

Construction of Aging Model and Compensation Based on Nonlinear Time Series Analysis

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Abstract. To solve the frequency aging problem of high-precision quartz crystal oscillator has been one of the most written topics in the international frequency control field. Aging prediction is a significant approach to the process of the long-term aging indicator problem. Aiming at the aging issue of the crystal oscillator, in this paper, we take the SC cut 10MHz OCXO as the aging experimental subject, construct a model by time series analysis method, fit the aging process of the crystal oscillator, predict the development trend, and compensate the accuracy of the crystal oscillator's frequency output with predicted result. The experimental results demonstrate that this model is well-performed in describing the frequency aging process of the quartz oscillator and predicting the frequency aging. Furthermore, verified by simulation, the effect of compensation is evident. The indicator, daily aging rate of crystal oscillator after compensation could reach the level of 10-11.

1. Introduction

In specific high-precision instruments, the frequency stability is considered an important indicator of overall performance of the system^[1]. Because of the relatively stable natural frequency within quartz crystal, oscillators made of it with high precision is widely used in the instruments mentioned above. However, there is a phenomenon of frequency aging in quartz crystal oscillator, which is, the physical phenomenon of the crystal's resonant frequency increasing or decreasing monotonically with working time^[2]. When the crystal is better temperature compensated^[3-6], the impact of aging drift will become the most important factor of the long-term frequency stability^[7] of crystal oscillator as the working hour extended. To solve the frequency aging problem of high-precision quartz crystal oscillator has been one of the most written topics in the international frequency control field. Aging prediction is a significant approach to the process of the long-term aging indicator problem. At present, different aging prediction models of crystal oscillator are purposed various literatures, which include logarithmic aging model^[8] (it reflects the overall tendency of the aging process moderately with terrible curve fitting), quadratic polynomial function aging model^[9] (it only reflects the short-term aging behavior of crystal oscillator with initial data, and stop changing monotonically over time), and some other composite function models consist of elementary functions^[10] (they are hard to realize because of the complexity within). Aiming at the

frequency aging drift problem of the crystal oscillator, in this paper we take the SC cut 10 MHz OCXO as the experimental subject. Based on the aging data obtained by actual measurement, we construct a mathematical model with the time series analysis method^[11] to observe the fitting degree and predict the development trend of the aging process, then calibrate the frequency output accuracy of crystal oscillator based on aging prediction results. The feasibility of this method to compensate crystal oscillator frequency drift is discussed in different existing prediction schemes. The experimental results show that the method is feasible, and the daily aging rate of SC cut OCXO after compensation could reach the level of 10^{-11} .

2. The Construction of Aging Models

There are several factors result in the aging of the crystal oscillator^[12]. They affect the frequency drift in different periods. This peculiarity of crystal oscillator causes the diversity in the way the crystal oscillator shows its aging characteristics^[13,14] in different phases. Therefore, try to find the aging drift rule of crystal oscillator straight from its representation is rather complicated. We regard the frequency drift ratio as a function of time, analyze the actual measurement aging data by time series analysis method, find the mathematical expression which reflects the aging process by modeling and describe the frequency aging drift tendency of the crystal oscillator in a kind of mathematic way.

Time series analysis method is a way to analyze time series based on the utilization of the past data, analogize or extend according to the development process, direction and tendency reflected by the time series, to predict the development trend of the experimental subject in the next period^[15]. In practical problems, many time series models are non-linear. Various analyzations and applications of different non-linear time series models have become a research frontier and most written topics in time series analyzation. Quartz crystal oscillator's aging is a physical phenomenon, which behaves as its resonance frequency slow-varying monotonically (increase or decrease) over working time, the amplitude of frequency change rate becomes flat at the same time as the time extended. The long term changing characteristic of the curve behaves as a non-linear tendency. In the research of existing literature, some scholars proposed the linear prediction model: such as $y = k \cdot x$, this model is simple and easy to set up, but it simplify the complex aging process of crystal to plain linear variation, which makes the error becomes greater; for more examples, the time logarithmic model $y = a + b \cdot \ln(x)$ and quadratic polynomial model $y = a \cdot x + b \cdot x^2$, in practical applications, if the model fitting data is not highly related to actual measurement data, the predicted error will be rather big. Such prediction compensation data overlaying on the actual frequency of the crystal oscillator will worsen the crystal oscillator's long-term stability indicator of frequency output accuracy. Therefore, based on the past model research, in this paper, we propose the exponential autoregressive time series model as our frequency aging drift model of quartz crystal oscillator after several experiments.

The general exponential autoregressive time series model is as shown below:

$$x_t = \sum_{j=1}^p f_j(x_{t-1}) \cdot x_{t-j} + \varepsilon_t$$

$f_j(x_{t-1})$ is the non-linear function of variable x_{t-1} . ε_t is an independent identically distributed white Gauss noise and $E(\varepsilon_t) = 0, E(\varepsilon_t^2) = \sigma^2$. We can obtain different exponential autoregressive model by substituting $f_j(x_{t-1})$ into different non-linear functions. Set $f_j(x_{t-1}) = a_j + b_j \cdot \exp(-r \cdot x_{t-1}^2), j = 1, 2, \dots, p$, we can obtain the exponential autoregressive model. In this paper, SC cut 10MHz OCXO WWH065 is chosen as the aging experimental subject. Its actual

measurement frequency aging drift data in 182 days serves as actual measurement data of aging characteristic curve of the crystal oscillator. We fit the actual data with different models. The outcome is shown in Fig.1, with power-up working time of the crystal oscillator on the x label and frequency change rate, which is $\frac{f - f_0}{f_0}$ on the y label:

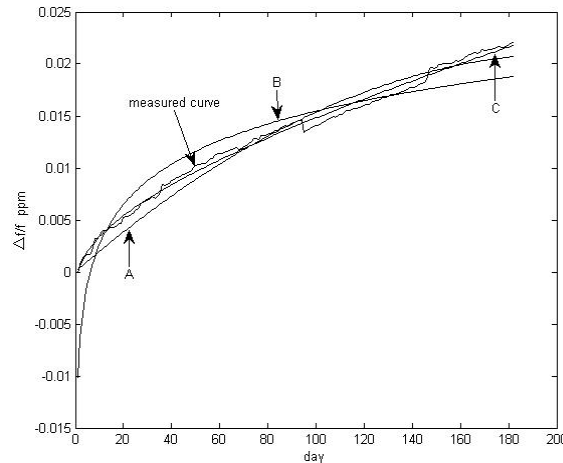


Fig.1 model fitting curve

In this figure, curve A represents the quadratic polynomial function model fit curve, curve B represents the time logarithmic function model fitting curve, curve C represents the exponential autoregressive time series model fitting curve proposed in this paper. As shown in the figure, curve C is most similar to the actual measurement data, the degree of fitting is superior to the two other models. Furthermore, we can obtain the fit relevancy of curve A, B, and C by fitting relevancy

computational formula $\rho = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^N (x_i - \bar{x})^2 (y_i - \bar{y})^2}}$, which is 0.9887, 0.9414 and 0.9979 respectively.

The data shows the exponential autoregressive time series model indeed fit the aging tendency curve of crystal oscillator better. Not only the fitting quality of the models in the existing literature is taken into consideration, but also the advantage in aging prediction.

3. Aging Prediction

In this paper, we take the actual measurement frequency aging drift data of our experimental subject, crystal oscillator WWH065 in 200 days as the actual measurement data to obtain the aging characteristic curve of this crystal oscillator. We fit the data respectively with the quadratic polynomial function model $y = a \cdot x + b \cdot x^2$, the logarithmic function model $y = a + b \cdot \ln(x)$, and the exponential autoregressive time series model, then predict the aging tendency in the upcoming 200 days. The fitting result is shown below in Fig.2, with the crystal oscillator power-up working time on the x label and frequency change rate on the y label:

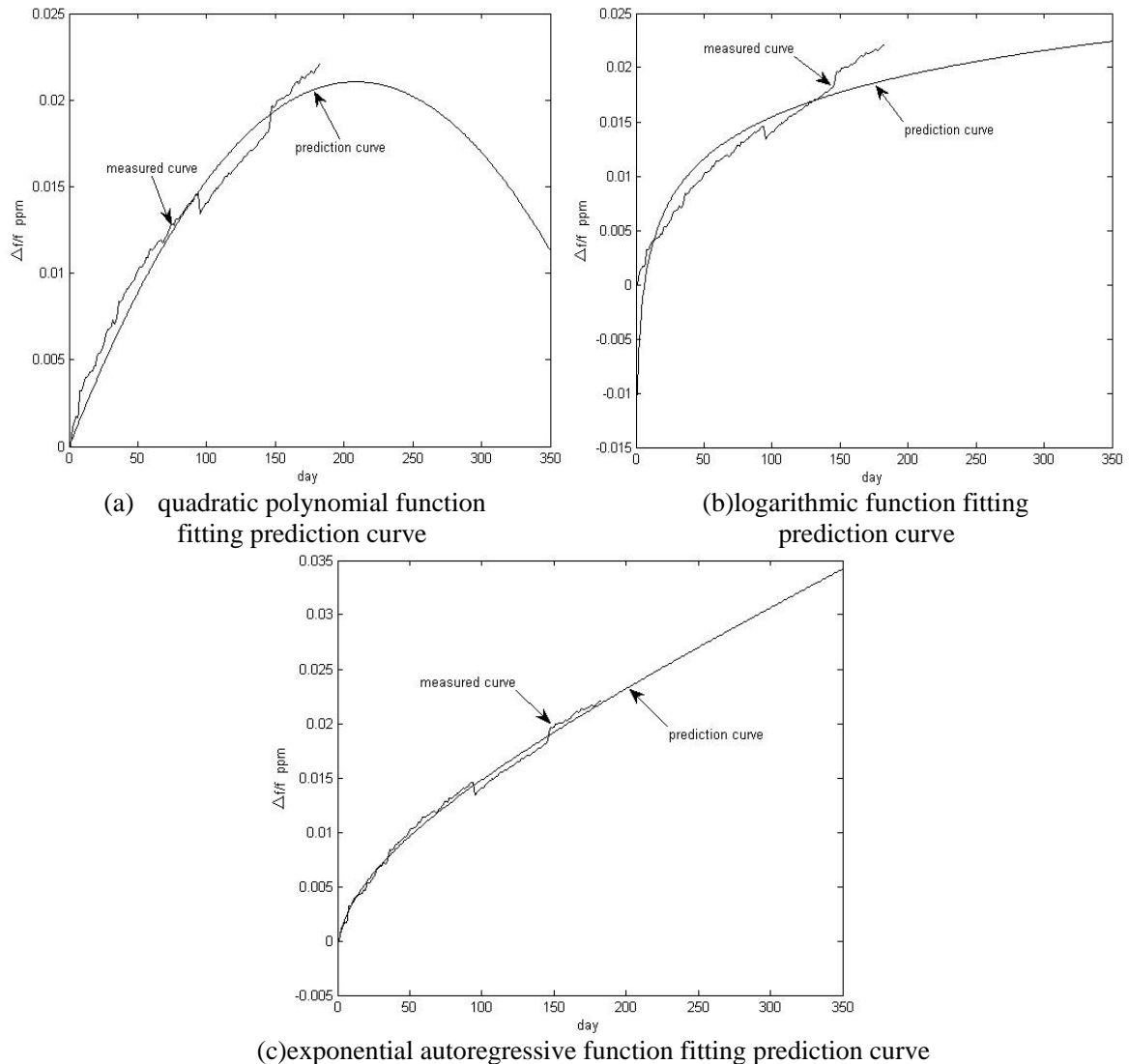


Fig.2 Model fitting prediction curves

As shown in Fig.2, with the increase of working time, the crystal frequency presents a tendency which changes monotonically^[17]. Compare the three model fitting prediction curves, we can draw a conclusion that the exponential autoregressive time series model is not only the closest to the actual data, which means it has an advantage in curve fitting quality but also the best in aging prediction, most suitable in reflecting the real frequency aging drift tendency of quartz crystal oscillator. Therefore, the last model is rather competent in describing the rule of crystal frequency aging.

4. Compensation Model Simulation

The main idea of prediction analysis method in solving the frequency aging of quartz crystal oscillator is: Based on the actual past data of oscillator frequency, predict the current frequency and future frequency, extract the essential compensation frequency by correlation computation with prediction aging rate data, output after overlying with the actual frequency of the oscillator. In the crystal oscillator with high stability, the frequency drift is improved over the power-up time. This fact that average frequency changes monotonically because of the aging phenomenon of frequency source's internal instrument has a characteristic similar to a system error. The frequency drift error can be obtained by time series prediction analyzation, with it the accuracy of frequency source can be corrected or calibrated.

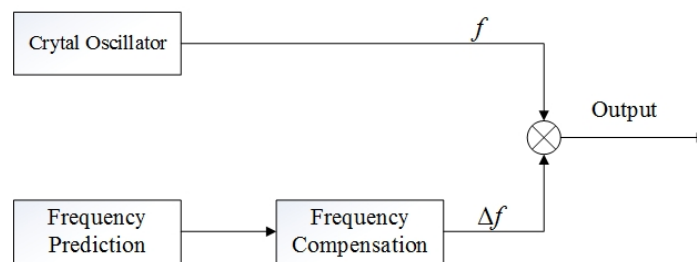


Fig.3 prediction compensation

In this paper, the frequency drift prediction curve of the crystal oscillator is obtained by the exponential autoregressive time series model. The compensation frequency is computed from the frequency drift data which extracted from the curve, before overlying to the output of the crystal oscillator. In order to compare the prediction aging drift data with actual measurement data more conveniently, we predict the upcoming frequency drift data by the data in the first 90 days of the crystal oscillator. After a series of computations, the result which presents the aging drift curve and the aging compensation curve of the constant temperature crystal oscillator is shown in Fig.4:

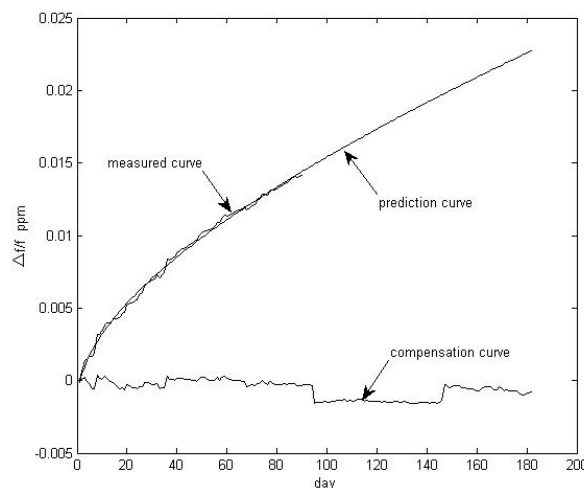


Fig.4 Aging drift curve and aging compensation curve of the crystal oscillator

In Fig.4, not only the crystal oscillator's change tendency of prediction aging drift and actual aging drift are manifested, so is the compensation result which obtained from the aging drift prediction data of mathematical model construction. We can find during the process of the computation that before the compensation, the maximum frequency aging drift rate is 0.02209ppm, that becomes 0.001597ppm after the compensation. The index, diurnal aging rate of crystal oscillator after compensation could reach the level of 10^{-11} . The result proves the exponential autoregressive time series aging model in this paper lower the error between frequency aging drift value and actual value generally, describe the frequency aging process of the crystal oscillator quite well. The prediction model could reasonably reflect the overall aging drift tendency of the constant temperature crystal oscillator, and make an effective prediction of the frequency aging.

5. Conclusion

Aiming at the frequency aging drift problem of the quartz crystal oscillator, we use the nonlinear time-varying exponential autoregressive prediction model in time series analysis method and compare it with some other models. The result shows that linear model is not suitable for the long-term change of the indicator of crystal oscillator, because the frequency drift value of the crystal oscillator presents a non-linear tendency with the extension of power-up working time; the quadratic polynomial function model fits the initial data, and could be used for short-term frequency

aging drift prediction. However, its range of application is bounded and will increase the error over time. The logarithmic model is applicable in the prediction model. Although it could reflect the overall tendency of the aging curve, the error between its fitting value and the actual value is quite significant. The exponential autoregressive time series aging model in this paper is applicable to various types of the crystal oscillator aging model with small errors. This model can describe the frequency aging process of the crystal oscillator quite well and prediction the frequency aging effectively. Verified by simulation, the result of the compensation is evident. The diurnal aging rate of crystal oscillator after compensation could reach the level of 10^{-11} .

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