

Research on Double Inverted Pendulum System Base on Hierarchical Fuzzy Sliding-mode Decoupling Control Method

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Introduction

Abstract. Hierarchical fuzzy sliding-mode decoupling control method is adapted in this paper, and the nonlinear system of a double inverted pendulum is decoupled into several subsystems, an adaptive law based on the Lyapunov function is used to tune the coupling factor of the hierarchical sliding-mode controller, a favorable decoupling performance is achieved under the condition of the system guaranteed stability by simulation experiments. Simulations are performed and demonstrated the feasibility of the proposed design method, which has been more predominant than adopting conventional fuzzy sliding-mode decoupling control in decoupling performance.

Introduction

Research has important practical significance which in the uncertain condition nonlinear system control problem. Sliding-mode control is the effective method to solve the problem of nonlinear, but there are sliding mode switching litter problem^{[1][2]}; Fuzzy sliding-mode control can reduce jitter, but sometimes can appear unstable phenomenon for the strong coupling system^[3]. In this paper, combining with the research puts forward the hierarchical fuzzy sliding mode design method of decoupling control. First, a class of nonlinear coupled system is decomposed into several subsystems, and define sliding surface for each subsystem. Then, using hierarchical fuzzy model controller to control the whole system, will not only ensure the decoupling system performance, but also has good stability and robustness characteristics.

Double Inverted Pendulum System Model

In this study, the control device is Googol Technology Company's linear double pendulum system. Linear double pendulum's schematic diagram as shown in fig. 1.

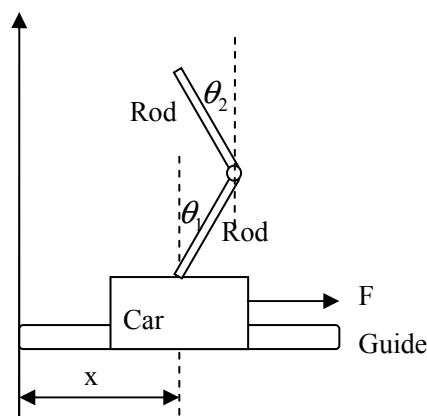


Figure 1. Structure of double inverted pendulum

It consists of guide rail, car, and two pendulums. A torque motor at one end of the guide rail, motor drive the car slides along the guide rail. Double inverted pendulum control system in the servo motor; pendulum rod 1 and pendulum pod 2 are installed on the optical encoder. Control of the double pendulum need 6 state variables, respectively are the pendulum position, pendulum speed, pendulum rod 1 position, pendulum rod 1 speed, pendulum rod 2 position, pendulum rod 2 speed. Obtained by the differential equation computing pendulum speed, a pendulum rod 1 and rod 2 speed. Then use state feedback to control the inverted pendulum. Car and two pendulum rod force is transmitted via a hinge heritability, the final steady-state performance of the system will not fall or sudden shock divergence^[4].

Hierarchical Fuzzy Sliding-mode Control Design

Single-input Multi-output Nonlinear System State Description. Generally the single-input multi-output nonlinear coupled system can be described as^[5]:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = g_1(x_1, x_2) + b(x_1, x_2)u(t) + \zeta_1(t) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = g_2(x_1, x_2) + b_2(x_1, x_2) + \zeta_2(t) \\ y(t) = c[x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T \end{cases} \quad (1)$$

Where $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ are the state variables; $g_1(x_1, x_2)$, $g_2(x_1, x_2)$, $b_1(x_1, x_2)$ and $b_2(x_1, x_2)$ are limited in the name of the nonlinear; $u(t)$ is the control input; $\zeta_1(t)$, $\zeta_2(t)$ are the comprehensive interference, and assume that they are bounded. Using hierarchical sliding mode control, define its control of the sliding-mode surface^[6] expressed as:

$$\begin{aligned} s_1 &= \dot{x}_1 + \lambda_1 x_1 = x_2 + \lambda_1 x_1 \\ s_2 &= \dot{x}_3 + \lambda_2 x_3 = x_4 + \lambda_2 x_3 \end{aligned} \quad (2)$$

Where, λ_1 and λ_2 are positives constant; by Eq.1 and Eq.2, according to the sliding-mode control condition of $\dot{s} = 0$, we can get by (x_1, x_2) and (x_3, x_4) composed of independent control law.

Hierarchical Fuzzy Sliding Mode Control. For the above-mentioned system, the decoupled sliding-mode surface is defined as^[7]:

$$s_r = s_1 - k^* s_2 \quad (3)$$

Where, k^* is the best coupling coefficient, its reaction coupling effect between the sliding-mode surfaces s_1 and s_2 . In order to improve the decoupling performance of the coupled system, can be estimated by fuzzy inference k^* .

Fuzzy Inference System:

R_i : If $s_r = A_{i1}$ and $s_r = A_{i2}$, then $k_{fx} = H$. Where, R_i is the i -th fuzzy rule $i=1, 2, \dots, m$; H_i is described by the single adjustable volume ϕ_i fuzzy evaluation; A_{ij} by the j input variables and i fuzzy rules to determine, and described by the membership function $\mu_{A_{ij}}(\cdot)$, Choose

$$\mu_{A_{ij}}(x_j) = e^{\ln(0.5)(w_s x_j - w_c)^2 w_d^2} \quad (4)$$

W_c is the Gaussian membership function center parameter value, W_d is the reciprocal from the center of this value, W_s used to measure x_{ij} .

Because k^* is positive, so the evaluation value \hat{k}_{fz} is also limited to a positive number, can be written as:

$$\hat{k}_{fz}(s_r, \dot{s}_r, \hat{\phi}) = \psi^T \hat{\phi} \quad (5)$$

Where, $\hat{\phi} = [\hat{\phi}_1 \hat{\phi}_2 \cdots \hat{\phi}_m]^T$ is the parameter vector; $\psi = [\psi_1 \psi_2 \cdots \psi_m]^T$ is regression parameter vector, and ψ is defined as:

$$\psi_i = \frac{\zeta_i}{\sum_{i=1}^m \zeta_i} \quad (6)$$

Where, $\zeta_i = \mu_{A_{ij}}(s_r) \cdot \mu_{A_{ij}}(\dot{s}_r)$ is the i-th fuzzy rule evaluation level; $\mu_{A_{ij}}(s_r)$ is the fuzzy membership function. According to the law of universal approximation^[5], then there is the optimal fuzzy logic system $k_{fz}^*(s_r, \dot{s}_r, \phi^*)$ as:

$$k_{fz}^*(s_r, \dot{s}_r, \phi^*) = \psi^T \phi^* \quad (7)$$

$$\phi^* = \arg \min_{\|\phi\| \in \Omega_\phi} \left\{ \sup_{s_r \in \Omega_r, \dot{s}_r \in \Omega_{\dot{r}}} [\hat{k}_{fz}(s_r, \dot{s}_r, \hat{\phi}) - k^*] \right\} \quad (8)$$

$$\text{Where, } \begin{cases} \Omega_\phi = \{\hat{\phi} \mid \|\hat{\phi}\| \leq M_\phi\} \\ \Omega_r = \{s_r \mid |s_r| \leq M_r\} \\ \Omega_{\dot{r}} = \{\dot{s}_r \mid |\dot{s}_r| \leq M_{\dot{r}}\} \end{cases} ; \quad M_\phi, M_r, M_{\dot{r}} \text{ are determined positive constants by the designer.}$$

The best parameter vector of the estimation error is:

$$\eta = \hat{\phi} - \phi^* \quad (9)$$

Define the Lyapunov Function:

$$V(s_r, \eta) = \frac{1}{2} s_r^2 + \frac{1}{2\alpha} \eta^T \eta \quad (10)$$

Where, α is a positive constant. According to the stability condition choose η . The parameter vector is retained in the interior of the constraint condition.

Define the function $E(t) \equiv (\varepsilon - \xi_{\max}) s_r(t) \leq -\dot{V}(t)$, obtain as:

$$\int_0^t E(\tau) d\tau \leq V(s_r(0), \eta(0)) - V(s_r(t), \eta(t)) \quad (11)$$

Formula Eq.9 to the right is limited, so as: $\lim_{t \rightarrow \infty} \int_0^t E(\tau) d\tau < \infty$. Because $\dot{E}(t)$ is limited, so $E(t)$ continuously. Use Barbalat lemma, can be obtained: $\lim_{t \rightarrow \infty} E(t) = 0$, so $s_r(t) \rightarrow 0$.

So, we can get control rule:

$$u = u_{eg} + u_n = (\hat{k}_{fz} \dot{s}_2 - \varepsilon_1 \operatorname{sgn}(\xi_r) - \delta \operatorname{sgn}(\xi_2 s_r) \dot{s}_2 - g_1 - \lambda_4 x_2) / b_1 \quad (12)$$

Simulation Results

Linear double inverted pendulum system selection as follows: $m_1 = m_2 = m = 1 \text{ kg}$, pendulum length are 0.12 m , $g = 9.8 \text{ m/s}^2$, $\xi = 0.05^\circ / \text{s}^2$, Gaussian membership function center parameter value

$w_c = [-1.5, -0.5, 0, 0.5, 1.5]$, set $w_s = 1$ and $w_d = 0.5$. The initial value of $\hat{\phi}_i$ and $\dot{\hat{\phi}}_i$ are 0.1, the initial condition are $x_1 = 30^\circ$, $x_2 = 0^\circ/s$, $x_3 = 10^\circ$, $x_4 = 0^\circ/s$, $x_5 = 0m$, $x_6 = 0m/s$, $i = 1, 2, \dots, 25$. For hierarchical fuzzy sliding-mode control, when $\lambda_1 = 5.5$, $\lambda_2 = 0.5$ and $\lambda_3 = 0.1$, to establish three hierarchical sliding-mode surfaces s_1 , s_2 , s_3 . When $\alpha = \bar{\alpha} = 0.05$, $\delta_1 = \delta_2 = 0.5$ and $\varepsilon_1 = 5$, adjust the optimum coupling coefficient \hat{k}_{fz1} and \hat{k}_{fz2} . When the value of $\hat{k}_{fz1} = 0.017$ and $\hat{k}_{fz2} = 0.672$, hierarchical fuzzy sliding-mode decoupling control and fuzzy sliding-mode decoupling control of the simulation results are shown in fig. 2. Simulation results show that by using hierarchical fuzzy sliding-mode decoupling control, all of the states assemble in zero places has the good performance of the system.

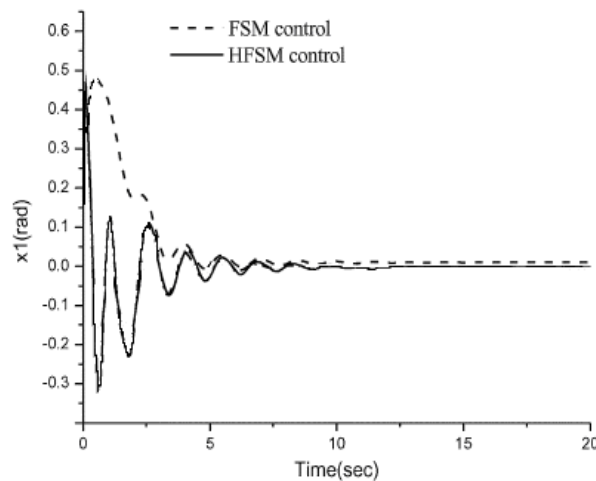


Figure 2. Angle1 curve of double inverted pendulum

Conclusion

Hierarchical fuzzy sliding mode control of single-input multiple-output decoupling of nonlinear coupled system provides a decoupling control method. This paper is based on Lyapunov function, constitute the hierarchical fuzzy sliding-mode controller. By adjusting the decoupling coefficient to adapt to the rules, on the premise of guarantee the stability of system, improve the coupling nonlinear system decoupling performance. For double inverted pendulum system, the simulation results show that the control method is feasible and has good control performance. And for general nonlinear coupling system research provides a new theoretical method.

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