

# A New Design Procedure For Slot Arrays Fed By Single-Ridge Waveguide

**Yang Bing, Xu Haitao, Gao Zhiguo, Chen Zhen**

*Institute No.25 of the Second Academy China Aerospace Science & Industry Corp. Beijing, 100854*

**Key words:** decoupling, single-ridge waveguide, VSWR

## Abstract

Design procedure for longitudinal slot arrays fed by air-filled rectangular waveguide including mutual coupling has been discussed and studied in many papers, and widely applied. Due to complexity of internal field, it is difficult to employ the same method on slot arrays fed by ridge waveguide. In this paper, a new design procedure is introduced for slot arrays fed by single-ridge waveguide, which simplified the procedure of the previous one. A  $1 \times 6$  array was constructed and tested, and the result validates the theory.

## 1 Introduction

Slot arrays fed by waveguide have been widely applied in radar and communication system, owing to a great number of merits, as high power capacity, low loss, simple feeding, easy manufacture, precise control of aperture distribution, and etc. Slots can be cut in the broad or narrow wall of the waveguide [2]. In this paper, longitudinal slots cut in broad wall are discussed. In 1948, Stevenson founded the basis theory of slotted waveguide in paper [1], and Oliner [2] has derived approximate expression of the admittance concerned to length and offset for the slot. Yee [3] has extended Oliner's variational solution, proposed the slot resonant length, accounting for the offset of the slot from the center of the waveguide. The design procedure for slot arrays fed by rectangular waveguide has been deeply and numerously studied including mutual coupling, since 70's by Elliott [6-8], which has been widely employed in waveguide slot array design field nowadays. Dolph-Chebyshev or Taylor polynomials [9] are usually applied to synthesize current distribution of slot array, for the sake of high gain and low

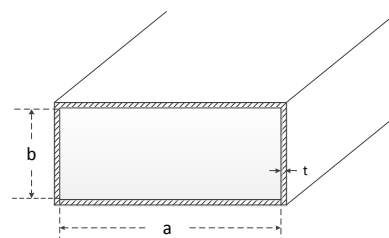
side-lobe.

However, the bandwidth of slotted array fed by rectangular waveguide is an obvious defect [10]. An effective method to broaden the bandwidth is to separate an array into several sub-arrays and feed them with a power divider [11]. Another option is to choose ridge waveguide instead of rectangular waveguide [12][13]. One design procedure for slot arrays fed by single-ridge waveguide [14], deduced from ones by the rectangular waveguides, in which eigenvalue of ridge waveguide is required. A new procedure is presented in this paper, which avoids complex computation.

## 2 New Design Procedure for slot arrays fed by rectangular waveguide

In [6][8], Elliott has demonstrated the design procedure of slot array fed by rectangular waveguide, which includes self-admittance simulation of single waveguide slot [1][2], reciprocity theorem, and mutual coupling between two slots, which has been approximated as two dipoles in space [7].

### 2.1 Traditional design procedure for slot array fed by rectangular waveguide



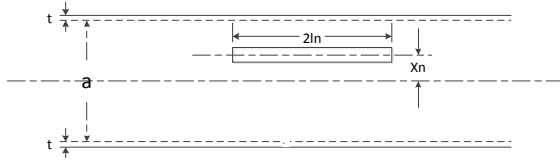


Figure 1. Rectangular waveguide (Up), longitudinal slot in the broad wall of a rectangular waveguide (Down)

Elliott deduced 2 equations (1) and (2) to calculate the lengths and offsets of each slot once the desired radiation pattern and input admittance were determined.

$$\frac{Y_n^a}{G_0} = K_1 f_n \text{sinc} l_n \frac{V_n^s}{V_n} \quad (1)$$

$$\frac{Y_n^a}{G_0} = \frac{K_2 f_n^2}{Z_n^a} \quad (2)$$

In which

$$K_1 = -j \left[ \frac{8}{\pi^2 \eta G_0} \left( \frac{a}{b} \right)^{\frac{1}{2}} \right] \quad (3)$$

$$f_n = \frac{\cos \beta l_n - \cos k l_n}{\text{sinc} l_n} \sin \frac{\pi x_n}{a} \quad (4)$$

$$K_2 = \frac{292(a/b)}{0.61\pi(\beta/k)} \quad (5)$$

Here,  $G_0$  is characteristic admittance of the waveguide,  $\eta$  is impedance of free space and  $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$ ,  $k$  and  $\beta$  are free-space wave number and propagation constant respectively.  $V_n^s$  is the mode voltage across the center of the slot, and  $V_n$  is the equivalent voltage as figure 2.

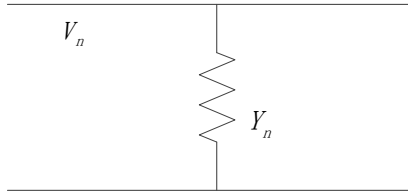


Figure 2. Definition of equivalent voltage

$Z_n^a$  is active impedance of  $n$ th slot, and

$$Z_n^a = Z_{nn} + Z_n^b \quad (6)$$

In which

$$Z_n^b = \sum_{m=1}^N \frac{V_m^s \sin k l_m}{V_n^s \sin k l_n} Z_{mn} \quad (7)$$

$Z_{nn}$  is self-impedance of  $n$ th slot.  $Z_{mn}$  is mutual impedance between the  $m$ th and  $n$ th slots, which is assimilated as half wavelength dipole and discussed in [7].  $V_m^s$  and  $V_n^s$  are mode voltages across shunt elements, respectively.

The design procedure could be summarized as: first of all, current distribution can be synthesized according to desired radiation pattern and side-lobe, thus initial admittances of each slot  $Y_{nn}$  with its offsets and lengths of each slot are acquired. Afterwards, equation (1) and (2) should be employed to compute active admittance  $Y_n^a$  of each slot. Once active admittance of the  $n$ th slot  $Y_n^a$  is achieved,  $Y_{nn}$  should be

adjusted to compensate the effect brought by mutual coupling, and offsets and lengths of each slot should be adjusted subsequently. Iterates the design procedure several times, final offsets and lengths of each slot of slot array are achieved.

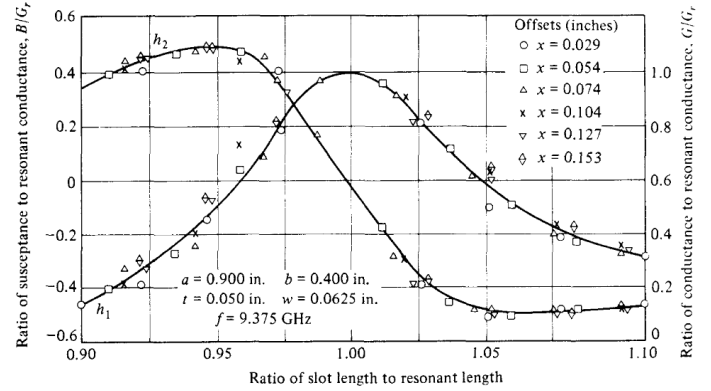


Figure 3. Normalized Self-Admittance Components for a Longitudinal Shunt Slot (by Stegen[4])

Usually after adjusted, self-admittances of slots won't be resonant anymore, due to effort of mutual coupling. Instead, active admittances are purely real and accord with the desired current distribution.

## 2.2 New method with assistance of electromagnetic calculating software

There are 2 equations (1) and (2) proposed to design slot array fed by rectangular waveguide by Elliott. But usually only (2) is used, (1) barely employed in design procedure. In [6], formula (1) is defined at resonant point, and  $Y_n^a$  is purely real. For single slot,  $Y_n^a$  is  $Y_{nn}$ , which can be calculated by electromagnetic software, in this paper, HFSS is employed. And the computational model is as figure 4.

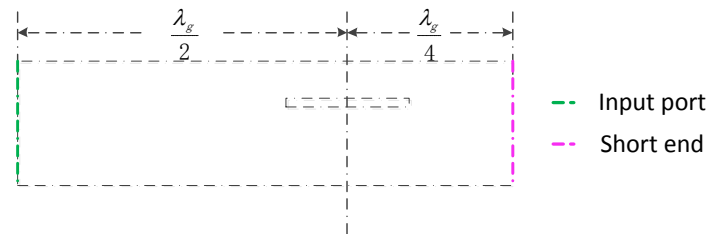


Figure 4. One-port model (short end) of slot fed by rectangular waveguide

As definition,  $V_n^s$  and  $V_n$ , also can be computed through electromagnetic software. The model is shown in figure 5.

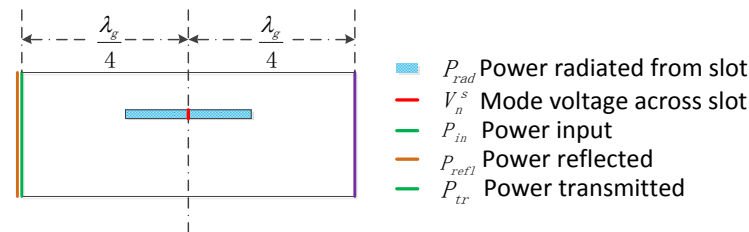


Figure 5. Two-port model (matched loaded) of slot fed by rectangular waveguide

Slot can be considered as a lumped circuit as figure 2.  $V_n^s$  can be read directly by HFSS. Relation of power radiated  $P_{rad}$  from slot gets from software and  $Y_{nn}$  is calculated from the model of figure 4. Therefore, equivalent voltage  $V_n$  of nth slot is obtained

$$V_n = \sqrt{\frac{2P_{rad}}{Y_{nn}}} \quad (9)$$

Thus,  $Y_{nn}$ ,  $V_n^s$  and  $V_n$  are all computed through computational electromagnetic software directly or indirectly. Put all these values in (1),  $K_1$  is achieved.

For example, a rectangular waveguide works at  $f_0 = 17.6\text{GHz}$ , of which width and height are  $a = 10.73\text{mm}$  and  $b = 3.5\text{mm}$  respectively. Calculated directly from (3), the result turns  $K_1^{dir} = 1.6428$ .

From formula (1) and electromagnetic software,  $K_1$  also can be calculated as aforementioned, written as  $K_1^{indir}$ , and the results are illustrated in table 1.

No.	Slot length(mm)	Slot Offset(mm)	$K_1^{indir}$
1	8.3931	1	1.6783
2	7.9	1	1.6524
3	8.6	1	1.6184
4	8.3931	1.3	1.6692
5	8.3931	1.5	1.6455

Table 1.  $K_1^{indir}$  computed in aid of software for slot fed by rectangular waveguide

Relative error is defined as (10):

$$\varepsilon_{K_1} = \left\| \frac{K_1^{dir} - K_1^{indir}}{K_1^{dir}} \right\| \quad (10)$$

And all results are less than 2%.

### 3 New Design Procedure for slot arrays fed by single-ridge waveguide

Compared to rectangular waveguide, ridge waveguide has smaller size and wider bandwidth. In [14], Kim and Elliott applied the same design procedure of last section on single-ridge waveguide, in which

$$K_1 = \frac{2k_T}{k_0} \sqrt{\frac{k_T^2}{\omega \mu_0 \beta_{10} G_0}} \quad (11)$$

$k_T$  is the eigenvalue of the single-ridge waveguide for the dominant mode  $TE_{10}$ . Computation of  $k_T$  is described in [5], which is complicated and obscured.

The new method for calculate  $K_1$  discussed above will be applied in later section to simplify the computation.

#### 3.1 New design procedure applied to compute $K_1$ of slot array fed by single-ridge waveguide

As previous,  $Y_{nn}$ ,  $V_n^s$  and  $V_n$  could be computed, and with equation (1),  $K_1$  could be calculated.

For example, a single-ridge waveguide, illustrated as figure 6,

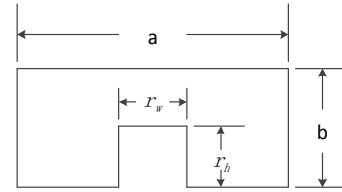


Figure 6. Geometry of single-ridge waveguide which works at  $f_0 = 17.6\text{GHz}$ , and  $a = 8.29\text{mm}$ ,  $b = 3.5\text{mm}$ ,  $r_w = 3\text{mm}$ ,  $r_h = 1.5\text{mm}$ .

No.	Slot length(mm)	Slot Offset(mm)	$K_1^{indir}$
1	9.018	1.5	1.5263
2	8.9	1.5	1.5200
3	8.8	1.5	1.4868
4	9.2	1.5	1.4737
5	8.8251	1.2	1.5567
6	9	1.2	1.5071
7	8.7	1.2	1.5505
8	8.6	1.2	1.5211

Table 2.  $K_1^{indir}$  for slot fed by single-ridge waveguide above  
Relative error is defined as

$$\varepsilon_{K_1} = \left\| \frac{\text{avr}(K_1^{indir}) - K_1^{indir}}{\text{avr}(K_1^{indir})} \right\| \quad (12)$$

In which  $\text{avr}(K_1^{indir})$  is average of  $K_1^{indir}$ , and calculated as,  $K_1^{indir} = 1.5178$ .

Error results are all less than 3%. Therefore,  $K_1$  of single-ridge waveguide can be approximated as a constant through the method aforementioned, instead of complex formula (11).

#### 3.2 Example of slot array fed by single-ridge waveguide designed with new method

One such case which can be instructive involves the design of a six-slot linear array, shown as figure 7.

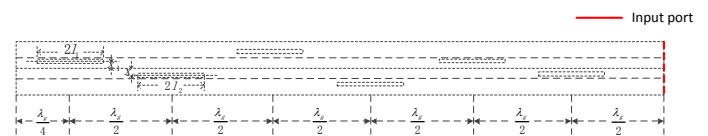


Figure 7. A  $1 \times 6$  array of longitudinal shunt slots in the broad wall of a single-ridge waveguide

The slot current distribution accords Taylor synthesis and the side lobe is designed at -25dB. Normalized current distribution is computed as:

$$0.1010 \quad 0.1701 \quad 0.2289 \quad 0.2289 \quad 0.1701 \quad 0.1010$$

Array works at  $f_0 = 17.6\text{GHz}$ , and the waveguide wavelength is  $\lambda_g = 22\text{mm}$ . Parameters of the ridge waveguide are

$$a = 10\text{mm}, \quad b = 3.5\text{mm}, \quad r_w = 4\text{mm}, \quad r_h = 1.7\text{mm}.$$

The slots' solution is:

$$\begin{aligned} x_1 &= 0.5868\text{mm} & 2l_1 &= 8.2710\text{mm} & g(x_1) &= 0.0557 \\ x_2 &= -0.9764\text{mm} & 2l_2 &= 8.4463\text{mm} & g(x_2) &= 0.1581 \\ x_3 &= 1.3436\text{mm} & 2l_3 &= 8.6033\text{mm} & g(x_3) &= 0.2862 \\ x_4 &= -1.3436\text{mm} & 2l_4 &= 8.6033\text{mm} & g(x_4) &= 0.2862 \\ x_5 &= 0.9764\text{mm} & 2l_5 &= 8.4463\text{mm} & g(x_5) &= 0.1581 \end{aligned}$$

$$x_6 = -0.5868\text{mm} \quad 2l_6 = 8.2710\text{mm} \quad g(x_6) = 0.0557$$

The radiation pattern is as figure 8:

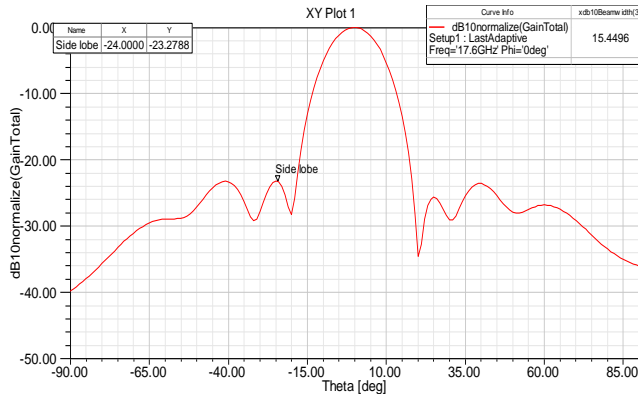


Figure 8. Pattern of the  $1 \times 6$  slot array example

We get from the figure that the side lobe is at -23.2788dB, nearly up to the request. At meanwhile, the desired normalized input admittance is equal to 1, but read from computational software,  $VSWR = 1.3866$ .

With the method aforementioned, we get new solution:

$$\begin{aligned} x_1 &= 0.7220\text{mm} & 2l_1 &= 8.2342\text{mm} & g(x_1) &= 0.0754 + j0.0192 \\ x_2 &= -1.1810\text{mm} & 2l_2 &= 8.4295\text{mm} & g(x_2) &= 0.2118 + j0.0469 \\ x_3 &= 1.5530\text{mm} & 2l_3 &= 8.6098\text{mm} & g(x_3) &= 0.3554 + j0.0622 \\ x_4 &= -1.5530\text{mm} & 2l_4 &= 8.6098\text{mm} & g(x_4) &= 0.3554 + j0.0622 \\ x_5 &= 1.1810\text{mm} & 2l_5 &= 8.4295\text{mm} & g(x_5) &= 0.2118 + j0.0469 \\ x_6 &= -0.7220\text{mm} & 2l_6 &= 8.2342\text{mm} & g(x_6) &= 0.0754 + j0.0192 \end{aligned}$$

The VSWR result reduces to 1.1499, which approaches the expectation. Nevertheless, effect of thickness of waveguide, precision of the computation and other factors which impact the accuracy should be taken into account.

## 4 Conclusion

In this paper, a new design procedure for longitudinal broadwall slots fed by a single-ridge waveguide is presented, which is less complex in contrast to the traditional one. The new design procedure derives from slot array fed by rectangular waveguide, and extends to arrays fed by single-ridge waveguide. The new method simplifies the mathematic computation of  $K_1$  with the aid of electromagnetic computational software. Two examples are illustrated later. The first one proves  $K_1$  is nearly a constant also for shunt slots of single-ridge waveguide. The latter one, that is a  $1 \times 6$  slot array, attains better VSWR after decoupling by the method. In conclusion, this new means assists us with in designing slot array fed by single-ridge waveguide.

## REFERENCES

- [1]. A. F. Stevenson, "Theory of slots in rectangular waveguides," *J. Appl. Phys.*, vol. 19, pp. 24-38, Jan. 1948.
- [2]. A. A. Oliner, "The impedance properties of narrow radiating slots in the broad face of rectangular waveguide, Part I-Theory, Part II-Measurements," *IRE Trans. Antennas Propagat.*, vol. AP-5, pp. 4-20, Jan. 1957.
- [3]. H. Y. Yee, "Impedance of a narrow longitudinal shunt slot in a slotted waveguide array," *IEEE Trans. Antennas Propagat.*, vol. AP-22, pp. 589-592, July 1974.
- [4]. R. J. Stegen, "Longitudinal Shunt Slot Characteristics," *Hughes Technical Memorandum* No. 261, Nov. 1951.
- [5]. J. P. Montgomery, "On the complete eigenvalue solution of ridge waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 547-555, June 1971.
- [6]. Robert S. Elliott, L. A. Kurtz, "The Design of Small Slot Arrays," *IEEE Trans. Antennas Propagat.*, vol. AP-26, Mar. 1978.
- [7]. Robert S. Elliott, William R. O'Loughlin, "The Design of Slot Arrays Including Internal Mutual Coupling," *IEEE Trans. Antennas Propagat.*, vol. AP-34, No. 9, Sep. 1986.
- [8]. Robert S. Elliott, "An Improved Design Procedure for Small Arrays of Shunt Slots," *IEEE Trans. Antennas Propagat.*, vol. AP-31, No. 1, Jan. 1983.
- [9]. R. S. Elliott, "Design of Line-Source Antennas for Narrow Beamwidth and Asymmetric Low Sidelobes," *IEEE Trans. Antennas Propagat.*, vol. AP-23, No. 1, pp. 100-107, Jan. 1975.
- [10]. Mazen Hamadallah, "Frequency Limitations on Broad-Band Performance of Shunt Slot Arrays," *IEEE Trans. Antennas Propagat.*, vol. 31, No. 7, Jul. 1989.
- [11]. J. C. Coetsee, J. Joubert, and W. L. Tan, "Frequency performance enhancement of resonant slotted waveguide arrays through the use of wideband radiators or subarraying," *Microwave Opt. Technol. Lett.*, vol. 22, no. 1, pp. 35-39, Jul. 1999.
- [12]. J. R. Pyle, "The Cutoff Wavelength of the TE<sub>10</sub> Mode in Ridged Rectangular Waveguide of Any Aspect Ratio," *IEEE Trans. Microwave Theory Tech.* vol. 14, No. 4, Apr. 1966.
- [13]. S. Hopfer, "The Design of Ridged Waveguides," *IRE Trans. Antennas Propagat.* vol. MTT-14, No. 4, Oct. 1955.
- [14]. David Y. Kim, Robert S. Elliott, "A Design Procedure for Slot Arrays Fed by Single-Ridge Waveguide," *IEEE Trans. Antennas Propagat.*, vol. 36, No. 11, Nov. 1988.