Free space quantum key distribution based on turbulence bubble model

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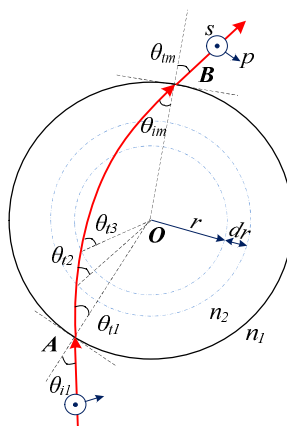
Abstract: Aiming at the transmission of polarization encoded single photons in turbulent atmosphere, the random turbulence is modeled as atmospheric bubbles with various refractive indexes so as to conceive the propagation process of single photon using geometric optics. The fluctuation of photon transmission ratio k has been quantitatively calculated which is revealed as the relationship between transmission ratio and incident angle or turbulence scale. Based on that, the quantum bit error rate(QBER) and polarization fidelity of single photons are analyzed respectively which are both caused by the misalignment of polarization state after continuous refractions. Then the three dimensional relationship of k and polarization error rate Q are simulated with random incident angle or turbulence scale.

Introduction

With the help of free space quantum key distribution (QKD) two distant parties can share secure random keys even between air and ground, which is regarded as a vital approach to establish secure aeronautical communication link and global confidential network. It has become one of the most promising and practical technology in quantum information field since proposed. So far the study of free space QKD is mainly about its feasibility analysis and security proof [1]. The Villoresi group from Padova realized transmission of single photons between satellite and ground station for the first time by using corner-cube retroflector which promoted the development of free space quantum communication [2]. In 2010 a 16km quantum teleportation has been reported by Pan's group from University of Science and Technology of China which broke the distance record of free space quantum communication [3]. The world's first airborne QKD experiment was implemented by Germany researchers in 2013 in aeronautical environment [4]. Nie analyzed the influence on free space QKD of sandstorm and haze as well as rainfall in detail [5,6], the relationship between quantum channel entanglement and turbulence intensity is established in [7] which serves as theoretical reference for the study of aeronautical quantum communication performance. Since polarization encoding is generally used in free space QKD [8], the signal photon will be influenced by many factors when transmitting in atmospheric channel where the refraction of dielectric interface contributes greatly to the polarization state change and finally induces quantum bit error rate (QBER) [9]. Therefore it is meaningful to figure out the polarization state change of photons after turbulence refraction and the consequent fluctuation of QBER.

The turbulence bubble model

As polarization encoded single photon propagates in the turbulent atmosphere [10], it is inevitable to be influenced by the refractions of turbulence which will impede the transmission and fidelity of polarization state. The light will be curved due to the inconsistent refractive indexes inside and outside the turbulence [11,12]. And from the microscopic perspective, the s component and p component of polarization state will be changed. In order to simplify the situation the turbulence can be abstracted as air bubbles with random size and refractive indexes. The refractive index inside the bubble will decrease with the enlargement of turbulence scale. Suppose that the refractive index of air is n_1 and the one inside the bubble is n_m. And according to [13] there is a relationship between n_m and bubble radius r which can be expressed as equation (1) and Figure.1 shows the diagram of photon path through continuous refractions.

Figure 1. Diagram of single photon path through continuous refractions

$$n_m = n_1 \sqrt{1 - b\left(\frac{r}{a}\right)^2} \tag{1}$$

Where a and b are both constant related to temperature and pressure of the turbulence. Suppose that the photon enters at A on the surface of bubble with incident angle of q_{i1} and then emerges out from B point. The refractive angle of the first refraction is q_{t1} which also equals the incident angle of the next refraction. The left incident angles and refractive angles are $q_{i1,i2...im}$ and $q_{t1,t2...tm}$ respectively. And O is the center of this bubble. If the wave vector of incident light i is $\vec{E}$ and its two components of incident light can be written as follow according to representation of polarized light and Fresnel principle [14], when the refractive index of turbulence is constant there will be:

$$\frac{\vec{E}_s^{t1}}{\vec{E}_s^{i1}} = \frac{2\cos q_{i1} \sin q_{t1}}{\sin(q_{i1} + q_{t1})} \tag{2}$$

$$\frac{\vec{E}_p^{t1}}{\vec{E}_p^{i1}} = \frac{2\cos q_{i1} \sin q_{t1}}{\sin(q_{i1} + q_{t1})\cos(q_{i1} - q_{t1})} \tag{3}$$

$$\frac{\vec{E}_s^{t2}}{\vec{E}_s^{t1}} = \frac{2\cos q_{i2} \sin q_{t2}}{\sin(q_{i2} + q_{t2})} \tag{4}$$

$$\frac{\vec{E}_p^{t2}}{\vec{E}_p^{t1}} = \frac{2\cos q_{i2} \sin q_{t2}}{\sin(q_{i2} + q_{t2})\cos(q_{i2} - q_{t2})} \tag{5}$$

The refractions also abide by Snell principle.

$$\frac{n_1}{n_2} = \frac{\sin q_{t1}}{\sin q_{i1}} = \frac{\sin q_{i2}}{\sin q_{t2}} \tag{6}$$

Then the transmission ratio of amplitude can be derived by comparing the components according to Eq.(2-5):

$$k = \left[\cos(q_{i2} - q_{t2})\cos(q_{i1} - q_{t1})\right]^2$$

$$= \left(\cos q_{i1} \sqrt{1 - \left(\frac{n_1 \sin q_{i1}}{n_2}\right)^2} + \sin q_{i1} \frac{n_1 \sin q_{i1}}{n_2}\right)^2 \tag{7}$$

However, when the refractive index of turbulence varies with the radius there will be:

$$\frac{\vec{E}_s^{t1}}{\vec{E}_s^{i1}} = \frac{2\cos q_{i1} \sin q_{t1}}{\sin(q_{i1} + q_{t1})} \tag{8}$$

$$\frac{\overset{\triangledown}{E}^{t1}_{p}}{\overset{\triangledown}{E}^{i1}_{p}} = \frac{2\cos q_{i1}\sin q_{t1}}{\sin(q_{i1}+q_{t1})\cos(q_{i1}-q_{t1})} \tag{9}$$

$$\frac{\overset{\triangledown}{E}^{tm}_{s}}{\overset{\triangledown}{E}^{tm}_{s}} = \frac{2\cos q_{im}\sin q_{tm}}{\sin(q_{im}+q_{tm})}(m=2,3,4\mathbf{L}) \tag{10}$$

$$\frac{\overset{\triangledown}{E}^{tm}_{p}}{\overset{\triangledown}{E}^{tm}_{p}} = \frac{2\cos q_{im}\sin q_{tm}}{\sin(q_{im}+q_{tm})\cos(q_{im}-q_{tm})}(m=2,3,4\mathbf{L}) \tag{11}$$

Therefore the transmission ratio is calculated as:

$$k = \left[\cos(q_{im}-q_{tm})\cos(q_{im-1}-q_{tm-1})\mathbf{L}\cos(q_{i1}-q_{t1})\right]^2 \tag{12}$$

$$\frac{n_{m-1}}{n_m} = \frac{\sin q_{tm-1}}{\sin q_{im}} \tag{13}$$

Figure.2 shows the relationship between k and the incident angle q_1 at different turbulence scales. It can be seen that no matter how large the turbulence is, when incident angle is 0 the photon penetrates the bubble directly along the diameter so the transmission ratio reaches its climax. However, this value descends rapidly when the incident angle exceeds a certain threshold. The solid line shows the situation of turbulence with 1m radius and dashed line is 5m while dotted line is 10m. It can be concluded from the trend that the larger the turbulence is, the faster its transmission ratio decreases. And the threshold of each line are 44.8 , 53.2 . and 57.5 respectively. When the incident angle exceeds the threshold, the fidelity of single photons becomes worse after continuous refraction. Figure.3 shows the relationship between k and r when the incident angle is 45 .
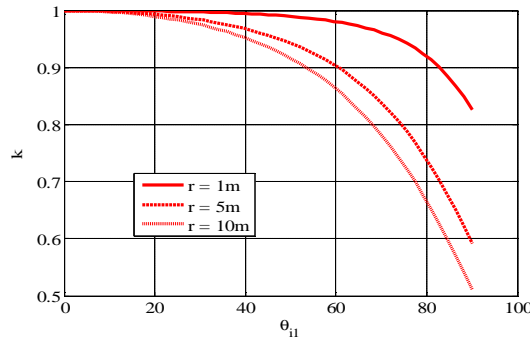

Figure 2. Relationship between transmission ratio and incident angle with different turbulence scale
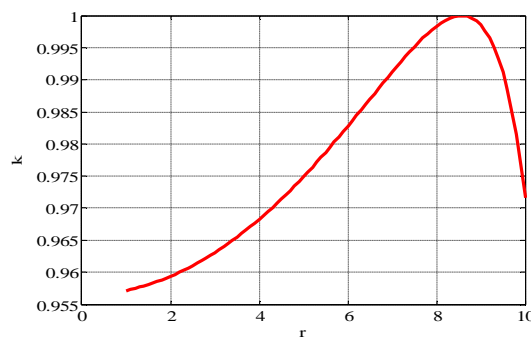

Figure 3. Relationship between transmission ratio and turbulence radius

The scale of turbulence and incident angle determine the fluctuation of transmission ratio to some extent. Here we firstly fix the incident angle at 45 . in order to figure out the optimal turbulence scale for transmitting photons. The result shows that 8m is the optimal radius. Whereas, in more general situation the value of k will varies gradually with the change of r and q_{i1}, which will induce apparent QBER. Figure.4 shows the relationship between k and incident angle and turbulence radius.
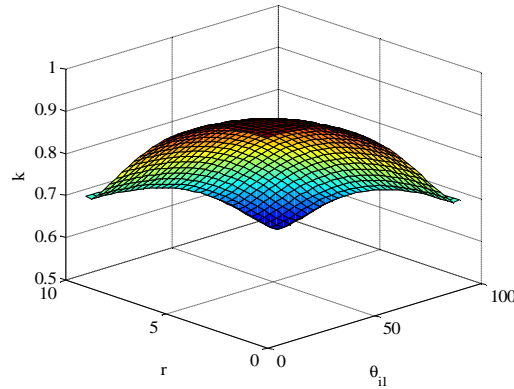
Figure 4. Relationship between k and incident angle and turbulence radius

In the scheme of polarization encoded BB84 protocol, Alice uses four different states to encode information on photons. The electric vector $\vec{E}$ of photons at any state can be projected on s direction and p direction respectively. Thus the polarization state before transmission is $a|P\rangle + b|S\rangle$. It will turn into $ap|P\rangle + bs|S\rangle$ after refraction of turbulence. According to the analysis of Figure.2 in general cases $s \neq p$ so the polarization state will be modified. The quantum bit error rate (QBER) due to polarization misalignment can be expressed as:

$$Q = \frac{(t_p - t_s)^2}{2(t_p^2 + t_s^2)} = \frac{(1-k)^2}{2(1+k^2)} \tag{14}$$

Figure.5 shows the simulation of Q when we take the optimal incident angle and implement the protocol at minimum turbulent influence. From this two diagrams it can be inferred that the propagating performance of single photons is unstable when n_2 changes randomly. The solid lines show the refraction of photons in turbulence with high refractive index. Although the energy will reduce greatly, the QBER increases slowly. The requirement of free space QKD can be satisfied only if the sending frequency is high enough. But the photons cannot penetrate with some probability due to total reflection and interrupt the regular key distribution process; the whole tendency coincides with the analysis of Figure.2.
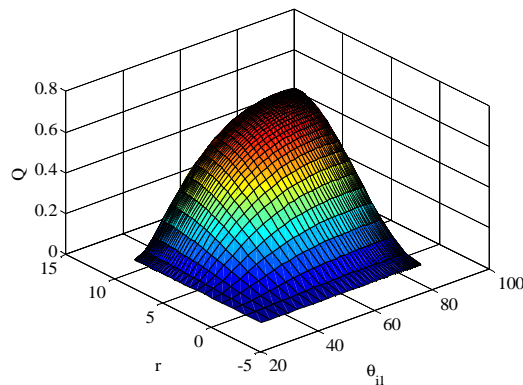

Figure 5. Relationship between Q and incident angle and turbulence radius

Conclusion

In general, we focus on the performance free space QKD in turbulent environment. The propagation process of polarized photons is abstracted as continuous refraction based on turbulence bubble model. Then the relationship between incident angle and transmission ratio and polarization QBER is simulated through analyzing change of polarization state. The upper bound of QBER is derived according to relationship between key rate and transmission ratio. Furthermore, the secure range of incident angle and refractive index is figured out. The results show that the performance of free space

QKD will be influenced by turbulence due to the refraction effect on polarization states. The requirement of QBER can be satisfied by optimizing parameters in future work.

References：

[1] Bonato,F.; Tomaello,A.; Deppo,V.D.; *et al.* Feasibility of satellite quantum key distribution [J] *New J Phys* 2009, 11, 045017.

[2] Villoresi,P.; Jennewein,T.; Tamburini,F.; *et al.* Experimental verification of the feasibility of a quantum channel between Space and Earth [J]. *New J Phys* 2008, 10, 3436-3440.

[3]. Jin,X.M.; Ren,J.G.; Yang,B.; et al. Experimental free-space quantum teleportation [J]. *Nature Photon* 2010, *4*, 376-381.

[4]. Nauerth,S.; Moll,F.; Rau,M.; et al. Air-to-ground quantum communication [J]. *Nature Photon* 2013, *7*, 382-386.

[5]. Min,N.; Jiaming,R.; Guang,Y.; et al. Influences of nonspherical aerosol particles and relative humidity of atmosphere on the performance of free space quantum communication [J]. *Acta Phys Sin* 2016, 65, 1-9.

[6]. Min,N.; Yun,W.; Guang,Y.; et al. Optimal mean photon number of decoy state protocol based on chameleon self-adaptive strategy under the background of rainfall [J]. *Acta Phys Sin* 2016,65, 23-30.

[7]. Min,N.; Kun,G.; Guang,Y.; *et al*. Effect of the Atmospheric Turbulence on the Performance of Free Space Quantum Communication [J]. *Acta Photonica Sinica* 2016, 45, 143-149.

[8]. Dong,L.; Wang,J.X.; Li.Q.Y,; *et al.* Teleportation of a general two-photon state employing a polarization-entangled state with nondemolition parity analyses [C]. *Quantum Inf Process* 2016, 2, 1-16.

[9]. Chong,Y.Y.; Ming,L; Peng,F.L,. Photon polarization in free-space quantum communication [J]. *Journal of Beijing University of Posts and Telecommunications* 2013, 36, 1-9.

[10]. Schmitt,T.; Weier,H.; Furst,M.; *et al*. Experimental demonstration of free-space decoy-state quantum key distribution over144 km [J]. *Phys Rev Lett* 2007, *98*, 010504.

[11]. Lylova,A.; Kudryashov,A.; Sheldakova,J.; *et al.* A new method of the real-time atmospheric turbulence modeling [J]. *International Conference Laser Optics* 2016, R, 4-12.

[12]. Yuksel,D.; Yuksel,H.; Geometrical Monte Carlo simulation of atmospheric turbulence [C]. *Proc Spie* 2013, 11, 8874.

[13]. Carrasco,C.A,; Fernandez,V.; Denisenko,N.; Free-space quantum communication [M]. In *Optical wireless communications*; Murat,U.; Springer International Publishing: Berlin, Germany, 2016, pp. 589-607.

[14]. Charnotskii,M.; Bit error rate of focused Gaussian beams in weak oceanic turbulence [J]. *JOSA A* 2015, 32, 1247.

[15] Lo,H.K.; Ma,X.; Chen,K.; Decoy state quantum key distribution [J]. *Phys Rev Lett* 2011, 94, 230504.