

The Application of Discovery Teaching Method in Normal University Mathematics Teaching

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Abstract. By the utilization of n order determinant in higher algebra as an example and based on Bruner's four principal theorems about math learning, the learning pattern and psychological mechanism of math concepts in higher normal universities are analyzed. The progress and approach of freshmen majoring in math transiting from studying under supervision to self-study are discussed. By six steps of discovery teaching method, which are request, assumption, situation creation, generalization and summarization, practice integration and application upgrading, approaches required for teachers of higher algebra to develop student's ways of thinking and skills in higher mathematics are analyzed. This, eventually, can achieve the goal of the transition for freshmen majoring in math from elementary ways of thinking in mathematics to higher ways of thinking in mathematics.

1. Introduction

The primary difficulty that the students just graduated from high school and enrolled by the department of mathematics of colleges and universities is to change to the thinking way of advanced mathematics. Advanced algebra is not only a required course for the students in mathematics program, but also a course constructed by an axiomatic system. In the meantime, it is a course that is offered for the freshmen at the first semester. Therefore, how to solve the changing of students' mathematical thinking way is a problem necessary for each algebra teacher to confront with. In this paper, by taking n -order determinant for example, the changing of the mathematical thinking way of the freshmen in the department of mathematics is discussed through discovery teaching method.

2. Overview of Discovery Teaching Method

2.1 Brief introduction to discovery teaching method.

Discovery teaching method, as a teaching methodology in a serious sense, was proposed in the *Process of Education* [1] by Jerome s. Bruner (1915-), who is a cognitive psychologist of the United States. In this method, students are required to discover truths like scientists under the careful guidance of teachers, aiming at "finding" the causal relationship and inner link of thing's changes, forming concepts and gaining principles. Bruner said that discovery included all forms of acquiring knowledge by using their own heads. The relationship between teaching and learning can be more accurately and fully reflected with the discovery method [2]. Bruner thinks that discovery method owns the following advantages.

- (1) Improving students' wisdoms, and bringing students' potentials into play
- (2) Promoting students to produce intrinsic motivation in learning, and strengthening their confidence

(3) Promoting students to learn how to discover groping methods, and training students to improve the ability to raise and solve questions and the attitude towards invention

(4) Student can get a better understanding and consolidation of what they have learnt and can apply it better because they make their own knowledge systematic and structural

With regard to mathematics learning and mathematics teaching, great numbers of mathematics learning experiments have been made by Bruner, from which four principle theorems are concluded [3] as follows. All these are of positive significance for guiding the mathematics teaching.

(1) Structure theorem: It means that the best way for students to start learning mathematics concept, principle or rule is to build up one of its representations

(2) Symbol theorem: It means that students can easily cognize and understand what they have learnt if the symbols suitable for the intelligence development of students can be applied in early structure and expressive forms

(3) Contrast change theorem: It means that contrast change method is also applied from the specific expression to abstract expression of concepts

(4) Contact theorem: It means that each concept principle and skill in mathematics has a close tie with other concepts, principles and skills

2.2 Learning mode and psychological mechanism.

From above introduction to discovery teaching method, it can be known that in this method, the students "discover" the causal relationship and inner link of thing's change, form concepts and acquire principles through their own exploration and study. However, in education psychology [4], the learning of students is divided into two types: accepting learning and discovery learning. In accepting learning, learning contents are directly presented in the form of final conclusion, and is a reproductive process in essence. In discovery learning, learning contents can be generated, and is essentially a high-level cognitive process including creation, re-thinking and critical behaviors. Sometimes, the two different learning ways are also called by us as "dependent learning" and "independent learning". Apparently, "dependent learning" or "independent learning" is an expression to the high-level skills in mathematics.

2.3 Operational steps of discovery teaching method.

Step 1 is to raise requirements, allowing students clearly know the purpose. Thinking sources from questions; discoveries can be triggered by doubts. Therefore, to apply the discovery teaching, it is necessary to provide the definite missions and goals of "discovery" first.

Step 2 is to make an assumption, allowing students clearly know the thinking direction.

Step 3 is to set up situation, forcing students to face up with contradictions. Only the inner "contradiction" is likely to arouse the power of students in seeking knowledge and making explorations. Therefore, after requirement and assumption, it is necessary to further set up situation and stimulate the "paradox" of students.

Step 4 is to guide students to classify data, list evidences, discover results, and draw up a conclusion according to real cases.

Step 5 is to combine the discovered conclusion with actual materials, making an enhancement to understanding. Once students find the inductive conclusions made by them are consistent with the "answers", they will feel unusually happy. However, if not consistent, guidance should be given by the teachers.

Step 6 is to apply the knowledge obtained from discovery in practice. This part is not only a sublimation of discovery teaching method, but also a subsequent requirement.

3. Taking n-order Determinant for Example

(1) Task and Purpose: Solve a common linear system in two unknowns as follows.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2, \end{cases} \quad (1)$$

Obviously, the following equation can be solved with Gaussian elimination method (high school mathematics knowledge).

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{12} a_{21}}, x_2 = \frac{a_{11} b_2 - a_{21} b_1}{a_{11} a_{22} - a_{12} a_{21}} \quad (2)$$

Obviously, equation (2) is the formula to solve equation (1).

(2) Making assumption: The analytic formula to solve the equation (2) is not easily remembered. Therefore, an easy memory method can be sought, and second-order determinant and diagonal calculation method are introduced.

Further, a

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, x_2 = \frac{a_{11} b_2 - a_{21} b_1}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \quad (3)$$

common linear system in three unknowns is solved as follows.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3. \end{cases} \quad (4)$$

Then, third-order determinant and diagonal calculation method are introduced.

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, x_3 = \frac{D_3}{D} \quad (5)$$

In equation (5),

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

D_1 , D_2 and D_3 are solved by replacing the first, second and third columns in D with the constant series in equation (4), as shown below.

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

It is also calculated with the above diagonal calculation method. The above results show that second-order and third-order determinant diagonal calculation methods are correct.

(3) Set up a situation: Can a fourth-order determinant be calculated with diagonal calculation method?

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = ?$$

The second-order determinant has an item of $2! = 2$; the third-order determinant has an item of $3! = 6$; the fourth-order determinant has eight items if calculated with diagonal calculation method,

and also there is $4!=24$. Obviously, the diagonal calculation method is not applicable to the fourth-order determinant (the diagonal calculation method refers to the method commonly used in teaching material [5], but is not a special diagonal calculation method [6,7]).

(4) Classification, induction and conclusion

Re-know the third-order determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

To solve question, the concepts of even permutation and reverse order number are introduced. All above line permutations are natural; the column permutations are as follow.

Even permutation $a_{11}a_{22}a_{33}$, $a_{12}a_{23}a_{31}$, $a_{13}a_{21}a_{32}$; inverse number is even and its mark is positive

Uneven permutation: $a_{13}a_{22}a_{31}$, $a_{12}a_{21}a_{33}$, $a_{11}a_{23}a_{32}$; inverse number is uneven and its mark is negative

Therefore, the following conclusion can be drawn up.

- The number of the elements of the third-order determinant is $3^2=9$
- The number of the expansions is $3!=6$
- Common item of expansions: $a_{1j_1}a_{2j_2}a_{3j_3}$ is the product of 3 elements from different row and column
- The mark is decided by the inverse number of column permutation: $(-1)^{\pi(j_1j_2j_3)}$

$$\text{e. Conclusion: } \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \sum_{j_1j_2j_3} (-1)^{\pi(j_1j_2j_3)} a_{1j_1}a_{2j_2}a_{3j_3}$$

(5) Drawing up a conclusion: The definition of the n -order determinant

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \sum_{j_1j_2\cdots j_n} (-1)^{\pi(j_1j_2\cdots j_n)} a_{1j_1}a_{2j_2}\cdots a_{nj_n}$$

(6) Practice: Calculating the n -order determinant with the definition

$$\begin{vmatrix} 0 & \cdots & 0 & 1 \\ 0 & \cdots & 2 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ n & 0 & \cdots & 0 \end{vmatrix} = ?$$

Solving: From the common item $a_{1j_1}a_{2j_2}\cdots a_{nj_n}$ of the n -order determinant, it can be known that the whole product is 0 as long as a factor is 0. Now, the factors not equal to 0 are considered, and thus there is $j_1 = n, j_2 = n-1, \cdots j_n = 1$. The above n -order determinant can be expressed as follows.

$$(-1)^{\pi(n, n-1, \cdots, 2, 1)} 1 \times 2 \cdots \times n = (-1)^{\frac{n(n-1)}{2}} n!.$$

4. Understanding of Mathematical Concepts is the Key to Changing to the Thinking Way of Abstract Axiomatic System

In the global five major ancient civilization countries (according to international view) except ancient Greek, mathematics was regarded as a tool for solving practical problems. However, in ancient Greek, mathematics was seen as a science for training people's mental quality [8]. The first axiomatic system (Euclidean geometry) was generated because of the first mathematical crisis; modern mathematics and its branches are all constructed on axiomatic system. College students in

mathematics program are necessary to change to the thinking way of the abstract axiomatic system. Thus, for a long time, everyone has his own different understanding of the question whether mathematics is a dynamic development field, or a static system including known concepts, principles and techniques.

At present, a common understanding, held by both teachers and students, is that mathematics is "objective truth". Mathematical knowledge is acquired by a few of scientists, experts and scholars through the explorations of serious procedures. We only accept and communicate the "objective truths" discovered by them. In the knowledge producing process, the participation of the subjective factors is rejected in the cognitive process. It is necessary for college mathematics teachers to set up a correct outlook on mathematics and teaching. The contemporary constructivism theory has provided a powerful theoretical support for great numbers of teachers. In the constructivism theory, cognition is a positive construction process of subject based on his own experience, but not the essential simple and passive reflection given by subject to object. It attaches high importance to the existing knowledge experience, and the learning initiative, sociality and situation. However, in Ausubel's cognitive structure migration theory [9], the characteristics of content and organization of relevant concepts in the students' existing cognitive structure will affect the learning of new knowledge. This plays a positive inspiration role in the mathematics teaching, and also provides a feasible way for college students to change to the high-level thinking way [10-13].

5. Conclusion

In this paper, through discovery teaching method and taking the definition of the n -order determinant for example, the learning of a new mathematical concept as a necessary component for teacher to directly provide definition and students to explore the new concept with the existing knowledge experience is discussed, and thus the definition of a new concept is obtained. Meanwhile, how to make students smoothly transition to the thinking way of advanced mathematics is discussed. Teachers should not only convey the products of the thinking way of advanced mathematics to students, and more importantly should allow students to acquire the thinking way of advanced mathematics. Therefore, it is necessary to emphasize the importance of learning the mathematical concepts at colleges and universities, as well as the paths to obtain the high-level mathematical skills. This is very crucial for the freshmen in mathematics program to change to the thinking way of the abstract axiomatic system.

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