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Sustainable Dual-Channel Supply Chain with Demand Uncertainty and Capacity Constraints

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Abstract
This paper focuses on the impact of consumer low-carbon awareness (CLA) and competition on order quantity in dual channel supply chain of one-manufacturer and one-retailer under the capacity constraints. Based on the economic order quantity model, this paper shows two scenarios: the centralized model and the decentralized model. And we can get the optimal order quantity and wholesale price from these scenarios.

Key words: *dual channel supply chain; demand uncertainty; sustainability; capacity constraints*

1 Introduction

An increasing number of companies focus on green products and put plans into practice (*G. Hua et al.*[1], 2011). Meanwhile, with the rapid development of e-commerce and increasing consumer acceptance of green products, many manufacturers are opening online shops to sell green products (*Q. Li et al.* [2], 2015). This is called a dual-channel supply chain, including an independent retail channel and an online channel (*S. K. Mukhopadhyay et al.*[3], 2008). Obviously, the online sales of green products can help manufacturer win the benefits associated with increasing demand for the products. This paper mainly answers the following problems: 1) in the sustainable dual-channel supply chain, when facing the demand from two channels with uncertainty, how the manufacturer makes the decisions of the production quantity. 2) When the production capacity is constraint, how the manufacturer distributes the production quantity between the two sustainable channels?

2 Notations and assumptions

We consider a supply chain consisting of one manufacturer, one retailer and one single product. The manufacturer, the leader of market, sells its product to both the retailer and the market directly. We assume that each one has two factors --price and the unit carbon emission reduction (CER)—influencing their demand coming from the market (*M. Laroche et al.*[4], 2011). We denote that the demand function of retail channel and direct channel for the product are D_r, D_d respectively and the demand

functions follow the following structure:

$$D_r = ma - p_1 + \beta(p_2 - p_1) + \theta e;$$
$$D_d = (1-m)a - p_2 + \beta(p_1 - p_2) + \theta e.$$

where a is initial market potential, β price difference and θ is consumer low-carbon awareness (CLA). CLA affects consumer willingness-to-pay of environmental friendly products and it can greatly changes with industries, consumers, and time so we can assume θ to be a random variable following a uniform distribution in the range of $[0, d]$. In order to simplify the presentation and analysis, we denote $u_1 = ma - p_1 + \beta(p_2 - p_1)$, $u_2 = (1-m)a - p_2 + \beta(p_1 - p_2)$,then $D_r = u_1 + \theta e$, $D_d = u_2 + \theta e$ so u_1, u_2 represent the deterministic demand of products 1 and products 2 influenced by price only. Table 1 summarizes the major notations in our following model development.

Table 1-- Parameters and decision variables in model

Parameters	
i	the retail channel and direct channel when $i = 1,2$ respectively
p_i	the sale price of product i
v_i	the unit salvage value of product i at the end of sale period
m	the retailer's proportion of initial market potential, and $1-m$ is the manufacturer's proportion
a	initial market potential
e	the unit carbon emission reduction(CER) of product
β	the sensitivity of switchovers toward price difference
k	the sensitivity of manufacturer's costs toward CER
θ	consumer low-carbon awareness(CLA)
d	the upper bound of CLA
π_r、π_m、π^s	the profit of retailer, manufacturer and the supply chain respectively
$\prod_r$、$\prod_m$、$\prod^s$	the expected profit of retailer, manufacturer and the supply chain respectively
decision variables	
Q_i	the order quantity of product i at the beginning of the sale period
w	the unit wholesale price charged by the manufacturer
r_i	return credit per unit paid by the manufacturer for returned goods

In this paper, we discuss the total order quantity Q_1+Q_2 is limited by A, and the percentage of the retailer is n,so the percentage of the manufacturer is $1-n$. A can be interpreted as the manufacturer's production capacity, and nA is the retailer's inventory capacity. Given the capacity constraint, it may not be feasible for the retailer to order according to the demand function. As such, the retailer needs to take

the capacity constraint into consideration when deciding order quantity for the product. We assume that $u_1 \le nA$, $u_2 \le (1-n)A$ to ensure that the deterministic demand can be satisfied.

3. Centralized model (M1)

The central planner's problem with a capacity constraint is described as follows:

$$\max_{Q_1,Q_2} \prod^{s}(Q_1,Q_2) = p_1Q_1 + p_2Q_2 + (v_1-p_1)\int_0^{\varphi_1}(Q_1-D_r)f(\theta)d_\theta + (v_2-p_2)\int_0^{\varphi_2}(Q_2-D_d)f(\theta)d_\theta - \frac{1}{2}ke^2$$

$$s.t.\ Q_1 \le nA,\ \ Q_2 \le (1-n)A.$$

where $\varphi_1 = \frac{Q_1-u_1}{e}, \varphi_2 = \frac{Q_2-u_2}{e}$.

With the Lagrangian multiplier, the problem can be written as follows:

$$L(Q_1,Q_2,\lambda_1,\lambda_2) = (p_1-\lambda_1)Q_1 + (p_2-\lambda_2)Q_2 + \frac{(v_1-p_1)(Q_1-u_1)^2}{2de} + \frac{(v_2-p_2)(Q_2-u_2)^2}{2de} - \frac{1}{2}ke^2 + \lambda_1 nA + \lambda_2(1-n)A$$

Let

$$\lambda_1^* = (nA-u_1)\frac{(v_1-p_1)}{de} + p_1$$

$$\lambda_2^* = [(1-n)A-u_2]\frac{(v_2-p_2)}{de} + p_2$$

Two critical capacity points

$$A_1 = \frac{dp_1e}{n(p_1-v_1)} + \frac{u_1}{n}$$

$$A_2 = \frac{dp_2e}{(1-n)(p_2-v_2)} + \frac{u_2}{(1-n)}$$

Satisfy $\lambda_1^*(A_1) = 0$、$\lambda_2^*(A_2) = 0$.

Theorem 1. In Centralized model with capacity constraints, the optimal order quantities (Q_1^A, Q_2^A) can be expressed as the follows, if

(1)$\max\left\{\frac{u_1}{n}, \frac{u_2}{(1-n)}\right\} < A_1 < A_2$,

i. if $A > A_2$, then $Q_1^A = Q_1^*$, $Q_2^A = Q_2^*$;
ii. if $A_1 < A < A_2$, then $Q_1^A = Q_1^*$, $Q_2^A = (1-n)A$;
iii. if $A < A_1$, then $Q_1^A = nA$, $Q_2^A = (1-n)A$;

(2)$A_1 < \frac{u_2}{(1-n)}$,

i. If $A > A_2$, then $Q_1^A = Q_1^*$, $Q_2^A = Q_2^*$;
ii. If $A_1 < A < A_2$, then $Q_1^A = Q_1^*$, $Q_2^A = (1-n)A$;
iii. If $A < A_1$, then $Q_1^A = nA$, $Q_2^A = (1-n)A$;

(3)$\max\left\{\frac{u_1}{n}, \frac{u_2}{(1-n)}\right\} < A_2 < A_1$,

i. If $A > A_1$, then $Q_1^A = Q_1^*$, $Q_2^A = Q_2^*$;
ii. If $A_2 < A < A_1$, then $Q_1^A = nA$, $Q_2^A = Q_2^*$;
iii. If $A < A_2$, then $Q_1^A = nA$, $Q_2^A = (1-n)A$;

(4)$A_2 < \frac{u_1}{n}$,

i. If $A > A_1$, then $Q_1^A = Q_1^*$, $Q_2^A = Q_2^*$;
ii. If $A_2 < A < A_1$, then $Q_1^A = nA$, $Q_2^A = Q_2^*$;
iii. If $A < A_2$, then $Q_1^A = nA$, $Q_2^A = (1-n)A$.

Theorem 1 presents the optimal solutions for each party given different levels of production capacity A.when the capacity A is sufficiently large $A \geq \max\{A_1, A_2\}$, the optimal order quantities ,the channel profit ,and the change of optimal solutions with m, e, d, β are the same as those in the centralized without a capacity constraint. When the capacity is less than one of the critical points(A_1, A_2), the optimal order quantity reduces to the capacity constraint point $(nA, (1-n)A)$, and the channel profit decreases.

4. Decentralized model(M2)

In this section, we present the manufacturers' and the retailers' optimal decisions in the decentralized model when there is a capacity constraint. The expected profit function of retailer and manufacturer are as follows:

$$\prod_r^d(Q_1) = p_1Q_1 + (v_1 - p_1)\int_0^{\varphi_1}(Q_1 - D_r)f(\theta)d_\theta - wQ_1;$$

$$\prod_m^d(Q_2, w) = wQ_1 + p_2Q_2 + (v_2 - p_2)\int_0^{\varphi_2}(Q_2 - D_d)f(\theta)d_\theta - \frac{1}{2}ke^2.$$

To solve the Stackelberg game model with manufacturer-leader, we can obtain the Theorem 2 and Theorem 3. We omit the proofs of the Theorems.

Theorem 2. In the decentralized model without capacity constraints, the optimal production quantities of retailer and manufacturer and the wholesale price are as follows:

$$Q_1^{d*} = \frac{dp_1e}{2(p_1-v_1)} + \frac{u_1}{2}; Q_2^{d*} = \frac{dp_2e}{p_2-v_2} + u_2; w^* = \frac{u_1(p_1-v_1)}{2de} + \frac{p_1}{2}.$$

Theorem 3. In decentralized model with capacity constraints, the optimal order quantities (Q_1^A, Q_2^A) can be expressed as the follows, if

(1) $\max\left\{\frac{u_1}{n}, \frac{u_2}{(1-n)}\right\} < A_1^d < A_2^d$,

i. If $A > A_2^d$, then $Q_1^{dA} = Q_1^{d*}$, $Q_2^{dA} = Q_2^{d*}$, $w^{d*} = w^*$.

ii. If $A_1^d < A < A_2^d$, then $Q_1^{dA} = Q_1^{d*}$, $Q_2^{dA} = (1-n)A$, w^{d*} is determined by the bargain power between the retailer and the manufacturer.

iii. If $A < A_1^d$, then $Q_1^{dA} = nA$, $Q_2^{dA} = (1-n)A$, w^{d*} is determined by the bargain power between the retailer and the manufacturer.

(2) $A_1^d < \frac{u_2}{(1-n)}$,

i. If $A > A_2^d$, then $Q_1^{dA} = Q_1^{d*}$, $Q_2^{dA} = Q_2^{d*}$, $w^{d*} = w^*$;

ii. If $A_1^d < A < A_2^d$, then $Q_1^{dA} = Q_1^{d*}$, $Q_2^{dA} = (1-n)A$, w^{d*} is determined by the bargain power between the retailer and the manufacturer.

iii. If $A < A_1^d$, then $Q_1^{dA} = nA$, $Q_2^{dA} = (1-n)A$, w^{d*} is determined by the bargain power between the retailer and the manufacturer.

(3) $\max\left\{\frac{u_1}{n}, \frac{u_2}{(1-n)}\right\} < A_2^d < A_1^d$,

i. If $A > A_1^d$, then $Q_1^{dA} = Q_1^{d*}$, $Q_2^{dA} = Q_2^{d*}$, $w^{d*} = w^*$;

ii. If $A_2^d < A < A_1^d$, then $Q_1^{dA} = nA$, $Q_2^{dA} = Q_2^{d*}$, w^{d*} is determined by the bargain power between the retailer and the manufacturer.

iii. If $A < A_2^d$, then $Q_1^{dA} = nA$, $Q_2^{dA} = (1-n)A$, w^{d*} is determined by the bargain power between the retailer and the manufacturer.

(4) $A_2^d < \frac{u_1}{n}$,

i. If $A > A_1^d$, then $Q_1^{dA} = Q_1^{d*}$, $Q_2^{dA} = Q_2^{d*}$, $w^{d*} = w^*$;

ii. If $A_2^d < A < A_1^d$, then $Q_1^{dA} = nA$, $Q_2^{dA} = Q_2^{d*}$, w^{d*} is determined by the bargain power between the retailer and the manufacturer.

iii. If $A < A_2^d$, then $Q_1^{dA} = nA$, $Q_2^{dA} = (1-n)A$, w^{d*} is determined by the bargain power between the retailer and the manufacturer.

Theorem 3 presents the optimal solutions for each party gave different levels of production capacity A. When the production capacity A is sufficiently large $A \geq max\{A_1, A_2\}$, the optimal order quantities, the channel profit are the same as those in

the decentralized without a capacity constraint. When the capacity is less than one of the critical points (A_1, A_2), the optimal order quantity reduces to the capacity constraint point $(nA, (1-n)A)$, the one's order quantities whose optimal order quantity reduces is not be affected by wholesale price. And the channel profit is less than that of the centralized model with a capacity constraint because of the double marginalization and the decentralized model without capacity constraint.

4. Conclusions

Based on the economic order quantity model, this paper shows two scenarios: the centralized model and the decentralized model. And we can get the optimal order quantity, wholesale price from these scenarios.

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