. Chinese Journal of Management Science, 2013, 21: 474-479.

[11] Zhao D Z, Lv X. Vender managed inventory model considering supplier's fairness preference in supply chain with dominant retailer[J]. Operation Research and Management Science, 2013, 22(3): 45-52

[12] Zhao D Z, Lv X. Vender managed inventory model considering supplier's fairness preference in supply chain with dominant retailer[J]. Operation Research and Management Science, 2013, 22(3): 45-52.