

Satellite Attitude Control with Actuator Failure

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Abstract. A method of attitude stabilization control, based on observer design is proposed for an on-orbiting spacecraft in the presence of partial loss of actuator effectiveness, external disturbance and actuator control input saturation problem. In this approach, observer is employed to estimate the value of actuator fault, and a Backstepping attitude controller is then designed to achieve fault tolerant control and external disturbance rejection. The Lyapunov stability analysis shows that the closed-loop attitude system is guaranteed to be almost asymptotically stable. The simulation results show the effectiveness of the derived control law.

Introduction

It is inevitable that the satellite components is in trouble, so it is one of the hot spots in the research of attitude control to design the control algorithm to achieve the fault tolerant control. A spacecraft attitude compensation controller is designed using adaptive control method in literature [13]. In literature [14], a fault tolerant attitude control strategy is proposed for the non singular terminal sliding mode control method. In literature [9], a passive fault tolerant controller is designed based on time delay method. The attitude tracking control can be realized in the case of 4 flywheels, but the controller must obtain the fault information of the actuators. Application of two stage Kalman filtering algorithm in literature [11]. The Backstepping control method has many advantages such as the decomposition of the complex nonlinear system into many subsystems and the simple design procedure of the controller. Therefore the backstepping can be used to design the attitude controller. Considering the problem of actuator failure, external disturbance and actuator control input constraint in this paper, a method of attitude stabilization control is proposed to achieve high precision and high stability attitude stabilization control.

Mathematical Model of the Satellite

For description method using Euler angles of satellite attitude exists the singular problems, this paper adopts MRPs describe the satellite attitude, and then the rigid satellite attitude mathematical model can be described by:

$$\begin{aligned}\dot{\sigma} &= \frac{1}{4} \left[(1 - \sigma^T \sigma) I_3 + 2\sigma^\times + 2\sigma^T \sigma \right] \omega \\ &= F(\sigma) \omega\end{aligned}\quad (1)$$

$$J \dot{\omega} + \omega^\times J \omega = \tau + d \quad (2)$$

where $\omega = [\omega_1 \ \omega_2 \ \omega_3]^T \in R^3$ is that coordinate system relative to the inertial coordinate system of angular velocity to project on the body coordinate system, $\tau = [\tau_1 \ \tau_2 \ \tau_3]^T \in R^3$ is the total control moment that is actually applied to satellite coordinate system, $d = [d_1 \ d_2 \ d_3]^T \in R^3$ is external disturbs matrix of the satellite. Matrix $J \in R^{3 \times 3}$ (positive definite and symmetric) is the moment of inertia of

the satellite array. In particular, Ep.(1) is the satellite attitude kinematics, and Ep. (2) is a rigid satellite attitude dynamics, and all of them is satisfied with Assumption 1.

Assumption 1: Assumes that the external disturbance d is bounded, and there is constant $d_{\max} > 0$ to make $\|d\| \leq d_{\max}$ to be established.

Considering actuator partial failure and the fault is modeled as a multiplicative factor, so the actuator failure acting on the total control torque of the satellite ontology is as follows:

$$\tau = \rho(t)\tau_c \quad (3)$$

where $\tau_c = [\tau_{c1} \ \tau_{c2} \ \tau_{c3}]^T \in R^3$ is actuator command control torque, $\rho(t) = \text{diag}([\rho_1(t) \ \rho_2(t) \ \rho_3(t)]^T) \in R^{3 \times 3}$ is actuator fault degree, $0 < \rho_i(t) \leq 1, i = 1, 2, 3$, and where $\rho_i(t) = 1$ and $\tau_i = \tau_{ci}$, actuator normal operation and control torque is consistent with the actual torque; If $0 < \rho_i(t) < 1$, $|\tau_i| = \rho_i(t)|\tau_{ci}| < |\tau_{ci}|$, actuator partial failure.

Controller Design

In this approach, observer is employed to estimate the value of actuator faults, and a Backstepping attitude controller is then designed.

Observer Design. Because the fault factor $\rho(t)$ is a diagonal matrix, Eq. (3) can be rewritten by

$$\rho(t)\tau_c = U\rho(t) \quad (4)$$

where $U = \text{diag}([\tau_{c1}(t) \ \tau_{c2}(t) \ \tau_{c3}(t)]^T) \in R^{3 \times 3}$, $\rho(t) = [\rho_1(t) \ \rho_2(t) \ \rho_3(t)]^T$, the satellite state dynamics Eq. (2) can be rewritten in the form of partial failure Eq.(3)

$$J\dot{\omega} = -\omega^\times J\omega + U\rho(t) + d \quad (5)$$

According to the attitude dynamics of spacecraft actuator fault Eq.(5), the design follows the observer to estimate the fault $\rho(t)$.

$$\begin{aligned} J\dot{\hat{\omega}} &= -\hat{\omega}^\times J\hat{\omega} + U\hat{\rho}(t) \\ &\quad -\Gamma(\hat{\omega} - \omega) - l_1 \text{sgn}(\hat{\omega} - \omega) \end{aligned} \quad (6)$$

$$\hat{\rho}(t) = l_2 \hat{\rho}(t - T) + l_3 (\hat{\omega} - \omega) \quad (7)$$

where $\hat{\omega}$ and $\hat{\rho}(t)$ are the respectively estimated value of ω and $\rho(t)$. $T \in R$ is the update time of the observer. $\Gamma \in R^{3 \times 3}$, $l_i \in R_+$ is Observer gain. In order to evaluate the performance of the observer Eq.(6) - Eq.(7) fault estimation, the energy index I_p is defined.

$$I_p = \frac{1}{t} \int_0^t [\|e\|^2 + \|\sigma\|^2] dt \quad (8)$$

Attitude Stabilization Controller Design. When $|\tau_i| \leq \tau_{\max}$ ($i=1, 2, 3$), application of observer Eq.(6) - Eq.(7) can realize the accurate estimation of the failure part $\rho(t)$. Variable $x_1 = \int \sigma dt$, $x_2 = \sigma$ and $x_3 = \omega$. According to Eq.(1) and Eq.(5), there are

$$\dot{x}_1 = x_2 \quad (9)$$

$$\dot{x}_2 = F(x_2)x_3 \quad (10)$$

$$J\dot{x}_3 = -x_3^\times Jx_3 + \hat{\rho}(t)\tau_c - \hat{\delta}(t)\tau_c + d \quad (11)$$

$$\text{where } \hat{\rho}(t) = \text{diag}([\hat{\rho}_1 \ \hat{\rho}_2 \ \hat{\rho}_3]^\top), \quad \hat{\delta}(t) = \text{diag}([\delta_1 \ \delta_2 \ \delta_3]^\top).$$

From the Eq.(9) to Eq.(11) shown in the form of the system structure, the standard Backstepping method can be used to design the controller.

To carry out the following transformation

$$z_1 = x_1 \quad z_2 = x_2 - \alpha_1 \quad z_3 = x_3 - \alpha_2 \quad (12)$$

Where $\alpha_1 \in R^3$ and $\alpha_2 \in R^3$ are the virtual control inputs for backstepping control. The following steps can be divided into the controller design

Step 1: According to the formula Eq.(12), available

$$\dot{z}_1 = \dot{x}_1 = x_2 = z_2 + \alpha_1 \quad (13)$$

Selected Lyapunov candidate function $V_1 = 0.5z_1^T z_1$ and design the virtual control Variable .where $\alpha_1 = -c_1 x_1$ and where c_1 is constant.

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T (z_2 - c_1 z_1) = -c_1 \|z_1\|^2 + z_1^T z_2 \quad (14)$$

So when $z_2 = 0$, $\dot{V}_1 = -c_1 \|z_1\|^2$, z_1 will be at the asymptotic convergence.

Step 2: according to the type of z_2 derivative Eq.(16) available.

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = F(x_2)x_3 + c_1 x_2 \quad (15)$$

Select another Lyapunov function $V_2 = V_1 + 0.5z_2^T z_2$ and α_2 for the design of virtual control input.

$$\alpha_2 = F^{-1}(x_2)(-z_1 - c_1 x_2 - c_2 z_2) \quad (16)$$

Where c_2 is constant. Apply (16), there are

$$\begin{aligned} \dot{V}_2 &= -c_1 \|z_1\|^2 + z_1^T z_2 + z_2^T [F(x_2)x_3 + c_1 x_2] \\ &= -c_1 \|z_1\|^2 - c_2 \|z_2\|^2 + z_2^T F(x_2)x_3 \end{aligned} \quad (17)$$

The same type from Eq.(17) to prove that when $z_3 = 0$, $\dot{V}_2 = -c_1 \|z_1\|^2 - c_2 \|z_2\|^2$. At this time z_1 and z_2 will converge.

Aiming at the fault spacecraft attitude control system Eq.(1), Eq.(5), the application of observer Eq.(6) design command control:

$$\tau_c = \text{Sat}(v, \tau_{\max}) \quad (18)$$

The controller τ_c input $v \in R^3$ is designed as

$$\begin{aligned}
 v(t) = & \hat{\rho}(t)^{-1} \left\{ J \left[\frac{dF^{-1}(x_2)}{dt} (z_1 + c_1 x_2 + c_2 z_2) \right. \right. \\
 & + F^{-1}(x_2) (\dot{z}_1 + c_1 \dot{x}_2 + c_2 \dot{z}_2)] \\
 & + x_3^\times J x_3 - c_3 z_3 - k_1 x_a - F^T(x_2) z_2 \\
 & \left. \left. - \frac{\chi_{Td}^2 \dot{z}_3}{\chi_{Td} \|z_3\| + \varepsilon \exp(-\beta t)} \right\} \quad (19)
 \end{aligned}$$

Where $\chi_{Td} = \tau_{\max} + d_{\max}$, $\varepsilon \in R_+$ is an arbitrary small constant. $k_1 \in R_+$, β and c_3 for control gain, $x_a \in R_+$ is the state variable of the following auxiliary system.

$$\begin{aligned}
 \dot{x}_a = & -k_2 x_a - \frac{\|\hat{\rho}(t)\|^2 \|\Delta u\|^2}{\|x_a\|^2} x_a - \hat{\rho}(t) \Delta u \\
 \Delta u = & \tau_c - v, \quad k_2 \in R_+. \quad \text{If} \quad (20)
 \end{aligned}$$

$$c_3 - 1 > 0, k_2 - \frac{k_1^2}{2} - \frac{1}{2} > 0 \quad (21)$$

The whole closed-loop attitude control system is stable.
Prove: By Eq.(11) and Eq.(16) available

$$\begin{aligned}
 \dot{z}_3 = & J^{-1} \left[-x_3^\times J x_3 + \rho(t) \tau_c + d \right] \\
 & + \frac{dF^{-1}(x_2)}{dt} (z_1 + c_1 x_2 + c_2 z_2) \\
 & + F^{-1}(x_2) (\dot{z}_1 + c_1 \dot{x}_2 + c_2 \dot{z}_2) \quad (22)
 \end{aligned}$$

If a Lyapunov candidate function is defined

$$V_3 = V_2 + \frac{1}{2} z_3^T J z_3 + \frac{1}{2} x_a^T x_a \quad (23)$$

The function of the time derivative of V_3 , and put Eq.(22) into

$$\begin{aligned}
 \dot{V}_3 = & -c_1 \|z_1\|^2 - c_2 \|z_2\|^2 - c_3 \|z_3\|^2 - k_2 \|x_a\|^2 \\
 & - \|\hat{\rho}(t)\|^2 \|\Delta u\|^2 - x_a^T \hat{\rho}(t) \Delta u \\
 & + z_3^T \{ \hat{\rho}(t) \Delta u - k_1 x_a - \hat{\delta}(t) \tau_c \\
 & + d - \frac{z_3 \chi_{Td}^2}{\|z_3\| \chi_{Td} + \varepsilon \exp(-\beta t)} \} \quad (24)
 \end{aligned}$$

According to the characteristic of saturation function, $\|-\hat{\delta}(t) \tau_c + d\| \leq \tau_{\max} + d_{\max}$. Can further prove

$$\begin{aligned}
 \dot{V}_3 \leq & -c_1 \|z_1\|^2 - c_2 \|z_2\|^2 - (c_3 - 1) \|z_3\|^2 \\
 & - \left(k_2 - \frac{k_1^2}{2} - \frac{1}{2} \right) \|x_a\|^2 + \varepsilon \exp(-\beta t) \\
 \leq & -\bar{m} \left(\|z_1\|^2 + \|z_2\|^2 + J_{\min} \|z_3\|^2 + \|x_a\|^2 \right) + \varepsilon \quad (25)
 \end{aligned}$$

Where $\bar{m} = \min \left\{ c_1, c_2, \frac{c_3 - 1}{J_{\max}}, k_2 - \frac{k_1^2}{2} - \frac{1}{2} \right\}$, $J_{\max} = \lambda_{\max}(J) > 0$,

According to the definition of V3,

$$\dot{V}_3 \leq -2\bar{m}V_3 + \varepsilon \quad (26)$$

From Eq.(26) to prove that V_3 is uniformly ultimately bounded

According to the formula Eq.(18) and

$$|\tau_i| = |\rho_i(t)| |\tau_{ci}| < |\tau_{ci}| \leq \tau_{\max} \quad (27)$$

The input limitation of satellite actuator can be solved.

Simulation

In order to verify the effectiveness of the proposed attitude control algorithm, the numerical simulation of attitude stabilization control for a rigid spacecraft is carried out in this paper. Satellite moment of inertia and constant failure respective as

$$J = \begin{bmatrix} 330 & 6 & 9 \\ 6 & 260 & 10 \\ 9 & 10 & 182 \end{bmatrix} kg / m^2 \quad (28)$$

$$\begin{cases} \rho_1(t) = 0.4, \text{ if } t \geq 8 \text{ sec} \\ \rho_2(t) = 0.5, \text{ if } t \geq 5 \text{ sec} \\ \rho_3(t) = 0.8, \text{ if } t \geq 10 \text{ sec} \end{cases} \quad (29)$$

The maximum torque of the actuator is $5N \cdot m$.
 $d = (\|\omega\|^2 + 0.05) [\sin 0.8t, \cos 0.5t, \cos 0.3t]^T N \cdot m$, The initial attitude of the satellite is $\sigma(0) = [-0.3 - 0.4 - 0.2]^T$ and its initial angular velocity is $\omega(0) = 0 \text{ deg/sec}$. The controller Eq.(18) with a fault observer Eq.(6) - Eq.(7) parameter selects the value of these parameters as $l_1 = 50$, $l_2 = 0.2$, $l_3 = 2$, $\Gamma = \text{diag}([800 \ 850 \ 1000]^T)$, $\gamma = 10$, $c_1 = c_2 = 0.5$, $c_3 = 1.25$, $k_1 = 0.55$, $k_2 = 1.5$, $\varepsilon = 0.001$ and $\beta = 0.55$.

The following is the simulation results:

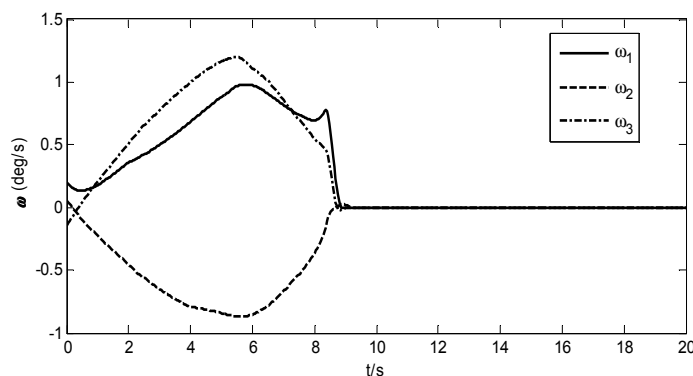


Figure 1. The velocity under the controller

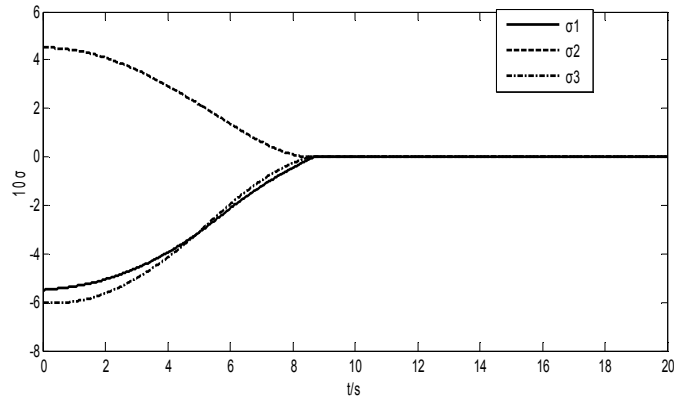


Figure 2. The attitude under the controller

The control method can realize attitude stabilization control. The observer Eq.(6) - Eq.(7) can accurately estimate the actuator fault in a short time, therefore the controller Eq.(18) is able to deal with the faults quickly and realize the attitude stabilization control.

Conclusion

The controller can realize the attitude stabilization control of the actuator and the external disturbance, and solve the problem of the actuator input limitation. But the design of the controller requires satellite attitude and angular velocity measurement information and can only handle single actuator faults.

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