Pipe Dream——A Water-Recycling Dream

Lang Liu

North China Electric Power University Baoding，Baoding 071000，China

2100639007@qq.com

Abstract: Our basic model is that to make some predictions for Billings next thirty years development. The major assumption is that the whole industry of the city can be divided into three parts: the agricultural area, the industrial area and the domestic area.

We firstly make some predictions based on the collected data for different walks. And the future 30 years water usage trend has been given out by using the least square method and multiple linear regression model. Then we build three basic models for the three major departments. The irrigation model establishes the ditch network for west crops region accompanied by the sprinkling irrigation model used for the livestock husbandry. Next step, the industrial water consumption model is applied to the current factories.

Keywords: Billings, the least square method, multiple linear regression model

1 Introduction

According to our former reaches and data collection work, nowadays, Billings has to face lots of problems involving the precious water resource. And the pressure mainly comes from three major dimensions:

- Firstly, there plenty of water wasted by unproper irrigating ways for crops and till 2015 the water usage for livestock husbandry and the agricultural crops shows no apparent differences.
- Secondly, as for industry area, the factories in inner land still have to tackle with the high costs coming from the constructing and maintaining the water pipelines.
- Thirdly, some water pipelines in domestic area have reused and unnecessary redundancy.

2 Model : Water Need Project In 30 Years

2.1 Model Overview

In order to devise an effective water project strategy, we firstly need to predict the total amount of water demanded and that should be supposed in the next 30years. The water demand and supply forecast is the preparation work for models aiming to

increase water resource. That is to say, our water-recycling plan, later movement and strategy are all based on the forecast of water.

2.2 Basic Model

2.2.1 The least square method[1]

We have a series of points that would like to put them into a curve. So we define:

$$f(x) = a_1 r_1(x) + a_2 r_2(x) + ... + a_m r_m(x)$$

where r_k is a set of linearly independent functions and a_k is undetermined coefficient. If the vertical coordinates of each point is $y_i (i = 1, 2, ..., n)$, then our goal is to get sum of squares of the least squares between $y_i (i = 1, 2, ..., n)$ and $f(x)$. So we define the following parameter:

$$J(a_1, ..., a_m) = \sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} [f(x_i - y_i)]^2$$

We order that $\frac{\partial J}{\partial a_k} = 0$, and get $\sum_{k=1}^{m} a_k [\sum_{i=1}^{n} r_i(x_i) r_k(x_i)] = \sum_{i=1}^{n} r_j(x_i) y_i (j = 1, ..., m)$, if we define :

$$R = \begin{bmatrix} r_1(x_1) & ... & r_m(x_1) \\ \vdots & \vdots & \vdots \\ r_1(x_n) & ... & r_m(x_n) \end{bmatrix}$$

$$A = [a_1, ..., a_m]^T, Y = [y_1, ..., y_n]^T, \quad R^T RA = R^T Y$$

then the final output is: $A = (R^T R)^{-1} R^T Y$

2.2.2 Multiple linear regression model[2]

We generally call

$$\begin{cases} Y = X\beta + \varepsilon \\ E(\varepsilon) = 0, COV(\varepsilon, \varepsilon) = \sigma^2 I_n \end{cases}$$

Gauss- Mark off linear model(K variables linear regression model),and this model is usually abbreviated as $(Y, X\beta, \sigma^2 I_n)$.

Where

$$Y=\begin{bmatrix} y_1 \\ \cdots \\ \cdots \\ y_n \end{bmatrix}, X=\begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix}, \beta=\begin{bmatrix} \beta_0 \\ \beta_1 \\ \cdots \\ \beta_k \end{bmatrix}, \varepsilon=\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdots \\ \varepsilon_n \end{bmatrix}$$

With the help of multiple linear regression model, we can figure out the weight of each factor and then predict the total water consumption in the future. Also, we can also forecast the living, industry and agriculture water consumption separately.

3 Solution to the problem

To do water forecast, we apply the least square method. We collect Billings' water resources data form the United States Environmental Protection Agency [1985-2010], including industrial production water, agriculture and animal husbandry water and residents living water . After the relationship between time and Billings' total water use per year, we firstly draw the chat and observe the changes in water consumption. Then together with multiple linear regression model, we predict the next following 30 years' consumption. The condition of water usage from 1985 to 2010[USGS 1985-2010] is list as follows.(Unit: mgal/d)

Table 2.Water consumption of each sector from 1985 to 2010(every 5 years)[3]

Time	Total	Industry	Agriculture	Living
1985	401.68	9.13	318.22	28.62
1990	440.38	9.68	399.07	37.35
1995	447.68	10.51	380.23	53.46
2000	483.55	9.04	434.34	34.97
2005	487.46	10.38	418.35	60.12
2010	501.59	9.86	413.93	45.17

Then, we can get a water consumption trends figure as shown below:

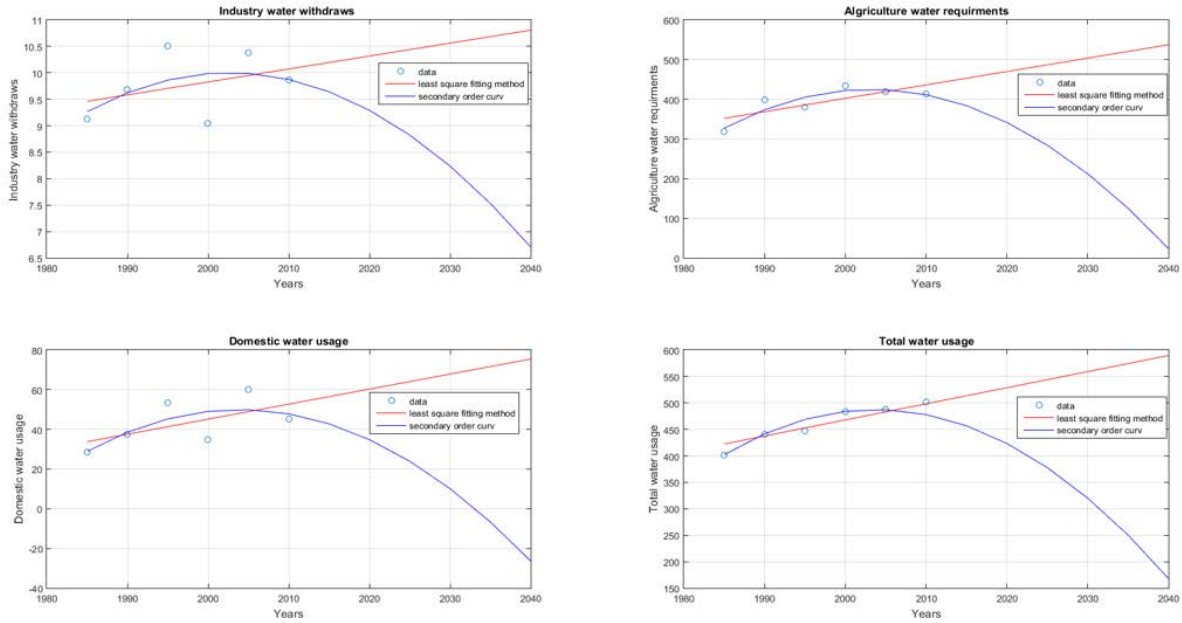

Figure 1.2. Predicted water consumption in next thirty years

With the help of the least square method and multiple linear regression model, we draw the linear and nonlinear fit function as shown below to reflect the next 30 years water demand. The results is shown in **Figure 1.2.**Then we discover that the least square method is better than the multiple linear regression model.

From the two linear and non-linear figures above, we can view that the different two curve reflect different trend of the water consumption. Take agriculture for example: according to the data in USGS, we can have an idea that agriculture and animal husbandry is one of the main industries of Billings.

However, in the figure above, the linear shows a upward trend, while the non-linear reflect an opposite trend. We analyze this phenomenon can attribute to too less data, so we can't successfully predict the actual water consumption trend. As for other curves, such as the domestic water consumption, the linear curve and the non-linear curve fit well with each other ,so we can draw a conclusion that parts of our prediction are correspond to reality but we still need to a more comprehensive, more accurate estimate of water consumption in Billings.

4 Model Improvement

The result we get from the least square method is not very ideal, and we believe the key reason lies in the lack of data. In other words, we only get the data from 1985 to 2010. So the six-group data is too less to forecast the thirty-year trends of future .In view of current situation, we devise a Grey Forecasting Model to get data with higher reliability, thus successfully overcoming the weakness of the least square method.

5 Reference

[1] Y. Yongsheng, W. Haiyan, W. Xuan, "A novel least-square method of source localization based on acoustic energy measurements for uwsn", *Signal Processing Communications and Computing (ICSPCC) 2011 IEEE International Conference*, pp. 1-5, Sept 2011.

[2] Liu Yan, "Mathematical model of mulitple linear regression", Journal of Shenyang Institute of Engineering (Natural Science), vol. 1, no. 2,3, pp. 128-129, 2005.

[3]Whole climate map of this area
http://www.drought.gov/drought/area/mt