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Kinematics Simulation of 4－DOF Manipulator

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Abstract. For a 4-DOF manipulator, The Kinematic Model of Manipulator is built by the method of D-H, Analysis the solution of forward and inverse kinematics of manipulator, The method that algebraic method combined with the geometric method is used to solve forward and inverse kinematics. MATLAB software is used to simulate and get the working space diagram of the manipulator. The Forward and Inverse kinematics and the trajectory planning are simulated by using Robotics Toolbox and this algorithm is proved to be feasible and accurate, which provides the theoretical basis for the subsequent analysis and research of manipulator.

1. Introduction

The kinematics of the manipulator is to establish the relationship between the position and attitude of the manipulator space and the movement of each joint, Which provides the method to analysis the control of the manipulator motion. The main research are divided into forward kinematics and inverse kinematics. Forward kinematics is to get the position and attitude of the end effector relative to the coordinate system by known structural parameters and motion parameters of each manipulator joint; and it is inverse kinematics to get the motion parameters of each manipulator joint by known structural parameters and the position and attitude of the end effector. which is a nodus of kinematic of manipulator. In this paper, The Kinematic Model is built with 4-DOF manipulator, the forward and inverse kinematic and the working space diagram of the manipulator and trajectory planning are analysis and simulated by using Robotics Toolbox.

2.The kinematic of manipulator

2.1 Model of manipulator

Four-degree-of-freedom manipulator is built up by the actuator and four rotate joints which consists of the waist joint, the shoulder joint, the elbow joint and the wrist joint. the lumbar joint is revolute joint, the other joints are bending joint. The structure of the manipulator is shown in Fig.1.

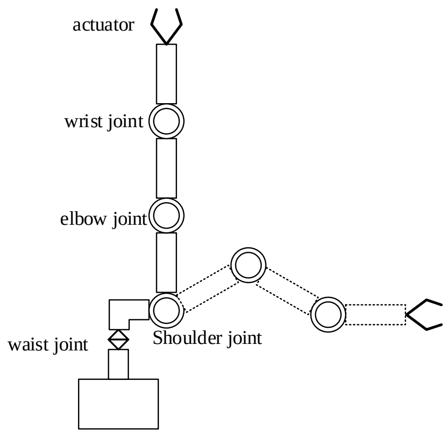

Fig.1 The structure of the manipulator

The kinematics coordinate system of 4-DOF serial manipulator is established by the method of Denavit-Hartenberg[1], as shown in Fig.2. θ_1、θ_2、θ_3、θ_4 is the rotation angle of four joints.

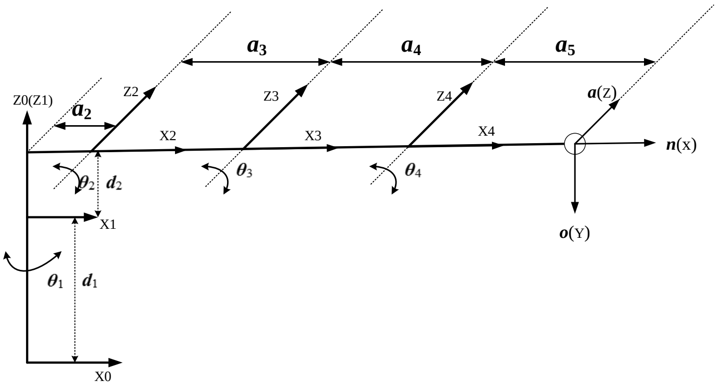

Fig 2. The kinematics coordinate system of 4-DOF serial manipulator

Joint 1 linked with base, so we take joint 1 and base as a whole to simplify computing, take the coordinate system of this joint as the base coordinate of manipulator. All joints are revolute joints and the axes of joint 2,3 and 4 are parallel, the end effector of manipulator is joint 5.

According to the structure and the kinematic coordinate system of the manipulator, we get the parameters of each joint and link.

Table 1 D-H parameters of joints

Joint i	θ_i /(°)	α_{i-1} /(°)	a_{i-1} /mm	d_i /mm	Range of θ_i /(°)
1	θ_1	0	0	0	-90~90
2	θ_2	-90	9	42	7~131
3	θ_3	0	77	0	-90~123
4	θ_4	0	97	0	-90~82
5	θ_5	0	133	0	0

The shortest distance between Z axes of adjacent joints is a_i, d_i is the shortest distance between X axes of adjacent joints. The angle between the Zi-1 axis when projection Z_i axis onto the plane of (Z_{i-1}, Y_{i-1}) is α_i, and θ_i is the angle between the Xi-1 axis after projection X_i axis onto the plane of (X_{i-1}, Y_{i-1}).

2.2 Forward kinematics

The forward kinematics of the manipulator is to get the desired position of the end effector of manipulator in the base coordinate system by known the angle of each joint. According to the method of D-H[2] and the kinematic coordinate system of the manipulator, the corresponding relationship between the two adjacent links is established.

The relative position and attitude transformation matrix of two adjacent links ${}^{i\text{-}1}_{\ i}T$ (i=1, 2, …, 5) can be obtained by this correspondence.

$$
\begin{aligned}
{}^{i\text{-}1}_{\ i}T &= Trans_z(d_i)Rot_z(\theta_i)Trans_x(a_{i-1})Rot_x(\alpha_{i-1}) \\
&= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{1}
$$

The transformation matrix of the end effector coordinate system relative to the base coordinate system can be obtained by multiply the coordinate transformation matrix of each adjacent links:

$$
\mathrm{T}={}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T\,{}^4_5T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1c_{234} & -c_1s_{234} & -s_1 & a_2c_1+a_3c_1c_2+a_4c_1c_{23}+a_5c_1c_{234}\text{-}d_2s_1 \\ s_1c_{234} & -s_1s_{234} & c_1 & a_2s_1+a_3s_1c_2+a_4s_1c_{23}+a_5s_1c_{234}+d_2c_1 \\ -s_{234} & -c_{234} & 0 & d_1-a_3s_2-a_4s_{23}-a_5s_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}
$$

In the formula (2), The position of the end effector of the manipulator is described by $p=[p_x, p_y, p_z]^{\mathrm{T}}$. $S_{ij}=\sin(\theta_i+\theta_j)$ 、 $C_{ij}=\cos(\theta_i+\theta_j)$ 、 $S_{ijk}=\sin(\theta_i+\theta_j+\theta_k)$ 、 $C_{ijk}=\cos(\theta_i+\theta_j+\theta_k)(i;\ j;k=1,2,3,4)$。

2.3 Inverse Kinematics

Inverse kinematics is to get the motion parameters of each manipulator joint by known structural parameters and the position and attitude of the end effector. The kinematics inverse plays an important role in the kinematics of robot, which is the precondition of kinematics analysis, motion space, offline programming and trajectory planning.

There are two kinds of methods to solve the inverse kinematics of manipulator, which are closed-form solution and numerical solution. The numerical solution has the disadvantages of large computation, slow speed. The closed-form solution is fast, efficient and suitable for real-time control. There are three kinds of methods of closed-form solution, which are iterative method, algebraic method and geometric method.

Iterative method is a conventional solution which solve the inverse kinematic by multiply the inverse matrix successively which is generated by adjacent joints. The inverse solution of the algebraic method[3,4] is based on the triangular transformation relationship between the parameters of the final transformation matrix, we can easily get one or several joint angle by substitute the corresponding values of corresponding parameters of the end effector matrix. The geometric method[5,6] is proposed by C.S.G.Lee in 1982,it is used to solve inverse kinematic by the triangle relation between the joint variable and links according to the structure.

2.3.1 Inverse kinematic method

（1）Solving θ_1

Obtained by formula (2).

$$\frac{n_y}{n_x} = \tan\theta_1, \quad \theta_1 = A\tan 2(n_y, n_x) \tag{3}$$

（2）Solving θ_3

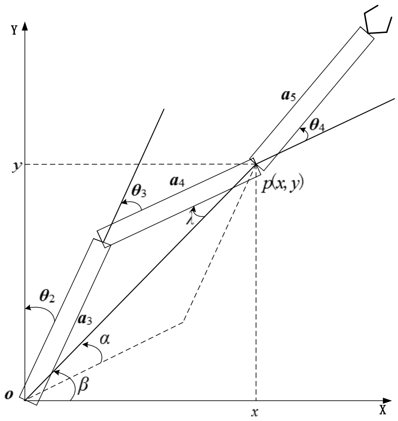

Fig.3 Partial joints of manipulator

As shown in Fig.3,this is the structure of the joints of 4-DOF manipulator except the waist joint, In the triangle consist of a_3, a_4 and op ,based on cosine theorem. θ_3 is obtained by formula（4）（5）.

$$x^2 + y^2 = a_3^2 + a_4^2 - 2a_3a_4\cos(180 - \theta_3) \tag{4}$$

$$\cos\theta_3 = \frac{x^2 + y^2 - a_3^2 - a_4^2}{2a_3a_4} \tag{5}$$

（3）Solving θ_2

In order to obtain θ_2, we first calculate the angle α and β, Figure 3 is shown.

$$\tan\beta = \frac{y}{x}, \quad \beta = A\tan 2(y, x) \tag{6}$$

$$\cos\alpha = \frac{x^2 + y^2 + a_3^2 - a_4^2}{2a_3\sqrt{x^2 + y^2}} \tag{7}$$

Obtained by formula(6)(7), $\theta_2 = \beta \pm \alpha$（$\theta_3 < 0$ is+， $\theta_3 > 0$ is－）

（4）Solving θ_4

Obtained by formula（2）.

$$\theta_2 + \theta_3 + \theta_4 = A\tan 2(n_z, o_z) \tag{8}$$

$$\theta_4 = A\tan 2(n_z, o_z) - \theta_2 - \theta_3 \tag{9}$$

Then the angle of inverse kinematic of the four joints of manipulator is obtained by the method above.

Algebraic method can easily solve one or a few joints angle. It eliminates the complexity of matrix calculation. It is intuitive and easy to understand and suitable for the manipulator which has no more than six freedom degrees and simply structure with geometric relationship between the joints.

The manipulator consists of four rotating and bending joints has clearly structure and few freedom. Combined with the method of geometric method with algebraic method can be easier get the relationship between angles according to the characteristics of these two methods, it can simplifies tedious mathematic operation in the process of inverse kinematics and quickly get the angle of each joints. First of all, the algebraic method is used to solve the value of θ_1 and $\theta_2+\theta_3+\theta_4$, then the value θ_2 and θ_3 is solved by the geometric method, finally θ_4 is obtained .

Each target pose may correspond to 4 sets of inverse solutions, select the only reasonable solution is needed. There is no uniform standard for chosen of the inverse kinematics solution. According to the actual situation, it is necessary to meet the requirements of the shortest route, the most power saving, obstacle avoidance and best force. Without considering the collision avoidance, the effective solution can be obtained by the range of each joint angle and the continuity of motion.

Since the first joint of the robot are relatively larger than the latter three. Here, the best solution is weighted ,when Δ obtain the minimum value we get the optimal solution.

$$\Delta = l_1\left|\Delta\theta_1\right| + l_2\left|\Delta\theta_2\right| + l_3\left|\Delta\theta_3\right| + l_4\left|\Delta\theta_4\right| \tag{10}$$

l_1、l_2、l_3、l_4 is weighting factor, due to the influence on the end effector which made by the angle change of the joint near the base is larger, so corresponding weighting factor is also larger. we set $l_1 = 1, l_2 = l_3 = l_4 = 0.5$. According to the principle of "less moving large joints and more moving small joints", the optimal solution can be obtained through MATLAB programming.

3. Reachable workspace of manipulator

Workspace is the description of the reachable workspace of the end effector of manipulator, which shape and size reflect the range of motion of the manipulator. it is important target to measure the structure and performance of the manipulator, which is of great significance in the design, control and operation of the manipulator. Reachable space is the set of all the positions of the reference points of the end effector. Monte Carlo method is used to analyze the workspace of the manipulator.

Monte Carlo method[7] is a method of using random sampling(pseudo-random number) to solve mathematical problems. According to the position vector of the point of end effector of the manipulator in the coordinate system, Using Rand function of MATLAB to generate a series of uniform pseudorandom numbers in [0,1], $\theta_i^{\max}$ is upper bound of joint angle, $\theta_i^{\min}$ is the lower bound of joint angle, *i* is the number of joints.

$$\left(\theta_i^{\max} - \theta_i^{\min}\right) \times Rand\left(N,1\right) \tag{11}$$

The random values of each joint angle are generated.

$$\theta_i = \theta_i^{\min} + \left(\theta_i^{\max} - \theta_i^{\min}\right) \times Rand(N,1) \tag{12}$$

The pseudo-random values of the generated joint variables are substituted into the forward kinematic equation, the corresponding position of the end point of the manipulator is calculated, extract the coordinates of x,y,z, Using the *plot3* function in MATLAB to draw the workspace of the manipulator and the projection of the workspace on each coordinate plane, The workspace of the manipulator is analyzed in multi-faceted, which provides a reference for the optimization of the workspace of the manipulator and the adjustment of its structural optimization. As shown in Fig.4.

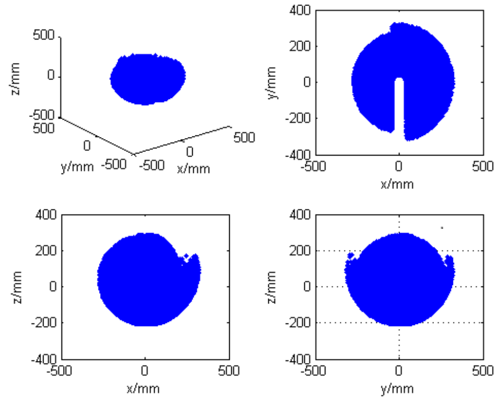

Fig.4 The three-dimensional workspace and the projection of the workspace on each coordinate plane of manipulator

As can be seen from Fig.4, the working space of the manipulator is an approximate ellipsoid and has a cylindrical notch, which is limited by the rotation range of the joints.

4. Simulation of kinematics of manipulator

4.1 Simulation of inverse kinematics

The inverse kinematic of the manipulator is verified by using the Robotics Toolbox[8,9] to create a robotic arm model. *link* function $\mathrm{L} = link([alapha \quad A \quad theta \quad D \quad sigma], 'standard')$ is used, $alapha$ is angle between joints, A is the length of link between joints, $theta$ is the rotation angle of a joint, D is offset distance between the adjacent joints in height. $sigma$ is the identification of the joint type, $sigma = 1$ means translational joint, $sigma = 0$ means pivot joint. $standard$ indicates the use of standard D-H parameters. As shown in Fig.5.

(*x*,*y*,*z*) indicates the position of the end of the manipulator, (*ax*,*ay*,*az*) represents the vector of *a* direction of the end effector. One group of angles joint angles are selected, which are [0°, 0°, -90°, 90°]. It can be calculated from Fig.5 that $x=a2+a3+a5=219mm, z=d2+a4=139mm$, which is in great agreement with the simulation results, which proves the accuracy of the model of forward kinematic.

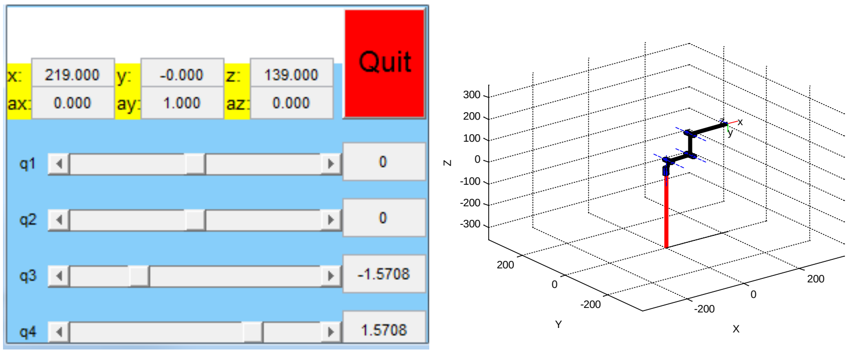

Fig.5 the verification of forward kinematic

The forward kinematic can also be verified using the *fkine* function in Robotics Toolbox, *qz*=[60°, 45°, -90°, 90°] is a set of initial angles of joint, The matrix of the position of end joint T which calculated by the set of angles consistent with the transformation matrix of end joint obtained by using *fkine(r,qz)* function, the correctness of the forward kinematics equation of the manipulator is verified.

The verification process of inverse kinematics is as follows: The joint angles we get from the inverse kinematics solution of the end position and attitude matrix obtained by the forward kinematics equation are verified the initial joint angles.

The inverse kinematics can be solved by *ikine* function in Robotics Toolbox. Take the above matrix as an example, the initial position of the robot is *q*=[0,*pi*/2,0,0], M=[1 1 1 1 0 0], M represents the actual number of joints。*qi*=*ikine*[*r*,T,*q*,M]=[1.0472 0.7854 -1.5708 1.5708] consistent with the initial joint vector *qz*. We can also verify the forward kinematics and inverse kinematics by programming in MATLAB, and the results are exactly the same.

4.2 Trajectory planning

Trajectory planning[9] and Simulation of manipulator based on Robotics Toolbox by using function $\lfloor q, q_3, q_4 \rfloor = jtraj(q_1, q_2, T)$. q representation the space trajectory planning of joint from state q_1 to q_2, q_3 and q_4 is differentiate means velocity and acceleration of trajectory planning, T represent length of time. The *fkine* function can be used to obtain the joint position, velocity and acceleration curve by trajectory planning. The space trajectory of the end effector of manipulator is shown in Figure 6. The simulation program is as follows:

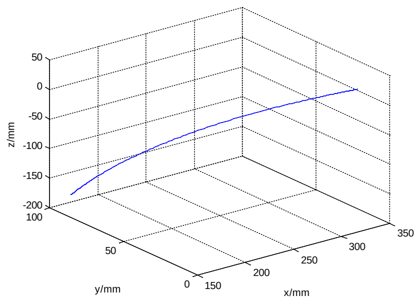

Fig.6 The space trajectory of the end effector of manipulator

It can be seen from the figure 7 and 8 that the end of the manipulator reaches the desired position and the curve of it is continuity and allay with time changes, which verifies the correct of the structural design of the manipulator.

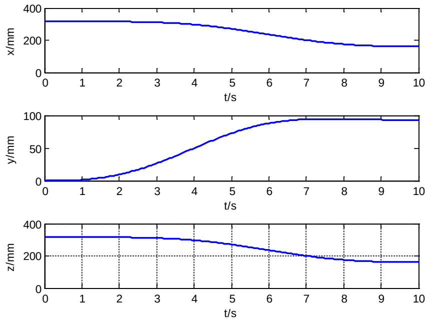

Fig.7 the curve of the end position of the manipulator

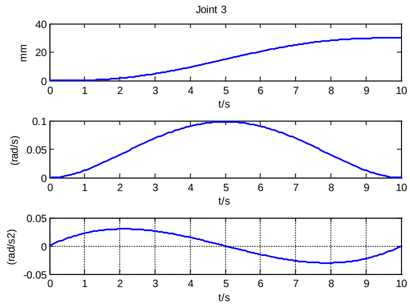

Fig.8 The curve of joint position, velocity and acceleration of joint 1

5. Summary

In this paper, for four degree of freedom manipulator, according to the D-H method, the forward and inverse kinematics of the manipulator is studied. Combined with the manipulator has less degrees of freedom and simple structure, using algebra method with geometric method to solve inverse kinematics, Simplify the complex of derivation process of inverse kinematic, makes the calculation process more simply and no theoretical error. The robot forward and inverse kinematics is verified by MATLAB programming, and the workspace and trajectory planning of manipulator are simulated by Robotics Toolbox. The simulation and verification results show that the method of modeling and Simulation of the manipulator is effective, and the inverse kinematics algorithm is correct and feasible. It is convenient to to realize this algorithm. It has certain universality for the manipulator with no more than six degrees of freedom and provides reference for further analysis and research on robot.

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