A Supplier Selection Approach based on the Combination of AHP and Rough set

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Abstract. The analytic hierarchy process (AHP) approach is often used in supplier selection. But due to its inherent subjectivity, the evaluation information based on human judgments often creates conflict and bears some kind of uncertainty. To overcome this, the paper proposed a model using the combination of rough set theory and analytic hierarchy process for supplier selection problem. Then a case study of a company dealing with selection of logistics supplier is used to illustrate this composite model using both rough set and AHP approach. The proposed model can provide guidelines and directions for managers involved in purchasing the logistics service to reduce the redundant criteria and consider both subjective and objective factors when evaluating the suppliers.

Introduction

Supplier selection is an important topic in the literatures of supply chains. The goals of the supply chains is to meet the customer's demands in the shortest time at least cost, therefore, the reduction of total costs and the delivery time is the key to success. Supply chain starts with selecting the right supplier for the raw materials and component parts. Analysis reveal that the cost of the raw materials and the component parts themselves carry around 70 percent of the total cost of the product (Weber et al. 1991), and a high percentage of production costs are related to the purchase of goods and services (Ghodsypour et al. 1998). The total cost will reduce considerably by reducing these costs depended on the supplier.

The selection of the best supplier is one of the key factors that contributes to the operational success of many firms (Jain et al. 2009), however it is also a complex issue, as it involves a large number of factors and decisions. These factors have complicated operational and financial implications. Therefore, supplier selection is a time-consuming process which needs many resources.

The supplier must satisfy the buyer on all the required criteria. Thus, supplier selection process becomes a multiple criteria decision making problem involving various criteria which may be quantitative as well as qualitative. Supplier selection is basically a multiple criteria decision-making problem. There are two aspects in the issue of the supplier selection. One aspect is the criteria for evaluation of suppliers, and the other aspect is the procedure or method of supplier selection (Chen et al. 2012).

The article is developed as follows: Firstly, a brief review of literature on supplier selection criteria and AHP approaches is presented. Secondly, the evaluation methodology based on the combination of rough set theory and AHP is proposed and a case study is followed. Finally, some remarks are concluded and future directions for the research are suggested in the last paragraph.

Literature Reviews

Supplier selection is based by a variety of criteria. Dickson (1966) identified 23 different criteria by doing a survey in purchasing agents and managers from USA and Canada. And the price, delivery and quality are particularly important factors among these criteria when making the order decisions. There are always some conflicting criteria on supplier selection process, Wind and Robinson (1968) propose that the lowest price may conflicts with the best quality when evaluating the suppliers, and trade-off between price and quality is needed to make suppliers selection decision.
When doing selection of suppliers, these criteria, including price, delivery, quality system, performance, production capacity, localization, human resources and information technology, are often used in the literature (Weber et al. 1991, Sevkli et al. 2007). And quality had highest frequency followed by delivery and cost when evaluating the suppliers (Inemek et al. 2009).

Many multi-criteria decision-making approaches have been applied in the supplier selection problem. Agarwal et al. (2010) presented a review of literature on multi-criteria decision-making approaches for supplier evaluation and selection. These approaches include: the analytic hierarchy process (AHP), the data envelopment analysis (DEA), the analytic network process (ANP), mathematical programming, dimensional analysis, fuzzy decision-making, neural networks and genetic algorithms.

A five-step AHP-based model was proposed to aid decision-makers in rating and selecting suppliers with respect to nine evaluating criteria (Muralidharan et al. 2002). Chan et al. (2004) applied the AHP hierarchy consists of six evaluating criteria and 20 sub-factors. Hou and Su (2007) developed a decision support system for the supplier selection problem in a mass customization environment based on the AHP model.

Kumar and Roy (2011) proposed an AHP model with three steps in vendors selection by the performance scores of various vendors. Bruno et al. (2012) developed a hierarchical model for supplier selection with 12 sub-criteria under 4 criteria. Moreover, some literatures applied the integrated approaches by using more than one technique jointly such as integrated AHP and DEA, integrated AHP and goal programming, etc.

On the basis of above, these literatures have a common problem which comes from the internal weakness of the AHP model. It is that the subjectivity of human opinions will strongly affect the weights of the criteria under the hierarchy of evaluation system. Thus it will create some kind of uncertainty and conflict when making assessment on supplier selection. To overcome it, this article proposes a model using the combination of rough set theory and AHP for supplier selection problem, and aims to add some objective component by rough set theory when making selection of suppliers.

Evaluation Methodology

Analytic Hierarchy Process (AHP). AHP is a method to support multi-criteria decision making, and was developed by Saaty (1980). AHP derives ratio scales from paired comparisons of criteria, and allows for some small inconsistencies in judgments. AHP is used in a wide range of applications such as the evaluation of suppliers, project management, hiring process or the evaluation of company performance, etc. In the AHP approach, the decision problem is structured hierarchically at different levels, with each level consisting of a finite number of decision elements. The highest level of the hierarchy represents the overall goal, the lower levels indicate the decision criteria and sub-criteria.

AHP uses pair-wise comparison of the same hierarchy elements in each level using a Saaty scale, indicating the importance of one element over another element within the hierarchy element, thus, to yield a relative weights of each element. By this way, The AHP model translates the subjective opinions influenced by human’s preferences or feelings, into measurable numeric relations. AHP helps to makes decisions in a more rational way and to make them more transparent and better understandable.

Rough Set. The rough set theory was developed by Pawlak (1982). It has emerged as a major mathematical method in appraising and assessment. It can evaluate the importance( or weights) of particular attributes( or criteria) in relationships between objects. Moreover it is used to reduce all redundant attributes and get minimal subsets of attributes that ensure a satisfactory approximation of the classification made by decisions.

(1) Information System and Indiscernibility Relation

Let \((U,A,F,V)\) be an information system, where \(U\) is a non-empty set of finite objects (the universe). Universe describes all the research objects, denoted by \(U=\{x_1,x_2, \ldots, x_n\}\). \(A\) is the set of
attributes belong to objects, denoted by \( A = \{ a_1, a_2, \ldots, a_m \} \). \( V \) is the set of values that attribute may take. \( F \) is the set of relations between attribute \( a \) and object \( x \). \( f(a, x) \) is the value of attribute \( a \) for object \( x \) in the universe \( U \).

The main concept of rough set theory is indiscernibility relation. Then \( x_i \) and \( x_j \) are indiscernible by the set of attributes \( R \), if and only if \( f(a, x_i) = f(a, x_j) \) for every \( a \in P \).

With any \( P \subseteq A \), there is an associated equivalence relation denoted by

\[
\text{IND}(P) = \{ (x_i, x_j) \in U \times U \mid \forall a \in P, f(a, x_i) = f(a, x_j) \}
\]

The relation \( \text{IND}(P) \) is called a indiscernibility relation. The partition of \( U \) is a family of all equivalence classes of \( \text{IND}(P) \) and is denoted by \( U / \text{IND}(P) \).

(2) Approximations and Positive Region
Suppose \( P \) is a non-empty subset of \( A \), the \( P \)-lower approximations of \( X \) are defined as

\[
P_X = \{ Y \subseteq U / \text{IND}(P) : Y \subseteq X \}
\]

\( P(X) \) is the largest union of the \( P \)-elementary sets included in \( X \). it is also the positive region of \( P \) denoted by \( \text{pos}_P(X) \).

The \( P \)-upper approximation is the union of all equivalence classes in \( U / \text{IND}(P) \) which have non-empty intersection with the target set \( X \).

\[
P_X = \{ Y \subseteq U / \text{IND}(P) : Y \cap X \neq \emptyset \}
\]

(3) Reduct and Core
A reduct is a subset of attributes \( R \subseteq PR \subseteq P \) such that the equivalence classes induced by the reduced attribute set \( R \) are the same as the equivalence class structure induced by the full attribute set \( P \).

The set of attributes which is common to all reducts is called the core. The core is the set of attributes which is possessed by every legitimate reduct, The core is the set of necessary attributes.

According to the rough set theory, the weights of criteria (the attributes in information system) can be obtained by the dependency of attributes as follow

\[
\gamma_a \doteq \frac{|\text{pos}_R(X) - |\text{pos}_{R-a}(X)|}{|\text{pos}_R(X)|}, \quad a \in R, \ R \subseteq A
\]

The Composite Model of AHP and Rough Set. Rough set theory can be employed to evaluate uncertain system. However, a key difference from AHP approach, and a unique strength, of using rough set theory is that it provides an objective form of analysis (Pawlak et al. 1995). Classical rough set analysis requires no additional information, external parameters or subjective interpretations to determine set membership. Instead it only uses the given data itself. (Düntsch and Gediga 1995). As a result, we combine the AHP and rough set theory in order to introduce more subjectivity to the rough set approach, and on the other hand, add more objectivity to AHP approach. Thus we provide the buying managers with a comprehensive model with which both objectivity and subjectivity are consider. The ratio between subjectivity and objectivity is defined as \( \mu_i \). The weights of the criteria are as follows:

\[
\omega_i = \mu_i \times a_i + (1 - \mu_i) \times b_i
\]

\( a_i \) stands for the weights evaluated by the rough set theory, and \( b_i \) stands for the weights calculated by AHP approach. Moreover, the rough set theory can help to set up a concise and complete hierarchy with different criteria on the various levels by reducing the redundant elements, thus to gain a more precise evaluation system on the supplier selection.
Case Study

The section is to propose an example for this combination model, which will help to solve the supplier selection problem based on multiple criteria. The case study methodology is used in a Chinese company dealing with selection of logistics suppliers.

The hierarchy of evaluation system consists of four criteria and their corresponding sub-criteria which are showed in Fig. 1

![Figure 1. The hierarchy of evaluation system](image)

There were 10 logistics suppliers available for selection which were denoted respectively by \(x_1, x_2, \ldots, x_{10}\). Criteria of the lowest level were denoted as \(a_{ij}\). For every criterion, the performance was classified as excellent, great, good and ordinary. and was expressed with 3,2,1 and 0 respectively. The letter of \(P\) is the value of decision attributes. Thus an information table was constructed as follows (Table 1)

<table>
<thead>
<tr>
<th></th>
<th>(a_{11})</th>
<th>(a_{12})</th>
<th>(a_{13})</th>
<th>(a_{14})</th>
<th>(a_{21})</th>
<th>(a_{22})</th>
<th>(a_{23})</th>
<th>(a_{24})</th>
<th>(a_{31})</th>
<th>(a_{32})</th>
<th>(a_{33})</th>
<th>(a_{34})</th>
<th>(a_{41})</th>
<th>(a_{42})</th>
<th>(P)</th>
</tr>
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</table>

**Calculation Based On Rough Set.** It is obvious that the values of attributes \(a_{11}\) and \(a_{32}\) are the same for all \(x_i\), so are attributes \(a_{13}\) and \(a_{41}\), \(a_{12}\) and \(a_{23}\), \(a_{22}\) and \(a_{21}\), and \(a_{42}\) and \(a_{41}\). According to rough set theory, there are redundant attributes in the information table. Therefore we reduce original attribute set to a more succinct but sufficient one including attributes \(a_{12}, a_{11}, a_{22}, a_{31}, a_{32}, a_{34}, a_{41}\) (Table 2)
Then the equivalent classes by reducing each attribute are calculated as follows:

\[
\begin{align*}
U/\text{IND}(P) &= \{\{x_1, x_3, x_5, x_{10}\}, \{x_2, x_4, x_9\}, \{x_6, x_7, x_8\}\} \\
U/\text{IND}(A) &= \{\{x_1\}, \{x_2, x_3\}, \{x_4, x_5\}, \{x_6, x_7, x_8\}, \{x_9, x_{10}\}\} \\
U/\text{IND}(A - a_{12}) &= \{\{x_1\}, \{x_2, x_3, x_5, x_6, x_7, x_8, x_9, x_{10}\}\} \\
U/\text{IND}(A - a_{21}) &= \{\{x_1\}, \{x_2, x_3, x_4, x_5, x_6, x_7, x_9, x_{10}\}\} \\
U/\text{IND}(A - a_{22}) &= \{\{x_1\}, \{x_2, x_3, x_4, x_5, x_6, x_7, x_9, x_{10}\}\} \\
U/\text{IND}(A - a_{31}) &= \{\{x_1\}, \{x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{10}\}\} \\
U/\text{IND}(A - a_{32}) &= \{\{x_1\}, \{x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{10}\}\} \\
U/\text{IND}(A - a_{34}) &= \{\{x_1, x_{10}\}, \{x_2\}, \{x_3, x_4, x_5, x_6, x_7, x_8, x_{10}\}\} \\
U/\text{IND}(A - a_{41}) &= \{\{x_1, x_3\}, \{x_2\}, \{x_4, x_5, x_6, x_7, x_8, x_{10}\}\}
\end{align*}
\]

Based on the above, the attribute \(a_{12}\) and \(a_{21}\) is redundant. And the core of set \(A\) is showed as:

\[
\text{core}(A) = \{a_{22}, a_{31}, a_{32}, a_{34}, a_{41}\}
\]

Then the weight of each attribute can be calculated:

\[
k_{ij} = \gamma_A(P) - \gamma_{A-x_{ij}}(P) = \frac{|\text{pos}_A(P)| - |\text{pos}_{A-x_{ij}}(P)|}{|U|}
\]

\[
c_{ij} = \frac{k_{ij}}{\sum k_{ij}}
\]

\[
c_{22} = 0.2, c_{31} = 0.2, c_{32} = 0.2, c_{34} = 0.4, c_{41} = 0.2
\]

**Calculated Based AHP.** Ten pair-wise comparisons were taken on 5 criteria consist of \(a_{22}, a_{31}, a_{32}, a_{34}, a_{41}\). The AHP scale is adopted as follows: 1- equal importance, 3- moderate importance, 5- strong importance, 7- very strong importance, 9- extreme importance (2,4,6,8 values in-between). Thus, the decision matrix was constructed as follows:
The principal eigen value of decision matrix is 5.262. The resulting weights for the criteria based on pair-wise comparisons is \( a_{22} = 0.253, a_{31} = 0.126, a_{32} = 0.142, a_{34} = 0.414, a_{41} = 0.066 \). Consistency Ratio CR is 5.8%, so the consistency requirement is satisfied. If we choose the ratio \( u_i = 0.5 \). Then the weights of the five criteria are (0.2265, 0.163, 0.171, 0.407, 0.133).

Conclusion

The supplier selection is a big challenge in supply chain management. The article proposed a model combining the subjective assessment with objective data on the basis of AHP and rough set theory, thus to gain a comprehensive consideration when making supplier selection decision. The ratio between subjectivity and objectivity is deserved to have a further research in the future.

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References
