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A Novel Damage Model of Fatigue Life Prediction for Metallic Structures

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Abstract. Fatigue failure is a mail form of engineering structures, especially for aircraft structures. Crack initiation and propagation threat to flight safety. So far, there has been a variety of methods in fatigue life prediction. Within them, damage mechanics method is a special engineering calculating method. Based on theory of damage mechanics, a new life prediction model for high circle fatigue of metal is proposed. The equation of damage evolution adopted here is concise and efficient, from which the formula for fatigue life prediction under general fatigue load could be given. Consequently, the damage parameter can be determined through standard fatigue experimental data. This new method simplifies the traditional method of life prediction, and reflects the performance of the material fatigue damage concisely and accurately.

1. Introduction

Fatigue failure is the most common form of failure in aerospace structures. Fatigue damage is apt to occur in the structures under long-term loadings. However, during structural service, structural fatigue damage is usually difficult to detect or monitor. Once the fatigue crack is formed, its subsequent expansion often exhibits an accelerated state, thus causing serious consequences [1-3]. At present, the traditional fatigue analysis methods have become more mature formed by experiment data and probability statistics [4, 5].

Conventional fatigue analysis often relies on a great deal of tests, especially the actual structural tests, requiring extensive experimental work, time and costs. Continuum damage mechanics is used to analysis the fatigue of structures or materials which based on the theory of elasticity combined with the law of material damage evolution [6, 7]. From the point of view of damage mechanics, both fatigue cracks initiation and its expansion can be classified as material damage, that is, damage initiation and evolution. When the damage extent reaches the critical value, the material fatigue failure will occur and loss of load capacity. Damage mechanics method provides theoretical guidance and practical analysis for structural life prediction and anti-fatigue optimization design [8-10]. It is reported that this method has also applied in solve some new problems such as impact defects and shape memory alloy fatigue [11, 12]. However, previous damage mechanics method has some disadvantages, such as too more parameters need to fitting, huge computing quantity, thus lead to inconvenience in engineering applications [13, 14].

This paper aims to establish a simple and efficient new model of damage mechanics, which abandon the previous evolution equation with threshold value, and to derive the general theoretical formula of fatigue life prediction. Finally, the parameters determination approach from the fatigue test data of the standard specimen was proposed.

2. Damage driving force and evolution equation

The damage extent was defined as:

$$D = \frac{E - E_D}{E} \tag{1}$$

Where, E is the Young's modulus of materials without damage, and E_D is the Young's modulus with damage. Fatigue damage causes local stiffness changing by micro-cracks forming. The damage extent D value ranges from 0 to 1. When D=0, the materials are perfect; when D=1, macro-cracks initial.

The constitutive equation of the material containing damage can be expressed as:

$$\varepsilon_{ij} = \frac{1}{1-D} C_{ijkl}\sigma_{kl} \tag{2}$$

Where, ε_{ij} is the strain component, σ_{kl} is the stress component, and C_{ijkl} is the fourth order flexible tensor.

For the isothermal process, the unit mass Gibbs free energy g will be:

$$g = \frac{\sigma_{ij}\varepsilon_{ij}}{2\rho} = \frac{C_{ijkl}\sigma_{kl}\sigma_{ij}}{2\rho(1-D)} \tag{3}$$

Where ρ is the material density, the damage driving force Y can be expressed as:

$$Y = -\rho\frac{\partial g}{\partial D} = \frac{C_{ijkl}\sigma_{kl}\sigma_{ij}}{2(1-D)^2} \tag{4}$$

The time-formed damage evolution equation can be written as:

$$\frac{dD}{dt} = bY^{\frac{m}{2}-1}\frac{dY}{dt} \tag{5}$$

Where, b and m are parameters to be determined by experiment data.

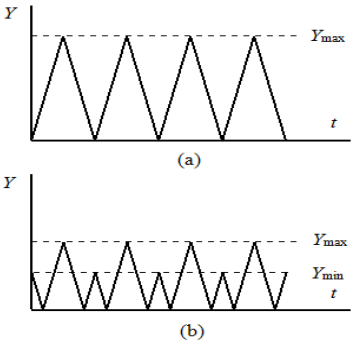

Fig. 1 The driving force under different loading mode

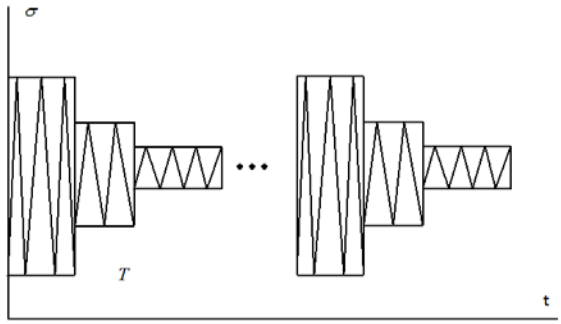

Fig. 2 periodically arranged block spectrum

From (4), the damage driving force Y is non-negative, so there are two cases of cyclic loading (shown in Fig. 1). The first case: the damage driving force has only one peak in one cycle (Fig. 1 (a)); while the second case: the damage driving force appears two peaks in one cycle (Fig. 1 (b)).

For case 1, define:

$$r_+ = \sqrt{\frac{Y_{\min}}{Y_{\max}}} > 0 \tag{6}$$

And for case 2, define:

$$r_- = -\sqrt{\frac{Y_{\min}}{Y_{\max}}} < 0 \tag{7}$$

Then for the two cases above, the cycle-formed evolution equation can be obtained by integral Equation (5):

$$\frac{dD}{dN} = \int_0^{t_{\max}} \frac{dD}{dt}dt = \frac{2b}{m}(1 - r_+^m)Y_{\max}^{\frac{m}{2}} \tag{8}$$

$$\frac{dD}{dN} = \int_0^{t_{\max}} \frac{dD}{dt}dt + \int_0^{t_{\min}} \frac{dD}{dt}dt = \frac{2b}{m}(1 + |r_-|^m)Y_{\max}^{\frac{m}{2}} \tag{9}$$

3. Fatigue life prediction method

In the elastic range, Equation (4) can be written as:

$$Y=\frac{W}{1-D}=\frac{\sigma_{\mathrm{e}}^{2}}{2(1-D)^{2}E} \tag{10}$$

Where, W is the elastic strain energy density and σ_{e} is the equivalent stress, which is defined as:

$$\sigma_e=\sqrt{(\sigma_1^2+\sigma_2^2+\sigma_3^2)-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)}$$

Where, σ_1, σ_2 and σ_3 are the principal stresses, μ is the Poisson's ratio.

Substituting Equation (10) into Equation (8) and Equation (9):

$$\frac{dD}{dN}=\frac{aRW_{\max}^{\frac{m}{2}}}{(1-D)^{\frac{m}{2}}}=\frac{aR\sigma_{e,\max}^{m}}{(2E)^{\frac{m}{2}}(1-D)^{m}} \tag{11}$$

Where, a=2b/m, and the cyclic characteristic parameter R is defined as:

$$R=\begin{cases}1-r_{+}^{m}\text{（case 1）}\\ 1+|r_{-}|^{m}\text{（case 2）}\end{cases}$$

For the constant amplitude load, the expression of the fatigue life N_{f} is obtained by integrating the Equation (11) directly:

$$N_f=\frac{(2E)^{\frac{m}{2}}(1-D_0)^{m+1}}{aR(m+1)}\sigma_{e,\max}^{-m} \tag{12}$$

Where, D_0 is initial damage. For the stress concentration specimen with notch or center holes, it can be proved that the strain energy density of the stress concentration point is approximately conserved in the damage process [10]:

$$W=W_0 \tag{13}$$

Where, W_0 is the strain energy density of the stress concentration point without damage. Substituting Equation (13) into Equation (11) and integrating it:

$$N_f=\frac{\left(1-D_0\right)^{\frac{m}{2}+1}}{aR\left(\frac{m}{2}+1\right)}W_{0,\max}^{-\frac{m}{2}}=\frac{(2E)^{\frac{m}{2}}\left(1-D_0\right)^{\frac{m}{2}+1}}{aRK^{m}\left(\frac{m}{2}+1\right)}\sigma_{n,\max}^{-m} \tag{14}$$

Where, K is the stress concentration factor, σ_{n} is the nominal stress.

For the variable amplitude spectrum, the load spectrum should be processed into a periodic cycle of the block spectrum (shown in Fig.2). Each period T contains j constant amplitude spectra with different amplitudes. Considering the Equation (11), (13) and (14), the periodic damage evolution equation is.

$$\frac{dD}{dT}=\sum_{i=1}^{j}\left(\frac{dD}{dN}\right)_i N_i=\frac{\sum_{i=1}^{j}a_iR_i\sigma_{0,\max,i}^{m}N}{(2E)^{\frac{m}{2}}\left(1-D\right)^{\frac{m}{2}}} \tag{15}$$

Therefore, the life of structures can be obtained by intigral Equation (15):

$$T=\frac{\left(2E\right)^{\frac{m}{2}}}{(\frac{m}{2}+1)\sum_{i=1}^{j}a_iR_i\sigma_{0,\max,i}^{m}N_i}$$

4. Summary

In this paper, a novel fatigue life prediction model of metallic structures is proposed in the frame work of damage mechanics theory. The model utilizes a simple and efficient damage evolution equation. The material damage parameters are more convenient to determine. This method has better applicability to the general engineering components and typically load spectrum. The model described in this paper provides a practical calculation method for engineering analysis.

5. Acknowledgments

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