

## Research of Cracking Moment in Negative Moment Area of the Steel-Concrete Continuous Beam

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**Abstract.** The cracking moment in negative moment area of the composite beam can solved by analyzing the strain difference between the steel beam and the concrete. The influence of shear slip between the steel beam and the concrete was considered to control cracks of the concrete more accurately. Verifying the feasibility of the approach which was put forward in the paper by comparing caculation results and experimental results.

### Introduction

Steel-concrete Composite Structure gives full play to the material properties of steel stretching resistance and concrete compression strength. It has the following good features: high bearing capacity, high stiffness, good seismic performance and dynamic performance, small component cross section size and fast construction. When Continuous Composite Beams are used, disadvantage conditions occur like:

Steel pressure to negative moment and tension to concrete flange plate, which leads to the cracks of the concrete slabs. With the cracks of the concrete slabs, composite beam stiffness will decrease, the stud and steel are easy to corrode and the durability of the bridge will reduce. Therefore, how to prevent concrete crack in the negative moment region and control the crack width of cracks become the key problems of bridge design of continuous composite beams. In the calculation of steel and concrete composite structure design, steel and concrete interface shear slip is usually ignored. Thus the conclusion interaction between each other is made. In fact, steel - concrete interface shear slip belongs to the partly interaction. Based on the elastic theory, the shearing slip under the reverse concentrated loads of the steel - concrete composite beams is analyzed in this paper. And the cracking moment of the negative bending regions is solved by differential strain. Compared with experimental data, the result that Negative bending moment the cracking moment can be solved by the strain difference of analysis of steel - concrete composite structure shear slip is believable.

### Theoretical

Differential strain calculation under simply supported composite beams reverse concentrated loads

**Equations.** Calculation of internal force of section

Composite sectional drawing under concentrated loads(fig.1) Cross section strain relationship(fig. 2)

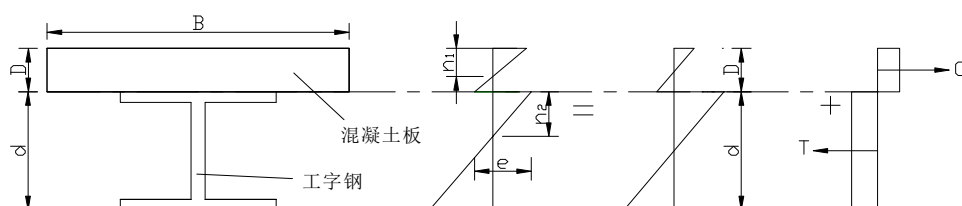


Figure 1. Section diagram of composite beam      Figure 2. Section strain diagram  
Internal force of section can be calculated by strain relationship in fig. 2:

Axial force of concrete slab:

$$C = kE_c A_c \left(n_1 - \frac{D}{2}\right) \quad (1)$$

$$T = kE_s A_s \left(\frac{d}{2} - n_2\right) \quad (2)$$

Axial force of steel beam :

Cross section bending moment :

$$M = k(E_c I_c + E_s I_s) + C\left(\frac{D}{2} + \frac{d}{2}\right) \quad (3)$$

Differential strain :

$$e = k(D + n_2 - n_1) \quad (4)$$

Where:

$k$  cross section curvature;

$E_c$ 、 $A_c$  the elastic modulus, area of concrete;

$E_s$ 、 $A_s$  the elastic modulus, area of steel beam.

Because  $C = T$  :

$$E_c A_c \left(n_1 - \frac{D}{2}\right) = E_s A_s \left(\frac{d}{2} - n_2\right) \quad (5)$$

Considering 1 and 5 :

$$k = \frac{C}{E_c A_c \left(n_1 - \frac{D}{2}\right)} \quad (6)$$

Considering 4、5 and 6:

$$n_1 - \frac{D}{2} = \frac{D + d}{2} \left(1 + \frac{E_c A_c}{E_s A_s} + e \cdot \frac{E_c A_c}{C}\right) \quad (7)$$

Considering 6、7、8 and 3:

$$\begin{aligned} M &= \frac{2C}{E_c A_c (D + d)} \left(1 + \frac{E_c A_c}{E_s A_s} + e \cdot \frac{E_c A_c}{C}\right) \cdot (E_c I_c + E_s I_s) + C \cdot \frac{D + d}{2} \\ &= \frac{C}{2} \left( \frac{(D + d)^2 E_c A_c E_s A_s + 4(E_c I_c + E_s I_s)(E_c A_c + E_s A_s)}{(D + d) E_c A_c E_s A_s} \right) \\ &\quad + 2e \left( \frac{E_c I_c + E_s I_s}{D + d} \right) \end{aligned} \quad (8)$$

The shear stiffness of stud shear force  $q$  and slip:

$$q = K \cdot S / l_a \quad (9)$$

Where:

$s$ : the amount of shear slip;

$l_a$ : shearing bolt spacing;

$K$ : bolt shear stiffness.

Stiffness calculation formula of the stud connectors from J.G .Nie. Tsinghua university :

$$K = 0.66 \times n_s \times V_u^{1/3}$$

Where:

$n_s$  : the columns number of the stud along the longitudinal axial beam

$$V_u = 0.43 \times A_s \times \sqrt{E_c \times f_{ck}};$$

$f_{ck}$  :cylinder compressive strength;

$A_s$  : the stud area;

$E_c$  :the concrete elastic modulus;

Composite strain difference calculation under vertical concentrated load

The strain difference :

$$e = \frac{ds}{dx} = \frac{l_a dq}{K dx} \quad (10)$$

The stress state of the steel - concrete composite beams in the interface: infinitesimal body diagram

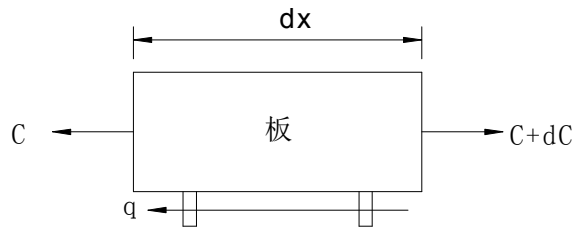


Figure 3. Composite beam force state sketch of interface

$$q dx + dC = 0 \quad (11)$$

$$q = -\frac{dC}{dx}$$

From which we have

$$\frac{dq}{dx} = -\frac{d^2 C}{dx^2} \quad (12)$$

Considering 12 、 10 and 8 gives

$$\frac{M}{\alpha} = \beta^2 C - C'' \quad (13)$$

Where:

$$\alpha = 2l_a \left( \frac{E_c I_c + E_s I_s}{D + d} \right) / K$$

$$\beta^2 = \frac{K}{4l_a (E_c I_c + E_s I_s)} \left( \frac{(D + d)^2 E_c A_c E_s A_s + 4(E_c I_c + E_s I_s)(E_c A_c + E_s A_s)}{E_c A_c E_s A_s} \right)$$

Under vertical concentrated load:



Fig.4 Composite beam under vertical concentrated load

$$C = C_1 e^{\beta x} + C_2 e^{-\beta x} + \frac{M}{\beta^2 \alpha} \quad (14)$$

Load boundary conditions of steel - concrete composite beams Under the action of concentrated

$$\text{is: } \begin{cases} x = 0, c = 0 \\ x = l, c = 0 \end{cases}$$

Take the boundary conditions to (14) :

$$\text{Come to the conclusion } \begin{cases} 0 = C_1 + C_2 + \frac{M}{\beta^2 \alpha} \\ 0 = C_1 e^{\beta l} + C_2 e^{-\beta l} + \frac{M}{\beta^2 \alpha} \end{cases}$$

$$\text{As: } C_1 = \frac{(1 - e^{-\beta l})}{e^{\beta l} - e^{-\beta l}} \frac{M}{\beta^2 \alpha}$$

$$C_2 = \frac{(e^{\beta l} - 1)}{e^{\beta l} - e^{-\beta l}} \frac{M}{\beta^2 \alpha}$$

Considering  $x = \frac{1}{2}l$ ,  $C_1, C_2$  as 14:

$$C = C_1 e^{\frac{\beta l}{2}} + C_2 e^{-\frac{\beta l}{2}} + \frac{M}{\beta^2 \alpha} \quad (15)$$

In the calculation, the area of the steel can be solved by the area of the concrete which is solved by transformed section method. When tensile stress on the surface of the concrete slab reach  $\varepsilon_t$ , the conclusion can be made that concrete slab cracks and the tensile strain is the cracking of concrete strain.

It can also be showed :

$$\varepsilon_t = kn_1 \quad (16)$$

The bending moment at this time is the cracking moment.

## Result

Comparing the results by calculating in this paper and test results (Table. 1) with the measured results of three experimental beams, calculation results and the measured results are almost the same. The result is shown below:

Table.1 cracking moment

beam number	type	$M_1$	$M_2$	$M_1 / M_2$
A	negative bending beam	37.6	36.1	1.04
B	negative bending beam	42.6	40.3	1.06
C	negative bending beam	49.3	46.7	1.06

## Conclusion

According to the calculating analysis of cross section internal force and composite interface strain, design formula of the cracking moment in the negative moment region of continuous composite beams is deduced. Calculation results and the measured results are almost the same by test. This method can be used for composite structure design and reference of construction.

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