Transmit Beamspace Design For Unitary ESPRIT In Two Dimensional Hybrid MIMO Phased-Array Radar

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Abstract
In this paper, a transmit beamspace design method for two dimensional Hybrid MIMO phased-array radar in application of direction of arrival (DOA) estimation is proposed. A new method to construct the virtual array rotational invariance property that enables the use of search-free DOA estimation techniques at receiver is introduced. Semidefinite relaxation is used to transform the proposed formulation into a convex one that can be solved efficiently. The unitary ESPRIT is also introduced. Numerical simulation validate the proposed approach.

1 Introduction
The concept of multiple-input multiple-output (MIMO) radar has been researched in many aspects [1]. It offers significant performance improvements such as higher sensitivity to detect targets, higher angular resolution, increasing number of detectable targets [2] and combating the signal scintillation [3]. However, MIMO radar sacrifices transmit coherent processing gain as compared to phased-array radar [4]. This can lead to signal-to-noise ratio (SNR) loss in the receive array which then influence DOA estimation accuracy when practical application. Focusing on the MIMO radar with collocated arrays which can increase aperture by forming virtual arrays, Hybrid MIMO phased-array radar is proposed to add transmit coherent processing gain in the traditional MIMO radar [5]. The essence of this technique is dividing the transmit array into several subarrays, then one waveform can be transmitted coherently from each element of the same subarray, different orthogonal waveforms are transmitted from different subarrays. Combining the advantages of MIMO radar and phased radar, Hybrid MIMO Phased-array radar can improve the performance of DOA estimation.

Recently, some algorithms have been developed for DOA estimation in the context of Hybrid MIMO Phased-array radar with linear M collocated transmit antennas and N receive antennas in one dimensional. The MUSIC algorithm proposed in [6] require an exhaustive search over unknown parameters and it will result in high computational complexity. The search-free ESPRIT-based algorithm of [7] needs rotational invariance property in the virtual array as a precondition, and it needs additional parameter matching algorithm when there are more than one targets in the space. Based on the research above, some scholars begin to research on the DOA estimation of two dimensional Hybrid MIMO phased-array radar, which has M×N collocated transmit antennas and M×N receive antennas. [8] utilizes MUSIC algorithm in multiple targets estimation, especially, the high computational complexity from the exhaustive search will be more lager in two dimensional array with more antennas. Hence, how to apply the DOA estimation on two dimensional Hybrid MIMO phased-array radar in a more efficient way is a problem which is waiting to be solved.

In this paper, we aim at improving DOA estimation quality of two dimensional Hybrid MIMO phased-array radar both in transmit beampattern design and DOA algorithm improvement. Firstly, we design the transmit weight matrix to force the energy on the desired targets spatial range under the constraint of uniform power distribution. We will also form the rotational invariance of the virtual array. Meanwhile, we wish to control the side lobe and form nulls in the interference direction of the transmit beampattern. Then, we will introduce how to use Unitary ESPRIT (which behaves more excellent than the traditional ESPRIT in many aspects, such as: lower computational complexity, higher estimation accuracy [9]) on the joint azimuth and elevation estimation. Finally, corresponding simulation results and analysis are given.

The paper is organized as follow. Two dimensional Hybrid MIMO phased-array radar signal model is briefly introduced in section II, transmit weight matrix design method is given in section III. In section IV, Unitary ESPRIT algorithm is introduced. In section V, simulation examples for the proposed method are drawn.

2 System model
Consider a uniform rectangular array (URA) two dimensional Hybrid MIMO phased-array radar system with \(M \times N_t\) antennas in the transmit array and \(M \times N_r\) antennas in the receive array. Where \(M\) is the number of antenna elements in column, and \(N_t\) is the number of antenna elements in row. The elements on any given column in the transmit array are assumed to be equally spaced. Where \(d_x\) and \(d_y\) are the distances between the adjacent antennas at each column and at each row respectively. Let the steering vector of the each transmit subarray represented as
The received data vector at receive means that need be the vector of predesigned is the time required for the signal to cover is the identity matrix of size can be divided into satisfy

\[
\mathbf{y} = \mathbf{H} \mathbf{a}(\theta, \phi) + \mathbf{n}
\]

where \(\mathbf{z}_i\) is an matrix of ones and zeros where \(mn\)th entry equals zero if the \(mn\)th element of the array is absent, \(\text{vec}()\) stands for the operator that stacks the columns of a matrix in one column vector, \(\odot\) stands for the Hadamard product, \((\cdot)\) denotes the transpose, \(\theta\) and \(\phi\) denote the elevation and azimuth angles, respectively. \(\mathbf{a}(\theta, \phi)\) and \(\mathbf{v}(\theta, \phi)\) are defined as follows

\[
\mathbf{a}(\theta, \phi) = \text{vec}(\mathbf{Z}_i \odot [\mathbf{a}(\theta, \phi) \mathbf{a}(\theta, \phi)])
\]

\[
\mathbf{v}(\theta, \phi) = [1, \text{vec}(\mathbf{u}(\theta, \phi) \mathbf{a}(\theta, \phi))], \ldots, \text{vec}(\mathbf{u}(\theta, \phi) \mathbf{a}(\theta, \phi))]
\]

\[
\mathbf{u}(\theta, \phi) = [1, \text{vec}(\mathbf{u}(\theta, \phi) \mathbf{a}(\theta, \phi))], \ldots, \text{vec}(\mathbf{u}(\theta, \phi) \mathbf{a}(\theta, \phi))]
\]

Let \(\mathbf{c}(t) = [\mathbf{c}_0(t), \ldots, \mathbf{c}_N(t)]^\top\) be the vector of predesigned independent waveforms which satisfy the orthogonality condition \(\int \mathbf{c}(t) \mathbf{c}^\top(t) dt = \mathbf{I}_N\). \(\mathbf{I}_N\) is the identity matrix of size \(N\) \times \(N\).

The complex envelope of the signals transmitted by \(k\)th subarray is

\[
s_k(t) = \sqrt{\frac{M \gamma}{K}} \mathbf{w}_k \phi_k(t)
\]

where \(\mathbf{w}_k\) is the transmit weight vector used to form the \(k\)th transmit beam. The 2D transmit beampattern can be written as

\[
G_k(\theta, \phi) = \frac{M \gamma}{K} \sum_{k=1}^{K} \mathbf{a}^\top_k(\theta, \phi) \mathbf{w}_k \mathbf{a}_k(\theta, \phi)
\]

Assume a target at direction of \((\theta, \phi)\), the signal reflected by the target is

\[
x(t, \theta, \phi) = \tau_k(\theta, \phi) \mathbf{b}(\theta, \phi) + \sum_{l=1}^{L} \tau_l(\theta, \phi) \mathbf{b}(\theta, \phi) + \mathbf{n}(t)
\]

where \(\tau_k(\theta, \phi)\) is the reflection coefficient associated with the target. \(\tau_k(\theta, \phi)\) is the required for the signal to cover from the distance between the first element of the \(k\)th subarray.

In addition to the desired targets, we also assume there are \(L\) interfering targets at \((\theta, \phi)\). The received data vector at receive array can be modeled as

\[
x(t) = \tau(t, \theta, \phi) \mathbf{b}(\theta, \phi) + \sum_{l=1}^{L} \tau_l(t, \theta, \phi) \mathbf{b}(\theta, \phi) + \mathbf{n}(t)
\]

where \(\mathbf{b}(\theta, \phi)\) is the \(MN\times1\) receive steering vector and \(\mathbf{n}(t)\) is the noise that is supposed to have zero mean. By applying matched filtering to the received data vector for each of the orthogonal waveforms \(\phi_k(t)\), the \(KM\times1\) virtual receive vector is

\[
\mathbf{y} = \mathbf{H} \mathbf{a}(\theta, \phi) + \mathbf{n}
\]

\[
\mathbf{y} = \int_{\mathbb{R}} x(t) \mathbf{w}^\top(t) dt
\]

where \(\mathbf{n}\) is the \(KM\times1\) virtual receive data \(y\) can be divided into \(K\) signal subarrays \(y_1, \ldots, y_K\) through the following

\[
\begin{align}
\mathbf{y}_1 &= \frac{M \gamma}{K} \beta \mathbf{x}(\theta, \phi) + \mathbf{n} \\
\mathbf{y}_2 &= \frac{M \gamma}{K} \beta \mathbf{x}(\theta, \phi) + \mathbf{n} \\
\mathbf{y}_K &= \frac{M \gamma}{K} \beta \mathbf{x}(\theta, \phi) + \mathbf{n}
\end{align}
\]

any two subarrays \(y_i, y_j\) satisfy

\[
y_i = e^{i\phi_1(\theta, \phi)} \mathbf{y}_j
\]

where \(i, k = 1, \ldots, K\). The rotational invariance property can be enforced by imposing the constraint \(|\mathbf{w}^\top_k \mathbf{a}(\theta, \phi)| = |\mathbf{w}^\top_k \mathbf{a}(\theta, \phi)|\) while designing the transmit weight matrix. If \(K=2\), the rotational invariance property can be enforced only by forcing \(|\mathbf{w}^\top_k \mathbf{a}(\theta, \phi)| = |\mathbf{w}^\top_k \mathbf{a}(\theta, \phi)|\), otherwise , it will be \(|\mathbf{w}^\top_k \mathbf{a}(\theta, \phi)| = |\mathbf{w}^\top_k \mathbf{a}(\theta, \phi)|\), where \(i = 1, 2, \ldots, K/2\). In addition to design the rotational invariance property in the virtual array, we can also optimize transmit beampattern by transmit weight matrix design. The main idea is to design the transmit weight matrix which aims at minimizing the difference between a desired transmit beampattern and the actual one to form energy focusing in the target spatial sector under the constraint of uniform power distribution, controlling sidelobe level, forming nulls in the interference orientation. Hence defining a new matrix \(H_1 = \mathbf{w}, \mathbf{w}^\top\), the optimization problem is formulated as:

\[
\min_{\mathbf{w}, \mathbf{w}^\top} \max_{\beta, \phi} \{ \mathbf{D}(\beta, \phi) - \frac{1}{M \gamma} \sum_{\beta, \phi} \mathbf{H}(\beta, \phi) \mathbf{H}^\top(\beta, \phi) \}
\]

\[
\text{s.t.} \sum_{\beta, \phi} \text{diag}(\mathbf{H}) = \frac{E}{M \gamma} \mathbf{I}_{MN}
\]

\[
\sum_{\beta, \phi} \text{Tr} [\mathbf{H}^\top(\beta, \phi) \mathbf{H}(\beta, \phi)] = \sum_{\beta, \phi} \text{Tr} [\mathbf{H}^\top(\beta, \phi) \mathbf{H}(\beta, \phi)] = 0 \quad i = 1, 2, \ldots, L
\]

\[
\sum_{\beta, \phi} \text{Tr} [\mathbf{H}^\top(\beta, \phi) \mathbf{H}(\beta, \phi)] = 0 \quad \theta = \phi, \theta \in \Phi
\]

where \(P(\beta, \phi)\) is the desired beampattern, \(E\) is the total power, \(\mathbf{D}(\beta, \phi)\) denotes the trace of a matrix, \(\text{diag}(\cdot)\) denotes the diagonal of a square matrix. \(H_1 \succeq 0\) means that \(H_1\) is positive semidefinite. The convex optimization problem (11) is solved using semidefinite programming (SDP) [10]. After denotes the optimal solution \(H_1^\text{opt}\), the actual transmit matrix \(\mathbf{w}_1\) need to be solved. We can achieve \(\mathbf{w}_1\) directly if \(\mathbf{H}_1^\text{opt}\) is rank one. Otherwise \(\mathbf{w}_1\) can be solved by randomization techniques [11] from \(H_1^\text{opt}\).

4 Unitary ESPRIT

Reconstruct the two matrix \(y_1\) and \(y_2\) from section III, we can achieve a new matrix as

\[
\mathbf{y} = [y_1, y_2]
\]

The first step is defining a specific unitary matrix as
The Monte Carlo trials is 500. To evaluate

\[ \text{Fig. 1 shows the overlapped transmit array.} \]

\[ \text{Now combine the matrix } \vartheta \text{ and } \varphi \text{ into a new matrix } P = \vartheta + j\varphi , \text{ then a diagonal matrix } \Sigma \text{ is obtained which is composed of the} \]

\[ \text{eigenvalues of the matrix } P . \text{ Finally the estimation of } \vartheta , \varphi \text{ is achieved as} \]

\[ \hat{\vartheta} = \arcsin \sqrt{u^2 + v^2}, \quad \hat{\varphi} = -\arctan \frac{v}{u} \]

\[ \text{where } u = 2/\pi \times \tan(\text{Re}(\Sigma)), v = 2/\pi \times \tan(\text{Im}(\Sigma)) . \]

5 Simulation results

In our simulations, we assume a 5×5 URA with \( d_x = d_y = \frac{\lambda}{2} \), where \( \lambda \) is the wavelength. The target spatial range is defined by \( \vartheta = [-60, -20], \varphi = [20, 60] \). The 2D transmit array is divided into \( K=2 \) subarrays that are fully overlapped [12] and each of them consists of 24 antennas. The noise is complex Gaussian with zero mean. There is an interference located in \( \vartheta = -10^\circ, \varphi = 80^\circ \). Fig.1 shows the overlapped transmit array.

\[ \text{Fig. 1: Fully overlapped subarrays of 5x5 URA when } K=2 \]

The optimization transmit beampattern obtained by solving (11) is shown in Fig.2 and Fig.3. It can be seen from the figure that the transmit power is focused within the desired 2D target spatial range with 30° width at each side of the mainlobe in the elevation domain and 25° at each side of the mainlobe in the azimuth domain. The sidelobes is constrained below -25dB. And it can form nulls in the interference location.

\[ \text{Fig. 2: Transmit beampattern: elevation aspect} \]

\[ \text{Fig. 3: Transmit beampattern: azimuthal aspect} \]

\[ \text{Fig. 4: RMSE over 500 trials, from -30 to 30 dB.} \]
the target locations, and $\hat{\beta} = |\hat{\theta}|$ are their estimations. As can be seen in Fig.4, the Unitary ESPRIT enjoys significant performance benefits compared to the traditional ESPRIT in RMSE.

6 Conclusion

The transmit beamspace design for search-free DOA estimation of two dimensional Hybrid MIMO phased-array radar is considered. In addition to the rotational invariance property design, we also consider the transmit beampattern under traditional requests, such as unit power distribution, sidelobe controlling, nulls forming. Our simulation results show the improved DOA performance for the proposed method as comparing to the traditional ESPRIT.

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