Research on The Efficiency of Commercial Bank Branches Based on Weight-restricted DEA Model

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Abstract: Financial industry is facing unprecedented challenges, especially fierce competition between commercial banks, so it is urgent for us to solve a problem of comprehensive analysis and appropriate suggestions for the development of commercial banks. This survey uses a commercial bank’s input and output data, provides an overall efficiency ranking of bank branches by calculating through the traditional DEA model(C²R), DEA Overlapping Efficiency Model and Weight-restricted DEA model. The experimental results provide the basis for understanding of each branch, which is affiliated to the commercial bank.

Keywords: Bank efficiency, DEA model, Input-output efficiency.

1. Introduction

Commercial banks can assess their strengths and weakness by implementing comprehensive performance evaluation of management. To promote commercial banks’ competitiveness, analysis of experimental results will provide insights into assigning the limited financial resources scientifically and rationally to decision-maker. As recognized by all economists in the western theory of economics, both the definition and description of bank efficiency are the contrasting relationship between investment and yield in their business activities.

With the deepening of reform and opening up, the financial industry is facing unprecedented challenges, especially between interbank market. The question of how to achieve optimal financial resources for a single unit within the bank is a problem that needs to be addressed. Twenty years after Farrell’s seminal work, and building on those ideas, Charnes et al. (1978), proposed data envelopment analysis (DEA)[1].

DEA (data envelopment analysis) Model is a commonly used method for evaluating company's operating efficiency[2]. However, the fact is that each decision unit is maximizing its efficiency rating index[3], which tends to use extreme and unreasonable weight distribution for the input and output indices, the traditional DEA method’s result is somewhat unsatisfactory and cannot identify strengths and weaknesses between all the DMU. DEA Overlapping Efficiency Model partially compensates for these shortcomings of the traditional DEA method[4].

Traditional DEA model and DEA Overlapping Efficiency Model are too objective for the efficiency evaluation of commercial banks[5]. This paper provide a weight-restricted DEA model, with more subjective factors of decision ideology and value judgement. Evaluation results can be more approximates the real facts by adding the appropriate weights constraint conditions[6].

The rest of this article is organized as follows. Section 2 describes the experimental dataset of a commercial bank. Section 3 explains the models for bank efficiency and how to calculate the efficiency values of bank branches. In section 4, we show the ranking list, compare our approach results and discuss the possible causes for the ranking results. Finally, we give concluding remarks in Section 5.

2. Datasets

We used seventeen sub-branches’ input-output data of a commercial bank, input data include two items: number of staff, expenditure, and output data include eight items: volume of business, deposit
and loan amount, number of public and private customers, number of credit card and debit card, income.

We pretreatment the raw data first before our experiments. That different evaluation index tend to have various unit and dimension somewhat affects the results of predictive analysis. In order to eliminate the interaction between dimension of indicators, the input-output data should be standardized. This paper takes raw data as DEA model input variables after unitary by Min-Max Normalization method.

3. Methodology

3.1 DEA method of C^2R model[7]

Consider a set of \( n \) DMUs, with each DMU \( j \) \((j = 1,2,...,n)\) using \( m \) inputs (consumption) \( x_{ij} (i = 1,\ldots,m) \) and generating \( s \) outputs (cost effectiveness) \( y_{rj} (r = 1,2,\ldots,s) \). For DMU \( j \), denote \( x_{ij} = X_j \) \((x_{1j},x_{2j},\ldots,x_{mj})\) and \( y_{rj} = Y_j \) \((y_{1j},y_{2j},\ldots,y_{sj})\) the amount of input and output jth DMU invest in the commercial production. Denote \( v_i \in V(v_1,v_2,\ldots,v_m) \) the weight of ith input type and \( u_r \in U(u_1,u_2,\ldots,u_r) \) the weight of rth output type. In addition, \( x_{ij} > 0 \), \( y_{rj} > 0 \), \( v_i > 0 \), \( v_r > 0 \), \( i = 1,2,\ldots,m; r = 1,2,\ldots,s; j = 1,2,\ldots,n \). The total virtual input of DMU \( j \) is defined as \( V^T \cdot X_j \) and its total virtual output is defined as \( U^T \cdot Y_j \), where subscript T means transposition. The C^2R model (Charnes et al., 1978), measures the efficiency of DMU \( j \) as the maximum of the ratio of its total output to its total input:

\[
\sum_{r=1}^{s} {u_r y_{rj}} / \sum_{i=1}^{m} {v_i x_{ij}}, \text{endow suitable weight to make } h_j \leq 1.
\]

Define the linear programming models of traditional DEA method(C^2R) as follows:

\[
\begin{align*}
\max_{v_i,u_r} & \quad Y_j^T u = E_u , \\
\text{s.t.} & \quad Y_j^T u - X_j^T v \leq 0, \quad 1 \leq j \leq n , \\
& \quad X_j^T v = 1 , \quad u \geq 0, v \geq 0 ,
\end{align*}
\]

Whether the assessment of DMU\( j \) is effective or not is relative to other DMUs. If the linear programming Eq(1) has an optimal solution with \( u^* > 0 \) and \( v^* > 0 \), and the corresponding target value \( E_u = 1 \), we said \( j \)th DMU is DEA efficiency, otherwise not.

3.2 DEA Overlapping Efficiency Model[8]

Actually, it is too inaccurate a problem to rely on the traditional DEA model, the efficiency value obtained from self-assessment cannot distinguish the strengths and weaknesses of each branch. In addition, the C^2R model calculates the weights of each DMU with the most favourable, weight distribution for each input and output is extraordinary disparity, which only takes the self-interest input-output indicators into consideration and ignore other DMUs’ indicators that are not to its advantage. This phenomenon makes the model reflect the merits of DMUs inaccurately.

In order to solve this problem, we introduce cross-evaluation mechanism. The principle of DEA Overlapping Efficiency Model is that using the optimal weight of each DMU\( i \) to calculate the other DMU\( k \)’s efficiency values and get cross-evaluation.

May solve directly regarding certain topics using the formula, but after some topics must carry on the ingenious distortion, can use the average value inequality solution.

The self-evaluation value \( E_{ik} \) \((1 \leq i \leq n)\) of DMU \( i \) is calculated by formula (1), given \( i \in \{1,2,\ldots,n\} \), \( k \in \{1,2,\ldots,n\} \). Solve the following linear equation:
\[
\begin{array}{ll}
\min & Y_k^T u \\
\text{s.t.} & Y_j^T u \leq X_j^T v, \\
& 1 \leq j \leq n, \\
& Y_i^T u = E_{ik}^T v, X_i^T v = 1, \\
& u \geq 0, v \geq 0.
\end{array}
\]

Again, the optimal solution of formula 2 is used to find the cross-evaluation value:

\[
E_{ik} = \frac{Y_k^T u_k^*}{X_k^T v_k^*} = Y_k^T u_k^*
\]

Finally, we form the cross-evaluation matrix by cross-evaluation values:

\[
E = \begin{pmatrix}
E_{11} & E_{12} & \cdots & E_{1n} \\
E_{21} & E_{22} & \cdots & E_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
E_{n1} & E_{n2} & \cdots & E_{nn}
\end{pmatrix}
\]

Which \( E_{ik} \) is self-evaluation and \( E_{ik} (i \neq k) \) is cross-evaluation. E’s column i is the evaluation value of other DUMs to DMU_i, the higher the value is, the better DMU_i is. For DMU_i, the strengths and weaknesses were being measured in the average value of E’s column i computed as:

\[
e = \frac{1}{n} \sum_{k=1}^{n} E_{ki}
\]

\( e_i \) is considered to be other DUMs’ total assessment of DMU_i, the higher \( e_i \) is, the better DMU_i is, thus solving the traditional DEA’s effective DMU separating problem.

### 3.3 Weight-restricted DEA

The focus of this article is Weight-restricted DEA model. After analyzing the domestic literature in existence, we find that the most of study on DEA is still using traditional DEA directly or improving DEA model, similar to DEA Overlapping Efficiency Model. Subjectively, the importance of output items causes the different weight, so some business of commercial bank should be given more attention and corresponding high weight. In order to make the result reasonable and acceptable, the major task of this paper is to add constraint weight condition to input-output data, making traditional DEA model and DEA Overlapping Efficiency Model more subjective.

If the decision-maker thinks the weight should satisfy the following t constraints:

\[
\begin{align*}
&u_1 y_{11} + \cdots + u_j y_{jr} \geq 0 \\
&\vdots \\
&u_1 y_{1r} + \cdots + u_j y_{jr} \geq 0
\end{align*}
\]

In matrix form:

\[
(u_1, \cdots, u_j) \begin{bmatrix}
b_{11} & \cdots & b_{1r} \\
\vdots & \ddots & \vdots \\
b_{r1} & \cdots & b_{rr}
\end{bmatrix} \geq 0
\]

So the DEA overlapping efficiency model with constraint conditions becomes:

\[
\begin{array}{ll}
\min & Y_k^T u \\
\text{s.t.} & Y_j^T u \leq X_j^T v, \\
& 1 \leq j \leq n, \\
& Y_i^T u = E_{ik}^T v, X_i^T v = 1, \\
& u \geq 0, v \geq 0, \\
& \mu^T B \geq 0.
\end{array}
\]
4. Result and discussion

In our work, we study sub-branches’ efficiency of a commercial bank during input-output data through C^2R Model and DEA Overlapping Efficiency Model, and their results were compared. In the end, we calculate efficiency value through Weight-restricted DEA Model. Through analysis and discuss, we find that the result obtained from Weight-restricted DEA Model is reasonable and acceptable by decision-maker.

Table 1 shows the results based on C^2R Model and DEA Overlapping Efficiency Model. We get a very close rank result by this two method. But compared with C^2R Model, DEA Overlapping Efficiency Model is able to distinguish the strengths and weakness of DMUs.

<table>
<thead>
<tr>
<th>Sub branch</th>
<th>C^2R rank</th>
<th>Sub branch</th>
<th>Overlapping DEA rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch-01</td>
<td>1</td>
<td>Branch-01</td>
<td>0.70232 4</td>
</tr>
<tr>
<td>Branch-02</td>
<td>0.84810</td>
<td>Branch-02</td>
<td>0.5754 15</td>
</tr>
<tr>
<td>Branch-03</td>
<td>0.89785</td>
<td>Branch-03</td>
<td>0.64386 10</td>
</tr>
<tr>
<td>Branch-04</td>
<td>1</td>
<td>Branch-04</td>
<td>0.74356 3</td>
</tr>
<tr>
<td>Branch-05</td>
<td>0.96776</td>
<td>Branch-05</td>
<td>0.67364 5</td>
</tr>
<tr>
<td>Branch-06</td>
<td>1</td>
<td>Branch-06</td>
<td>0.77095 2</td>
</tr>
<tr>
<td>Branch-07</td>
<td>0.86329</td>
<td>Branch-07</td>
<td>0.54443 16</td>
</tr>
<tr>
<td>Branch-08</td>
<td>0.86848</td>
<td>Branch-08</td>
<td>0.59256 13</td>
</tr>
<tr>
<td>Branch-09</td>
<td>0.90782</td>
<td>Branch-09</td>
<td>0.65886 8</td>
</tr>
<tr>
<td>Branch-10</td>
<td>0.86263</td>
<td>Branch-10</td>
<td>0.58366 14</td>
</tr>
<tr>
<td>Branch-11</td>
<td>1</td>
<td>Branch-11</td>
<td>0.64848 9</td>
</tr>
<tr>
<td>Branch-12</td>
<td>0.71270</td>
<td>Branch-12</td>
<td>0.2938 17</td>
</tr>
<tr>
<td>Branch-13</td>
<td>0.98916</td>
<td>Branch-13</td>
<td>0.59882 12</td>
</tr>
<tr>
<td>Branch-14</td>
<td>1</td>
<td>Branch-14</td>
<td>0.65946 7</td>
</tr>
<tr>
<td>Branch-15</td>
<td>1</td>
<td>Branch-15</td>
<td>0.61969 11</td>
</tr>
<tr>
<td>Branch-16</td>
<td>1</td>
<td>Branch-16</td>
<td>0.80979 1</td>
</tr>
<tr>
<td>Branch-17</td>
<td>1</td>
<td>Branch-17</td>
<td>0.6698 6</td>
</tr>
</tbody>
</table>

Comparison between C^2R and Overlapping DEA Model as follows Fig 1:

![Fig 1 Comparison between CCR and Overlapping DEA Model](image)

Table 2 add the subjective conditions of decision-makers, so the result has changed, and under a certain weight constraint, efficiency and ranking has changed, but the results can reflect actual situation, and easily accepted by policy makers.
Table 2 Experiment result of Weight-restricted Model

<table>
<thead>
<tr>
<th>Sub branch</th>
<th>Weight-restricted Model</th>
<th>rank</th>
<th>Sub branch</th>
<th>Weight-restricted Model</th>
<th>rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch-01</td>
<td>0.271129419</td>
<td>5</td>
<td>Branch-10</td>
<td>0.158526814</td>
<td>16</td>
</tr>
<tr>
<td>Branch-02</td>
<td>0.234492073</td>
<td>8</td>
<td>Branch-11</td>
<td>0.281369014</td>
<td>4</td>
</tr>
<tr>
<td>Branch-03</td>
<td>0.223084928</td>
<td>9</td>
<td>Branch-12</td>
<td>0.161291921</td>
<td>15</td>
</tr>
<tr>
<td>Branch-04</td>
<td>0.987401452</td>
<td>1</td>
<td>Branch-13</td>
<td>0.559987119</td>
<td>3</td>
</tr>
<tr>
<td>Branch-05</td>
<td>0.234907723</td>
<td>7</td>
<td>Branch-14</td>
<td>0.254511999</td>
<td>6</td>
</tr>
<tr>
<td>Branch-06</td>
<td>0.183182102</td>
<td>12</td>
<td>Branch-15</td>
<td>0.220407608</td>
<td>10</td>
</tr>
<tr>
<td>Branch-07</td>
<td>0.129016028</td>
<td>17</td>
<td>Branch-16</td>
<td>0.174168985</td>
<td>13</td>
</tr>
<tr>
<td>Branch-08</td>
<td>0.169951282</td>
<td>14</td>
<td>Branch-17</td>
<td>0.783620783</td>
<td>2</td>
</tr>
<tr>
<td>Branch-09</td>
<td>0.20638388</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The output data weight of table 2 result, which calculated through Weight-restricted Model, is determined by commercial bank’s staff. After adding weight restriction, the rank of each branch is rearranged, the DMU with more output of high-weighted item is ranked front.

5. Summary

This paper proposes Weight-restricted DEA Model to compute the efficiency values of a commercial Bank’s branches. We also calculate efficiency values through the traditional C²R Model and DEA Overlapping Efficiency model. Experiments, obtained from C²R Model and Overlapping Efficiency DEA model, is very similar. Also we find that Weight-restricted DEA can better reflect the actual situation, which is consistent with our intuitive knowledge and acceptable by decision-maker.

Acknowledgments

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References