ATLANTIS PRESS

Advances in Engineering Research, volume 114

International Conference on Advances in Materials, Machinery, Electrical Engineering (AMMEE 2017)

An Analysis of Optical Flow 8×8 Patches

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Keywords: optical flow, high-contrast patches, Klein bottle.

Abstract. We apply computing topological method to qualitatively analyze space of 8×8 optical flow patches. We experimentally prove that there exist subspaces of 8×8 optical flow patches that are topologically equivalent to a circle. For the space of 8×8 optical flow patches, we cannot find its subspaces having homology as that of the Klein bottle.

1. Introduction

Optical flow is the apparent motion of objects in a visual scene originated by the relative motion between the viewer and the scene [1]. Roth and Black [2] studied the spatial statistics of optical flow, and obtained a rich prior model of optical flow. Adams, Atanasov and Carlsson [3] used the nudged elastic band technique to analyse optical flow data, they discovered a new topological feature for 3×3 optical flow patches. The authors of [3] shown a similar topological features for optical flow 3×3 patches with that of range image. The authors of papers [4, 5] studied optical flow patches for $n=3,\ 4,\ 5,\ 6,\ 7$ and got similar results as the case $n=3$.

In this paper, we expand the size of optical flow patches to 8 and detect the topological features of spaces of 8×8 optical flow patches. By the methods of the paper [6], we find that there exist density subsets of 8×8 optical flow patches that are topologically equivalent to a circle. And we will show that the Klein bottle feature of 8×8 optical flow patches may disappear.

2. The Spaces of Optical Flow Patches

Our space is created from the Roth and Black optical flow database [2]. We randomly choose high-contrast 8×8 patches from the optical flow database. Our spaces X_8 is set of 8×8 patches produced by similar way to [6, 7, 8].

One 8×8 patch is arranged as
$$\begin{pmatrix} (u_1,v_1) & (u_9,v_9) & \cdots & (u_{49},v_{49}) & (u_{57},v_{57}) \\ (u_2,v_2) & (u_{10},v_{10}) & \cdots & (u_{50},v_{50}) & (u_{58},v_{58}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (u_7,v_7) & (u_{15},v_{15}) & \cdots & (u_{55},v_{55}) & (u_{63},v_{63}) \\ (u_8,v_8) & (u_{16},v_{16}) & \cdots & (u_{56},v_{56}) & (u_{64},v_{64}) \end{pmatrix},$$

here u represents optical flow in the horizontal direction and v represents the vertical direction. Each 8×8 patch is considered as a vector $\boldsymbol{x}=(u_1,\ldots,u_{64},v_1,\ldots,v_{64})\in\mathbb{R}^{128}$.

For simplifying calculations, we randomly select 50,000 patches from X_8, denoted by XS_8 .

3. Results for $XS_8(k,p)$

Persistent homology is a tool to identify topological features of a space by using a finite sampled points of the space. We apply software package Javaplex to calculate persistent homology, for more details, please refer to [6, 9, 10, and 11].

We think over core subsets $XS_8(200,30)$, and calculate its barcodes, Figure 1 gives a sample of PLEX result for it. There is a long Betti0 line and a long Betti1 line in the plots, that is: $\beta_0=1$, $\beta_1=1$, with the topology of a circle. We run one hundred trials on $XS_8(200,30)$, the result is stable. For core subsets $XS_8(100,30)$, we obtain the similar result.

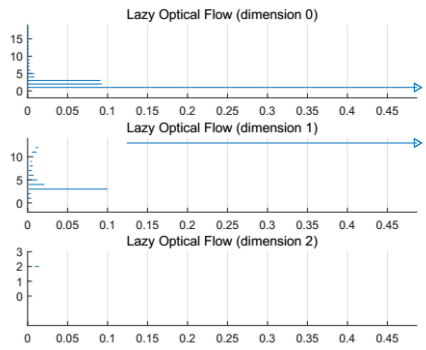


Figure 1. Barcodes for $XS_8(200,30)$

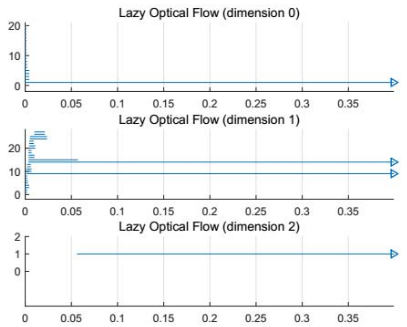


Figure 2. Barcodes for $K_8(200)$

4. Computing methods

For 3×3 patches of optical flow there exists a two-dimensional subspace with homology of the Klein bottle [4], as increasing of the size of patches, the Klein bottle feature of the spaces gradually weakens. For how large size of optical flow patches, the Klein bottle feature vanishes? To study the problem we give an outline of producing a theoretic Klein bottle model. We take a set $\wp$ of two variables polynomials with form of $a_2(a_1x+b_1y)^2+b_2(a_1x+b_1y)$, here $(a_1,b_1)\in S^1,(a_2,b_2)\in S^1$, and S^1 denotes the unit circle in the plane. Now we define two mappings: $g:S^1\times S^1\to\wp$ is defined by $(a_1,b_1,a_2,b_2)\mapsto a_2(a_1x+b_1y)^2+b_2(a_1x+b_1y)$ ([6]), the other is $h_8:\wp\to S^{127}$ by a composite of evaluating the function at each planar grid $G_8=\{-7,...,-1,0,1,...,8\}\times\{-3,-2,...,3,4\}$ subtracting the mean and normalizing.

As the proof in [6], the images $im(h_8|\wp)$ is homeomorphic to the Klein bottle.

We uniformly take 200 points $(\{x_1,\ldots,x_{200}\})$ on the unite circle, all possible tuples (x_i,x_j) form a point set on the torus $S^1\times S^1$. Then, we map each of the 40000 points into S^{127} by the mapping $h_8\circ g$, the image is denoted by $K_8(200)$. Figure 2 is the PLEX result for $K_8(200)$, it gives $\beta_0=1$, $\beta_1=2$ and $\beta_2=1$, Betti numbers of the Klein bottle. Therefore $K_8(200)$ is an appropriate approximation of the Klein bottle in S^{127}.

5. Results for X_8

In this section, we will show that the Klein bottle feature of 8×8 optical flow patches may disappear. We describe two methods to get the subspaces of X_8 as following.

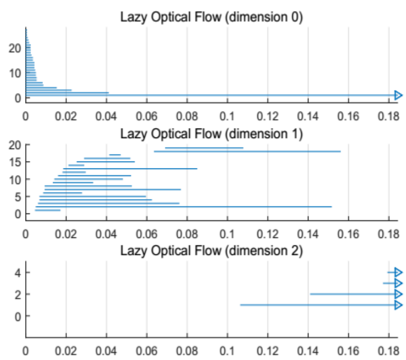
(i) For any point p of $K_8(200)$ we calculate the Euclidean distance from p to every point of X_8, and then take t closest points to the point p. The constructed subspace of X_8 is denoted by $Kopt_8(200,t)$.

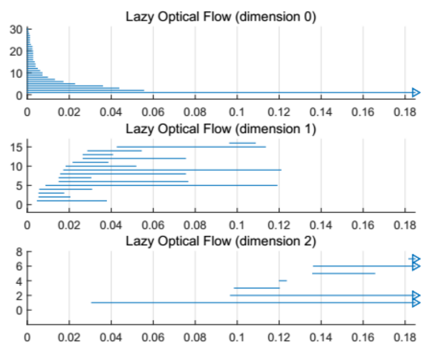
(ii) For any point of X_8, we compute the Euclidean distance from p to the set $K_8(200)$, then we resort points of X_8 according to increasing of their Euclidean distances to $K_8(200)$, then we take the top t percent of the closest distances, and denote the subspace of X_8 as $XP_8(200,t)$.

To detect whether a subspace of X_8 has the homology of the Klein bottle, we utilize the subspace $Kopt_8(200,11)$. We run 100 experiments, there are only 29 times PLEX barcodes giving Klein bottle feature, and barcode intervals with the homology of the Klein bottle are very short. Figure 3 shows that $Kopt_8(200,11)$ has the homology of the Klein bottle for the parameter values from 0.108 to 0.141. Figure 4 gives another PLEX result for $Kopt_8(200,11)$, which has no the homology of the Klein bottle.

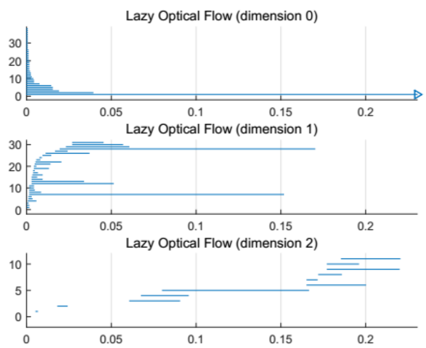
We also run many time experiments on $Kopt_8(200,t)$ for t=13, 9, 7, 5, 3, 1, we get similar results as for $Kopt_8(200,11)$.

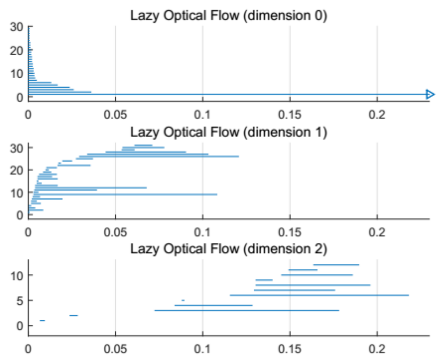
Now we consider subsets $XP_8(200,t)$ for t=10, 15, 20, 25, 30, 35, and 40, we run many experiments on them with various parameters, we cannot find that they have the Klein bottle feature. But for $XP_8(200,25)$, its PLEX results seldom give the Klein bottle feature in very small intervals. If we consider the union $XP_8(200,25) \cup Kopt_8(200,1)$, we do 160 experiments on it, there are 31 experiments with the Klein bottle feature (i.e. $\beta_0 = 1$, $\beta_1 = 2$, $\beta_2 = 1$), and some barcode intervals with $\beta_0 = 1$, $\beta_1 = 2$, $\beta_2 = 1$ are very short, but the others have no the Klein bottle feature. Figure 5 gives $\beta_0 = 1$, $\beta_1 = 2$, $\beta_2 = 1$ in [0.085, 0.1505] for $XP_8(200,25) \cup Kopt_8(200,1)$, but Figure 6 shows no the Klein bottle feature for it.





Figures 3, 4. Barcodes for $Kopt_8(200,11)$





Figures 5, 6. Barcodes for $XP_8(200,25) \cup Kopt_8(200,1)$

Hence we may conclude that the Klein bottle feature of the spaces X_8 disappears.

6. Conclusion

In this paper we use persistent homology method to discuss topological qualitative analysis of 8×8 optical flow patches. We show that the spaces of high contrast 8×8 patches have core subsets modeled as a circle. The Klein bottle's feature of small optical flow patches gradually disappear as increasing of the optical flow patch size. For 8×8 optical flow patches, the Klein bottle's feature may disappear.

Acknowledgements

The project is supported by the National Natural Science Foundation of China (Grant No. 61471409).

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