The Evaluation of Bus Lines

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Abstract: This paper analyzes and evaluates the efficiency of bus lines from the aspects of line layout, hardware facilities, passenger’s comfort, convenience and vehicle’s speed. We use DEA evaluation method based on AHP constraint, through multi-objective decision model, to decompose the complex issues into parts.

Key words: DEA evaluation method; Bus line; Mathematical model

1. Introduction

This paper selects DEA evaluation method based on AHP constraint to evaluate the bus lines, in order to find the shortcomings of the bus line in the construction and operation through a comprehensive analysis and evaluation, to provide decisive basis for the optimization of bus lines.

2. The establishment of the model

Through analytic hierarchy process, comparing the importance of the evaluation factors to determine the weight and constructing a judgment matrix. Firstly, matrix is normalized by column; then the regularized elements are added by row; finally, the sum of each row is normalized.

2.1 Calculated value of the weight of the indicators

<table>
<thead>
<tr>
<th>Category</th>
<th>Target layer</th>
<th>Indicator layer</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>input indicators (v)</td>
<td>The structure of lines(X1)</td>
<td>The length of line x(11)</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-linear coefficient x(12)</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average distance</td>
<td>0.08</td>
</tr>
</tbody>
</table>
between stations \( x(13) \)

The probability that a station is well equipped \( x(21) \)

The number of buses \( x(22) \)

Full load rate at peak time \( y(11) \)

Delivery speed \( y(21) \)

Departure frequency \( y(22) \)

The rate of punctuality \( y(31) \)

The accuracy of information \( y(32) \)

<table>
<thead>
<tr>
<th>Hardware facilities (X2)</th>
<th>Comfort (Y1)</th>
<th>Fastness (Y2)</th>
<th>Convenience (Y3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x(13)</td>
<td>0.37</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>x(21)</td>
<td>0.75</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>x(22)</td>
<td>1.00</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>y(11)</td>
<td>0.83</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>y(31)</td>
<td>0.17</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>y(32)</td>
<td>0.17</td>
<td>0.75</td>
<td></td>
</tr>
</tbody>
</table>

### 2.2 The basic principle of DEA method

Suppose that there are \( n \) decision units DMU, and each decision unit has \( m \) kinds of input indicators and \( s \) kinds of output indicators. \( x_{i,j} \) indicates that the \( j \)-th DMU which corresponds to the value of the \( i \)-th input index, \( y_{i,j} \) indicates the value of the \( j \)-th DMU corresponding to the \( r \)-th output index. \( v_i \) is the weight vector for the \( i \)-th input index, \( u_r \) is the weight vector of the \( r \)-th output index, \( i = 1, \cdots, m; j = 1, \cdots, s \).

Corresponds to a set of weight coefficients:

\[
\begin{align*}
& v = (v_1, \cdots, v_m)^T, \\
& u = (u_1, \cdots, u_s)^T,
\end{align*}
\]

Each decision unit DMU has a corresponding efficiency evaluation index:

\[
h_j = \frac{u^T y_j}{v^T x_j} = \sum_{r=1}^{s} u_r y_{ij} \cdot \cdots \cdot \cdots j = 1, \cdots, n
\]

Select weight factors \( u, v \) appropriately, make it meet: \( h_j \leq 1, j = 1, \cdots, n \).

It can be seen from the above formula, to get the desired integrated efficiency value, the input indicator should be extremely small and the output indicator should
be extremely large, so the corresponding type of indicators should be transformed accordingly.

To evaluate the efficiency of the \( j_{0} \)-th decision unit DMU, we consider the
weight coefficient \( u, v \) as variable, make the \( j_{0} \)-th decision indicator as target, and consider the efficiency indexes of all decision units as constraints, then build the following optimization model (vector form):

\[
\begin{align*}
\max & \quad \frac{u^T y_{j_0}}{v^T x_{j_0}} \\
\text{s.t.} & \quad \frac{u^T y_j}{v^T x_j} \leq 1, \quad j = 1, 2, \ldots, n \\
& \quad v = (v_1, v_2, \ldots, v_m)^T \geq 0 \\
& \quad u = (u_1, u_2, \ldots, u_m)^T \geq 0
\end{align*}
\]

The optimal solution can be obtained from the model, and the maximum efficiency index can be obtained. The higher the efficiency index is, the better the decision unit DMU is.

The optimization model is a fractional programming problem, using the Charnes-Cooper conversion, it can be transformed into an equivalent linear programming problem.

Make: \( t = 1 / v^T x_{j_0} \); \( \omega = t \cdot v \); \( \mu = t \cdot u \), \( p \) will transformed into an equivalent linear programming model \((P_{c,R})\):

\[
\begin{align*}
\max & \quad h_{j_0} = \mu^T y_{j_0} \\
\text{s.t.} & \quad \omega^T x_j - \mu^T y_j \geq 0, \quad j = 1, 2, \ldots, n \\
& \quad \omega^T x_{j_0} = 1 \\
& \quad \omega = (\omega_1, \omega_2, \ldots, \omega_m)^T \\
& \quad \mu = (\mu_1, \mu_2, \ldots, \mu_m)^T
\end{align*}
\]

Introducing slack variables and residual variables, we get the following equation:
\[
\begin{align*}
\min \theta \\
\text{s.t.} \sum_{j=1}^{n} \lambda_j x_j - s^- & = \theta x_{j0} \\
\sum_{j=1}^{n} \lambda_j y_j - s^+ & = y_{j0} \\
\lambda_j & \geq 0, j = 1, 2, \ldots, n \\
s^- & = (s_1^-, s_2^-, \ldots, s_m^-)^T \geq 0 \\
s^+ & = (s_1^+, s_2^+, \ldots, s_m^+)^T \geq 0
\end{align*}
\]

In the formula: \( \theta \) is the evaluation index of the \( j_{th} \) decision unit DMU, \( n \) is the number of decision units DMU; \( S^- \)、\( S^+ \), respectively, for the slack variables and residual variables.

2.3 DEA evaluation method based on AHP constraint

The evaluation index system contains more indicators, there is a certain degree of difficulty in the application of DEA method analysis. Therefore, we need appropriate integration of indicators. To make DEA analysis with comprehensive indicators, the indicators are treated as follows:

\[
Z^k_i = \sum_j \omega_j \sigma^k_j
\]

In the formula

\( Z^k_i \) —— The value of the i-th comprehensive index in the k-th decision unit;

\( \omega_j \) —— The weight of the j-th sub-indicator in the i-th comprehensive indicator;

\( \sigma^k_j \) —— The value of the j-th sub-index of the k-th decision unit about the i-th comprehensive indicator.

Using the AHP method, after meeting the consistency test, the relative weight of each index is obtained, and calculate the comprehensive index of each decision unit.

3. Conclusion

In this paper, we use DEA evaluation method based on AHP constraint to evaluate the bus line. DEA evaluation method is simple, easy to use. It is used to evaluate decision units with multiple input and output indicators, which is different from the previous assessment method for dealing with individual outputs. The AHP: through a person's subjective intent to construct a judgment matrix, comparing multiple influencing factors, it embodies the basic characteristics of human decision-making thinking. For the model we have built, the optimization and resource allocation of the bus line were defined from a number of aspects and we combine AHP with DEA. This model is equally applicable for some of the similar problems in other areas. The scope of the
model is very broad, for example, in the choice of travel attractions and other common problems.

4. References