

# Optimal Capacitor Switching Method Based On Minimum Spinning Tree Theory in Distribution Systems

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**Abstract.** According to the radial structure operation characteristics of distribution systems, this paper proposes a new method based on minimum spanning trees for capacitor optimal switching. Firstly, taking the minimal active power loss as objective function and not considering the capacity constraints of capacitors and source, this paper uses Prim algorithm among minimum spinning trees algorithms to get the power supply ranges of capacitors and source. Then with the capacity constraints of capacitors considered, capacitors are ranked by the method of breadth-first search. In term of the order from high to low of capacitor ranking, capacitor compensation capacity based on their power supply range is calculated. Finally, IEEE 69 bus system is adopted to test the accuracy and practicality of the proposed algorithm.

## 1. Introduction

The optimal capacitor switching of distribution systems is to seek the optimal switching capacity of the capacitors to achieve the goal of power loss reduction and voltage equalization, considering the constraints of branch loadability and voltage. From the perspective of electrical characteristic, it can also be considered for finding the optimal power sources for reactive power load. Do not number your paper: All manuscripts must be in English, also the table and figure texts, otherwise we cannot publish your paper. Please keep a second copy of your manuscript in your office. When receiving the paper, we assume that the corresponding authors grant us the copyright to use the paper for the book or journal in question.

The optimal capacitor switching is a kind of non-linear integer-programming. There are many kinds of solution methods, but all these methods are based on Mathematical Programming (MP) and Artificial Intelligence algorithms (AI).

The MP method [1-5] is fast in calculation and good in convergence, but it is easy to fall into local optima. [1] did some research on the real-time optimization of capacitor switching. It built a successive linear integer programming model, and solve it by using dual relaxation method and successive discretization method. [2, 3] adopt feasible direction method to solve the optimal capacitor switching, which has high calculation efficiency. [4] gave a quadratic programming method of solving the real-time optimization problem of capacitor switching. Kron reduction technique is used in the solving process, but the credibility of the optimal solution lacks the theoretical basis. [5] obtained the capacity of the capacitor switching by iterative solving the optimal matching injection current according to loop analysis method and circuit superposition theorem. In order to obtain the optimal solution, the AI method [6-11] is applied, but can not meet the requirements of real-time control because of the large amount of calculation.

Based on above, this paper tries to find another way to avoid the drawbacks of existing methods. It combines the radial structure operation characteristics of distribution systems with minimum spanning trees, and proposes a new method of capacitor optimal switching.

## 2. Fundamental Principles

### 2.1 The Power Supply Ranges of Capacitors and Source.

Without the capacity constraints of capacitor and power source, if the power loss is minimum when the reactive power of Load  $L_i$  is supplied by Capacitor  $C_i$ , then there will be only one path between any two nodes, according to the radial structure operation characteristics of distribution systems. And if Load  $L_{i+1}$  which is on the same branch of  $L_i$  but far away from  $C_i$  is supplied by  $C_i$ , and the power loss is minimum, the power supply path must pass through  $L_i$ . Therefore, if we treat the node of capacitor or power source as the root of the tree, with minimum power loss as objective function, then we can get the power supply ranges of capacitors and source by keeping expanding every tree until all the nodes are contained.

In this paper, Prim algorithm among minimum spinning trees algorithms is used to get the power supply ranges of capacitors and source. In order to analyze the supply ranges of capacitors and source, the path from the capacitor node to the source node including the branches and nodes that passing through is called the trunk path. And we can call the branch which connected to but not belongs to the trunk path the branch path. The load on the branch path can be equivalent to load connected to corresponding node of the trunk path. We can call it load equivalent. Further, the equivalent load is equal to the sum of the load of the node, the load on the branch path and the power loss which is produced by the branch path load pass through the branch.

The process of the Prim algorithm is: Start with the source node  $s_0$  and capacitor node  $c_i, i=1,2,\dots,n$ , in which  $n$  is the number of capacitors.  $S \leftarrow \{s_0, c_i\}$ , and every time a branch  $E=(v_j, v_k)$  is add, this branch is made to be the minimum arc of the cut set which is formed by the node-set  $S$  and its complementary set  $V \setminus S$ , in which  $v_j \in S, v_k \in V \setminus S$ . Add node  $v_k$  to node-set  $S$ , which means  $S \leftarrow \{v_k\}$ , and keep expanding  $S$  until containing all the nodes.

The equivalent load, arc weights and capacity calculation of the capacitor switching are all based on the assumption that the node voltage invariant, and will just be one iteration of the calculation process for capacitor switching optimization.

The power loss  $P_{\text{lossq}}$  produced by the added reactive power load  $Q_{\text{load}}$  of a node each time is:

$$P_{\text{lossq}} = \sum_{i=1}^m [I_i + Q_{\text{load}}e / (e^2 + f^2)]^2 \times r_i \quad (1)$$

In which,  $I_i$  is the reactive current of the  $i^{\text{th}}$  branch which from the load node to the source or capacitor;  $e, f$  are separately the real and imaginary parts of the node voltage which located  $Q_{\text{load}}$ ;  $r_i$  is the resistance of branch  $i$ ;  $m$  is the total number of all the branches that from  $Q_{\text{load}}$  node to the capacitor node.

For the load supplied by power source, the arc weight can be determined by (1).

For the capacitor, it injects active power while injecting reactive power to the power system. So the saving power loss is:

$$P_{\text{lossp}} = \sum_{i=1}^n [(Q_{\text{load}} + P_{\text{lossq}})f / (e^2 + f^2)]^2 \times r_i \quad (2)$$

In which,  $n$  is the number of all the branches between the capacitor node and the source node.

For the load supplied by capacitor, the arc weight can be determined by the following equation:

$$W = P_{\text{lossq}} - P_{\text{lossp}} \quad (3)$$

Thus, according to (1) and (3), we can calculate the weight of each arc in the cut set which formed by the node-set  $S$  and its complementary set  $V \setminus S$ . The arc with the minimum weight is considered as the minimum arc.

When the weights of all the cuts formed by the node-set  $S$  and its complementary set  $V \setminus S$  are calculated each time, if node  $v_k$  of branch  $E$  is on the trunk path, the loads of all the branch paths from  $v_k$  to  $s_0$  or  $c_i$  should be considered. It means that all the node loads of the branch paths of this trunk path are equivalent loads. It is easy to find out if the arc is the minimum arc, the loads of all the branch paths

from  $v_k$  to  $s_0$  or  $c_i$  should be supplied by the same power source or capacitor with node  $v_k$ . And if  $v_k$  is not a node of the trunk path, the special treatment if not required.

So, we can get spanning tree with the root node of power source or capacitor separately. And the spanning tree shows their power supply range.

### 2.2 The Calculation of Capacitors Switching Capacity.

The compensating reactive current  $I_{cap}$  of capacitor:

$$I_{cap} = Q_{cap}e / (e^2 + f^2) \tag{4}$$

In which  $Q_{cap}$  is the rating of the capacitor.

The reactive current  $I_{load}$  produced by load is:

$$I_{load} = (Q_{load}e - P_{load}f) / (e^2 + f^2) \tag{5}$$

In which  $P_{load}, Q_{load}$  are active and reactive power of the load.

The method of optimal capacitor switching in this paper adopts the principle of reactive current balance. It means that the compensating reactive current  $I_{cap}$  of capacitor is equal to the sum of all the reactive current  $I_{load}$  of the loads within the range of its power supply. There are two rules of calculating the capacitor switching capacity when considering the capacity  $Q_{cap}$  constraints of capacitors:

For multiple capacitors, the compensating reactive current  $I_{cap}$  of downstream capacitor should be calculated first, and then the upstream capacitor;

For single capacitor, the compensating reactive current  $I_{cap}$  of capacitor should be used to compensate the load reactive current within its downstream power supply range, and then the upstream.

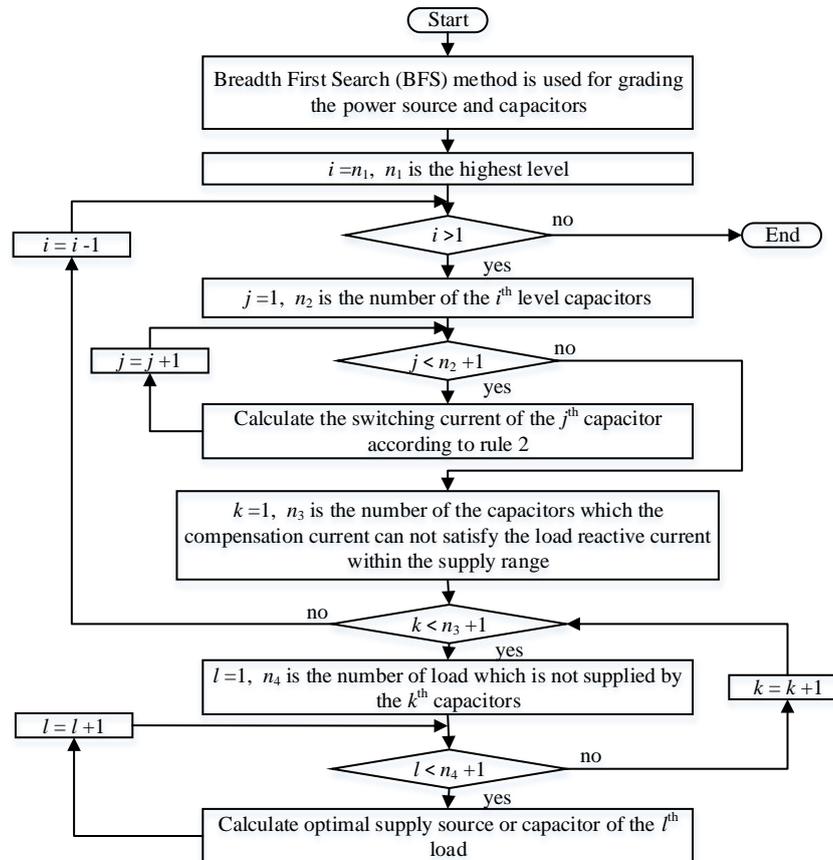


Fig.1 Flow chart of capacitor compensation process

In Fig. 1, the calculation process of the optimal capacitor or source for supplying load of number  $l$ .

With rule 1, Breadth First Search (BFS) method is used for grading the power source and capacitors, and deciding the compensation order of the capacitors. First, only the nodes of power source and capacitors are reserved, and all the load nodes are taken out. According to the BFS method, the power

source node is labeled as the first level, and label the capacitors within the downstream of the first level and adjacent as the second level; label the capacitors within the downstream of the second level and adjacent as the third level. So continue, until all the capacitors are labeled. The flow chart of capacitor compensation process is shown in Fig. 1.

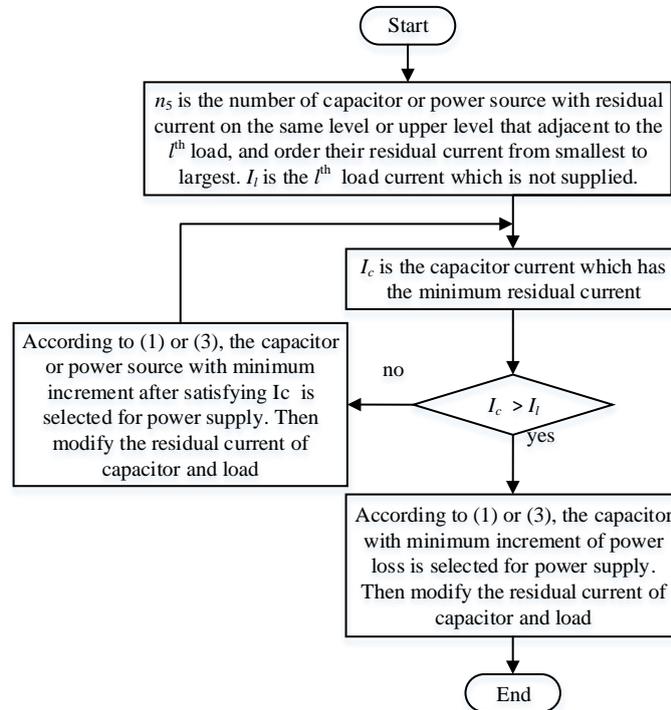


Fig.2 Flow chart of optimal capacitor or source for supplying load of number  $l$

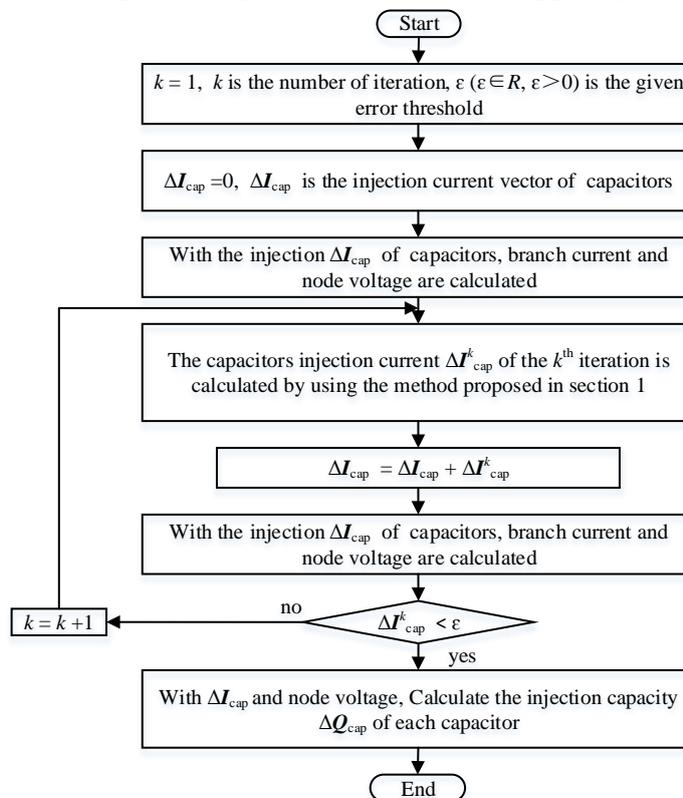


Fig.3 Flow chart of optimizing capacitor switching

### 3. Flow Chart of Optimizing Capacitor Switching Algorithm

In the first section, the node voltage is assumed to be constant when the power supply range of source or capacitors are determined and compensation capacity of capacitor is calculated. But when the capacitor switches, the node voltage will change. Meanwhile, the reactive current will change either. For this reason, an iteration method is used in this paper for obtaining the power supply range of source or capacitors and the compensation capacity of capacitor. The flow chart of algorithm is shown in Fig.3.

The convergence of the algorithm is shown below:

In the first iteration, the compensation current of the capacitor is largest since that the load reactive current should be compensated. In the second iteration, the load reactive current has been compensated yet, but the compensation current of the capacitor should be recalculated on account of the change of load reactive current caused by the voltage variation. The compensation current is smaller than the compensation current of the first iteration apparently, and the voltage variation after compensation is smaller than the voltage variation of the first iteration. In the third iteration, the reduction of node voltage variation will lead to the reduction of load current variation. The compensation current of capacitor will continue to be reduced, and so will the node voltage variation after compensation. By analogy, the algorithm will converge quickly.

The iteration of the algorithm is based on the balance relationship of supply reactive current and load reactive current within the supply range of capacitor. It is kind of direct optimizing method which is different from the mathematical programming method or intelligent search method with appropriate step for optimization. And it can converged after a few iterations.

### 4. Example and Analysis

The algorithm in this paper is realized by c++, and it is executed on the computer with Intel Pentium (D) CPU 2.80MHz. The IEEE 69 bus system is used for testing. Reference capacity is 100MVA, and reference voltage is 12.66kV. The specific data can be found in [2].

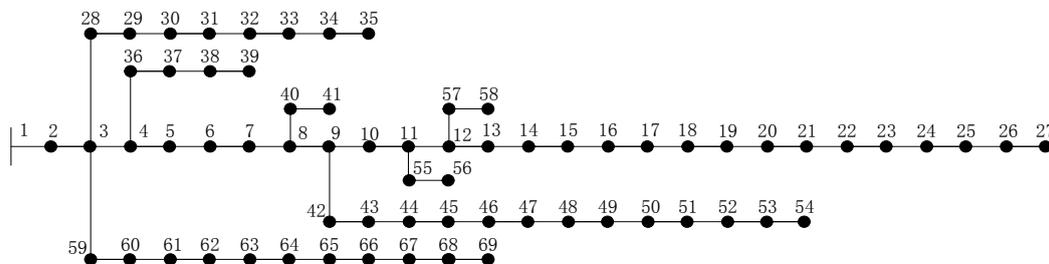


Fig.4 IEEE 69 bus system

Table 1 The iteration results for reactive power injection of capacitors

| Node number of capacitor | Number of iteration |        |        |
|--------------------------|---------------------|--------|--------|
|                          | 1                   | 2      | 3      |
| 19                       | 0.3                 | 0.3    | 0.3    |
| 47                       | 0.239               | 0.318  | 0.341  |
| 52                       | 0.9999              | 0.9999 | 0.9999 |

Table 1 shows that the algorithm has a good convergence. It converged in three times.

In each iteration, the supply range of capacitor and source remained the same which means that the effect of supply range by voltage variation can be neglected. The data after the third iteration is shown in Table 2.

**Table 2 The supplying range of capacitors and source**

| Node number | Node of capacitor or power source | level | Reactive injection current (p.u.) | Sum of load current (p.u.) |
|-------------|-----------------------------------|-------|-----------------------------------|----------------------------|
| 1           | 1~8,28~35, 36~41,59~69            | 1     |                                   |                            |
| 19          | 10~27,55~58                       | 2     | 0.0031                            | 0.00397                    |
| 47          | 9,42~46,48,49                     | 2     | 0.0105                            | 0.00324                    |
| 52          | 50,51,53,54                       | 3     | 0.0106                            | 0.0124                     |

According to Table 2 and rule 1, the compensation current of the capacitor on node 52 was calculated first, and the sum of the load current within its control range was larger than its rated compensation reactive current. According to rule 2, the rest load on the node 50 was not be supplied by the capacitor on node 52, but was supplied by the capacitor on node 47. Then the compensation current of the capacitors of node 19 and 47 was calculated. The rated compensation current of the capacitor on node 47 is larger than the sum of the load reactive current within its control range, and all the loads within the supply range can be supplied by the capacitor. The rated compensation current of the capacitor on the 19<sup>th</sup> node is smaller than the sum of the load reactive current within its control range, and all the loads within the supply range can be supplied by the capacitor. According to rule 2, the loads of node 10, 11, 55, 56, 57, 58 and part of the load on node 12 are supplied by power source and capacitor on node 47. The optimal power supply can be obtained by figure 2. The switching for capacitors is shown in Table 3.

**Table 3 The switching for capacitors**

| Node number of capacitor | Capacity (Mvar) | Round (Mvar) | Number of switching group | Capacity of each group (Mvar) | Maximum capacity (Mvar) |
|--------------------------|-----------------|--------------|---------------------------|-------------------------------|-------------------------|
| 19                       | 0.3             | 0.3          | 6                         | 0.05                          | 0.3                     |
| 47                       | 0.341           | 0.33         | 11                        | 0.03                          | 1                       |
| 52                       | 0.9999          | 0.99         | 33                        | 0.03                          | 1                       |

**Table 4 The results of switching for capacitors**

| index                | Before optimization(kW) | After optimization(kW) | Power loss reduction(kW) |
|----------------------|-------------------------|------------------------|--------------------------|
| Power loss           | 225.303                 | 148.022                | 77.28                    |
| Minimum node voltage | 0.909                   | 0.931                  |                          |

Table 3 shows that the algorithm proposed in this paper has the same result with [11] which uses evolutionary programming algorithm, but the algorithm of this paper only adopts simple graph method and has higher computational efficiency. There is no problem of convergent to local optimal solution relative to mathematical programming method.

## 5. Summary

According to the radial structure operation characteristics of distribution systems, the minimum spanning trees method for power supply ranges determination of capacitor has obvious physical significance, and gives the optimal capacity of the capacitor. The proposed algorithm has a clear concept, simple implementation and good practical application prospect.

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