How To Weaken Zika Virus?

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Abstract. This paper mainly focuses on providing a plan for using Wolbachia to influence the mosquito population with Zika Virus. In our SEIR Model we first obtain and solve a series of differential equations to predict the trend of Zika spreading, based on this model, we build a Wolbachia Spread In Mosquitoes Model to find how Wolbachia invades the population of mosquito. By building a series of delay differential equations, we successfully predict the Zika virus spreading trend after introducing modified mosquitoes. The results shows that at least 450000 modified mosquitoes are needed to meet the constraints.

1. Introduction
Zika virus is an arbovirus transmitted by Aedes aegypti mosquito, Its incubation period (from exposure to symptoms) is not clear, may be a few days\(^{[1]}\).Only about 20% of those infected with the virus will show mild symptoms.However, pregnant women are infected, often lead to severe infant defects(as shown in Figure 1).

![Fig. 1: How Zika virus spread by mosquitoes](image1)

![Fig. 2: How Wolbachia spread in mosquitoes](image2)

At present, there is no vaccine and prevention technology for the Zika virus, So in addition to man-made measures to reduce mosquito bites, mosquitoes carrying the Wolbachia bacterium, which growing in the laboratory\(^{[2]}\), release into the environment. It can reduce the population of mosquitoes and mosquitoes’ capacity to transmit Zika to humans(as shown in Figure 2).

2. Model Theory
2.1 The SEIR Model
In this model, human population of Puerto Rico is divided into six classes: susceptible (\(S_h(t)\)), exposed (\(E_h(t)\)), symptomatically infected (\(I_{h1}(t)\)), convalescent (\(I_{h2}(t)\)) and recovered (\(R_h(t)\)) \(^{[3]}\) at time \(t > 0\),and the mosquito population is divided into three parts: susceptible (\(S_v(t)\)), exposed (\(E_v(t)\)),and infectious (\(I_v(t)\)), respectively. We mark \(N_h\) as the total number of humans, \(N_v\) as the total mosquito population, so:

\[
N_h = S_h + E_h + I_h + R_h
\]  
(1.1)

If we assume \(N_v\) as the total mosquito population,there is:

\[
N_v = S_v + E_v + I_v
\]  
(1.2)
Both \( N_h \) and \( N_v \) are assumed to be constant. We use the SEI type of structure for mosquitoes and SEIR type of structure for humans. Based on the assumptions, the ZIKV transmission dynamics between humans and mosquitoes are governed by the following model equations:

\[
\frac{dS_h}{dt} = -ab \frac{I_v}{N_h} S_h - \beta \frac{\kappa E_h + I_h}{N_h} S_h \\
\frac{dE_h}{dt} = \theta \left( ab \frac{I_v}{N_h} S_h + \beta \frac{\kappa E_h + I_h}{N_h} S_h \right) - v_h E_h \\
\frac{dI_h}{dt} = v_h E_h - \gamma_h I_h \\
\frac{dR_h}{dt} = \gamma_h I_h \\
\frac{dS_v}{dt} = \mu_v N_v - ac \frac{\eta E_h + I_h}{N_h} S_v - \mu_v S_v \\
\frac{dE_v}{dt} = ac \frac{\eta E_h + I_h}{N_h} S_v - \left( v_v + \mu_v \right) E_v \\
\frac{dI_v}{dt} = v_v E_v - \mu_v I_v
\]

Where:
- \( a \) is the mosquito biting rate.
- \( b \) is the transmission probability from an mosquito to a susceptible human per bite.
- \( \beta \) is the transmission rate from symptomatically infected humans to susceptible humans.
- \( \kappa \) is the relative human-to-human transmissibility of exposed humans to symptomatic.
- \( \theta \) is the proportion of symptomatic infections.
- \( \eta \) is the relative human-to-mosquito transmission probability of exposed to infected.
- \( 1/\mu_v \) is the mosquito lifespan. (Days)

2.2 Results of the SEIR Model

To solve the further questions, we first predict how the numbers of the infected public (as shown in Figure 3) and pregnant women (as shown in Figure 4) value with time going.

![Fig. 3: Trend of the infected’s number](image1)
![Fig. 4: Trend of infected pregnant women’s number](image2)

From the figure above, we can see that the number of the infected rises fast under no control. Therefore, it’s necessary to introduce some "modified mosquitoes" to improve this situation.

2.3 Results of the SEIR Model

In our model, we consider the dynamics of reproductive adult mosquitoes. Let \( R_F(t) \) denote the number of released females at time \( t \), and \( R_M(t) \) the number of released males, both infected. We
assume that the decay rate for either $R_F(t)$ or $R_M(t)$ increases with its own size and the total population size $N_v(t)$ due to strong competition between adults. Let $\delta_j$ be the decay rate constant. Then

\[
\frac{dR_F(t)}{dt} = -\delta_j R_F(t) N_v(t) \tag{2.1}
\]

\[
\frac{dR_M(t)}{dt} = -\delta_j R_M(t) N_v(t) \tag{2.2}
\]

Let $R_F(t)$ and $R_M(t)$ denote the numbers of uninfected reproductive females and males, $R_F(t)$ and $R_M(t)$ denote the numbers of infected reproductive females and males other than those from releasing, respectively. Then the total number $N_v(t)$ has the decomposition

\[
N_v(t) = I_F(t) + I_M(t) + U_F(t) + U_M(t) + R_F(t) + R_M(t) \tag{2.3}
\]

The same, mosquitoes that are susceptible become mild after being infected by the Wolbachia.

\[
S_v(t) = \left( \mu_v N_v - ac \frac{E_h}{N_h} S_v - \mu_s S_v \right) \frac{U_F(t) + U_M(t) + R_F(t) + R_M(t)}{N_v(t)} \tag{2.4}
\]

\[
E_v(t) = \left( ac \frac{E_h}{N_h} S_v - (v) \mu_t S_v \right) \frac{U_F(t) + U_M(t) + R_F(t) + R_M(t)}{N_v(t)} \tag{2.5}
\]

To meet the constraints, we analysis how the ratio of Wolbachia-infected mosquitoes change while the take $\tau$ different values.

In the figure 5 above, the blue, orange and green curve represents the values of $\tau$, the ratio of mW/fW, they are respectively 20%, 40%, 60%, 80%, 100%. In the picture, we find that the ratio of Wolbachia-infected mosquitoes rise fastest when the $\tau$ reach 1. Therefore, we can combine the parameters blow and the differential equations above to predict the trend that Zika spread in the Puerto Rico. (As shown in picture 6)
3. Conclusion

According to the message, as of right now, the number of newly infected Puerto Ricans is estimated to be 600 per day, including 22 pregnant women, then, we can reach the total number of Zika infected public after 5 years.

References