The Design of Toll Plaza

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Keywords: Highway, Target programming model

Abstract. Highway is an important transportation infrastructure. In the article I use the target programming model to design a better toll plaza. Consider a toll highway having L lanes of travel in each direction and a barrier toll containing B tollbooths (B > L) in each direction. Meanwhile important considerations included in my model are the toll plaza area, the throughput (number of vehicles per hour passing the point where the end of the plaza joins the L outgoing traffic lanes) and the accident prevention. From my design, it can find that, when I assume L = 3, B = 8, the toll plaza area is 9780.32 square meter.

1. Introduction

Expressway[1] is an important transportation infrastructure, which plays an important role in coordinating urban and rural development and so on. However, along with people’s higher pursuit of transportation speed and quality, the capacity of expressway toll station often can’t match with the capacity of expressway, which leads to frequent congestion at the toll station, even leads the main road traffic disruption. Therefore, our task is to establish a model, through the calculation, to design the charging area, so that the shape and size more reasonable and it has higher vehicle throughput, all these under the premise lower construction costs.

2. The design of the solution

2.1 Selection of design index

Through the analysis for the requirements of the subject, I design the toll plaza to consider the factors including accident prevention, throughput and cost. The cost of the toll plaza is directly proportional to the size of area. The design[2] is related to the vehicle "fan-out" square transition angle, throughput, accident prevention, the arrival rate of vehicles and let the line rate, so we choose the square gradual change rate \( p \) and square area \( S \) two quantitative indicators to determine the shape of the toll plaza. In the analysis of the problem, we assume \( L = 3, B = 8 \).

2.2 Analysis of design index

2.2.1 Toll plaza gradual change rate

The entrance of the toll plaza is the role of the transition section to guide the vehicle naturally and smoothly into and out of the toll plaza, the gradual change rate requires gently vary, in order to easy to drive. Otherwise it’s easy to make drivers result in an offset in the process of driving and result in improper driving operations so it can endanger the traffic Safety. Toll plaza gradual change rate \( p \) is the ratio of the width of the transition section of the main entrance to the length of the transition section, which

\[
p = \frac{B - L}{Q}
\]

The gradient section of toll plaza’s exit and entrance are used to guide the vehicle naturally into and out of the toll plaza, which is required to be as smooth as possible so that the vehicles are able to merge into the regular travel lanes more conveniently.

The relationship between the gradual change rate of toll plaza and traffic accidents is shown as the following table:
Table 1 The relationship between expressway toll plaza gradual change rate and the traffic accidents

<table>
<thead>
<tr>
<th>Number of accidents/time</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>11</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toll square gradual change rate</td>
<td>0.143</td>
<td>0.165</td>
<td>0.201</td>
<td>0.250</td>
<td>0.250</td>
<td>0.339</td>
</tr>
</tbody>
</table>

Using MATLAB to fit, get the following graph curve:

![Graph showing the relationship between toll plaza gradient rate and traffic accidents.](image)

Figure 1 The relationship between the expressway toll plaza gradual change rate and the traffic accident curve

According to Figure 1, along with the increment of the gradient rate, the traffic accidents will increase, which compromise the toll plaza. By processing the data with regression analysis, obtain the correlation model between the toll plaza’s gradient rate and the number of traffic accidents.

\[ Y = 1.4231e^{0.0664\rho} \]  

(2)

In this model, the correlation coefficient \( R^2 = 0.9938 \), showing that the model has a high correlation. According to the model, traffic accident rate is proportional to the gradient rate of toll plaza. Insufficient length of gradient session means high gradient rate which leads to the lack of sight distance. Under the circumstances, it cannot meet the safety requirements for drivers to change lanes thus leading to traffic accidents.

2.2.2 Area of the toll plaza

![Original toll plaza plan and designed toll plaza plan.](image)

The original toll plaza plan and the toll plaza we designed as shown above, according to the China Transportation Group data show that the corresponding size of the figure are \( L = 3, B = 8, x_1 = 3.2 \text{meter}, x_2 = 4.4 \text{meter}, Y = 100 \text{meter}, \tan \alpha = \frac{1}{6} \).

Next, we calculate the most value of original toll plaza area, the border value of the toll plaza in the case show as follows:
Figure 4 toll plaza area under the floor plan

Figure 5 toll plaza area under the maximum floor plan

Figure 4 is described in the case of the minimum toll plaza floor plan, according to the plan, we can calculate the original toll plaza minimum area $S_{\text{min}}$ for:

\[
S_{\text{min}} = x_2 Y + x_1 \frac{x_2 - x_1}{\tan \alpha} \times 2 = 6469.12 \text{ m}^2
\]

(3)

Maximum area for: $S_{\text{max}} = x_2 \times \left( Y + \frac{x_2 - x_1}{\tan \alpha} \times 2 \right) = 45867.008 \text{ m}^2$

(4)

the toll plaza area we designed $S$:

\[
s = 6000.64 + 573.44 \times \frac{\sqrt{1 - P^2}}{P}
\]

(5)

Also, the minimum area of the toll plaza does not change with the change of $\beta$, that is, the area minimum of the toll plaza is fixed, it cannot less than $S_{\text{min}}$.

Taking the gradual change range of 0 to 1 in a limited number of points, using MATLAB simulation, the relationship between $S$ and $P$ as shown below:

Figure 6 Square area with the gradual change rate $P$ changes

As can be seen in the above figure, the area of the toll plaza is negative proportional to the gradient rate.

3 Design of the solution

Our goal is to satisfy the basic conditions, the square area and gradient rate as small as possible. Next, I use the target planning model\(^4\) to solve the shape and size of the toll plaza.

(1) Positive and negative deviation variables

Set up $\sum_{i=1}^{l} (i=1,2, \cdots, l)$ is the number of objective function, which the positive deviation variable $d_i$ max\{ $f_i$, $d_i^0$, 0\} indicates the part where the decision value exceeds the target value, negative deviation variable $d_i$ min\{ $f_i$, $d_i^0$, 0\} indicates the part where the decision value does not reach the target value. $d_i^0$ is indicated by the target value of $f_i$.

(2) Priority factor
This planning problem has two objectives, in order to achieve these two goals, there are primary and secondary points, the priority objective is given priority factor $p_1$, the subordinate objects are given priority factor $p_i$.

(3) Objective function of target planning

The objective function of the objective programming is constructed by the positive and negative deviation variables and given the corresponding priority factor which is constrained by each goal. There are three basic forms:

a. The number $i$ goal require that the target value be reached exactly, that is to say, the positive and negative deviation variables should be as small as possible. Then we can list the following relationship:

$$\min w_i d^-_i + w_i d^+_i$$

(6)

b. The number $i$ goal requires no more than the target value, that is to say, it is allowed the target value is not reached, but the positive deviation variable should be as small as possible. Then we can list the following relationship:

$$\min w_i d^+_i$$

(7)

c. The number $i$ goal requires more than the target value, that is to say, the amount of excess is not limited, but the negative deviation variable must be as small as possible. Then we can list the following relationship:

$$\min w_i d^-_i$$

(8)

4. Conclusions

Through the analysis of the model, I can establish the mathematical model of this problem:

$$\min p_1(d^+_1 + d^-_1) + p_2(d^+_2 + d^-_2)$$

(9)

$$S = x_2 \tan \alpha + \frac{(x_1 + x_2)(x_2 - x_1)}{2} \times \frac{1}{p} \sqrt{1 - \frac{p^2}{4}} \times \frac{1}{2}$$

s.t.

$$S \geq S_{\min} = 6469.12$$

$$d^-_i \geq 0, d^+_i \geq 0, S \geq 0, p \geq 0 (i = 1, 2)$$

(10)

Prioritization is determined by expert assessment, then I calculate the problem by MATLAB and obtain the optimal solution is $p = 0.15$. By substituting the formula, we can get $S = 9780.32$.

References


[3]. Wang Jing. Design and research of toll station in Hunan Expressway [D]. Central South University, 2006