Product Line and Channel Structure Design with Responsive Pricing

Xiaolu Zuo

China Merchants Group Postdoctoral Research Station, Guangdong Shenzhen 518067, China

Abstract. This paper studies a manufacturer’s product line and channel structure decisions when downstream channel(s) using holdback pricing strategy. Products have two quality levels: high quality and low quality, and the manufacturer has five product-channel strategies: (1) Duopoly channel with high quality products ($DH$); (2) Duopoly channel with low quality products ($DL$); (3) Duopoly channel with mixed quality products ($DM$); (4) Monopoly channel with high quality products ($MH$); (5) Monopoly channel with low quality products ($ML$). The product-channel is designed by making technology adoption and resource allocation decisions for the products. We characterize equilibrium quantity-pricing decisions of two channels and the optimal resource allocation decisions. We find that profit gap of different technology adoption strategies increases as market size raises. Product-channel strategy is influenced by the distribution of potential market demand to a large extent.

Keywords: Product line, channel structure, pricing; quality, game theory.

1. Introduction

Product line and channel structure design, which are relatively inflexible marketing strategies due to rapid innovation or short product life cycles, are interrelated with each other. Pricing, as a responsive marketing strategy, is constrained by fixed product line and channel structure.

When new production technology comes out with manufacturer’s successful adoption, products have two quality levels: high quality ($H$) and low quality ($L$). Manufacturer decides how many product lines to offer, whether to adopt new technology for each product line or not and how to allocate resource to different product lines. Here we consider five product-channel strategies based on product quality and channel structure: (1) Duopoly channel with high quality products ($DH$); (2) Duopoly channel with low quality products ($DL$); (3) Duopoly channel with mixed quality products ($DM$); (4) Monopoly channel with high quality products ($MH$); and (5) Monopoly channel with low quality products ($ML$).

As e-commerce continues to expand, increasingly, $DH$, $DL$ and $DM$ become commonly used strategies as manufacturers have been selling through online channels, in addition to retail channels (Chen et al., 2008; Banciu et al., 2010). Examples using $MH$ strategy can be found in luxury goods sellers, who focus on the upper echelon and offer limited editions of products (Branch, 2004). Since investment decisions need to be made years before demand is realized (Anupindi and Jiang, 2008), in this study we use responsive pricing strategy.

Assume the manufacturer uses production technology researched and developed by others in the industry. The manufacturer has three technology adoption strategies: Not adopting new technology ($N$); adopting new technology for both channels ($Y$); and adopting new technology for one channel and not adopting for the other channel ($M$). We use $\theta$ to capture the stochastic result of technology adoption, with value equals to 1 or 0. When $\theta=1$, technology is successfully adopted with probability $\gamma$; otherwise, when $\theta=0$, technology adoption fails with probability $1-\gamma$. We use $\gamma \in [0,1]$ to stand for successful technology adoption rate, indicating the maturity of new industrial technology or manufacturer’s capability of adopting new technology.

In this study, we would like to consider the following questions. (1)What’s the optimal technology adoption strategy? (2)How should the manufacturer manage channel competition by allocating resource between the channels? (3)What is the optimal quantity and pricing decisions of each channel?
2. Related Literature

We extend the product line literature by showing that the combination of technology choice and resource allocation decision determines product line. Product diversification is an important means to practice price discrimination and manage diversified demand (Bernad et al., 2010). Kim and Chhajed (2002) consider a monopolist offers two multi-attribute products to serve a market with two customer segments. Ferguson and Koenigsberg (2007) investigate a monopoly’s production line decision by deciding a firm’s production and pricing decisions. Lacourbe et al. (2009) find that variable costs drive vertical differentiation, which is consistent with our analysis.

Responsive pricing is used by many industries so as to better manage supply and demand and increase profits (Liu and Zhang, 2013). Tomlin and Wang (2008) find that responsive pricing benefits the firm much more than recourse allocation. Biler et al. (2006) find that considering price postponement at the planning stage leads to a significant increase in profits. Petruzzi and Dada (1999), and Dana and Petruzzi (2001) all consider the quantity-and-price problem in a single-product newsvendor setting. Bish and Wang (2004) consider a two-product newsvendor model, but they do not consider the problem in a competitive situation.

3. The Model

The manufacturer is risk neutral and maximizes expected profits. The basic model is illustrated in figure 1. The manufacturer produces two substitutable and horizontally differentiated (e.g. different colors, designs, etc.) products, indexed by \( a \) and \( b \), which are sold by different sales channels and engaging in competition in the same market. New production technology, which can improve product quality, occurs during the production stage with successful adoption rate \( \gamma \in [0,1] \). The product is considered “new” if it is produced using new technology, or “old” if it is produced without technology update. The unit production cost of new product and old product is \( f_c \) and \( d_c \), respectively, and \( f_c > d_c \).

\[
\begin{align*}
\text{Manufacturer} & \quad \text{Retailer a} \\
& \quad \text{Retailer b} \\
& \quad y, y \in [0, K] \\
& \quad K - y \\
& \quad 0 \leq q_a \leq y \\
& \quad 0 \leq q_b \leq K - y \\
& \quad \text{Unit production cost of new product: } f_c \\
& \quad \text{Unit production cost of old product: } d_c \quad (c_j < c_i)
\end{align*}
\]

Fig. 1 Basic model structure

A three stage decision is made in the analysis (figure 2). The total production resource for the two products is \( K \), the manufacturer allocates resource between the products and each resource produces one product. Assume product \( a \)’s resource quantity is \( y \), \( 0 \leq y \leq K \) and product \( b \) is allocated with resource quantity \( K - y \). When market demand and successful technology adoption rate are realized, competitive channels make their pricing decisions, constrained by the allocated resource, where the production quantity of product \( a \) and product \( b \) is denoted by \( q_a \) and \( q_b \), respectively.

\[
\begin{align*}
\text{Technology adoption strategy} & \quad \text{Resource allocation} \\
\text{Manufacturer makes technology adoption decisions for two product lines} & \quad \text{Manufacturer allocates production resource for two product lines} \\
\text{Quantity and pricing decisions} & \quad \text{Sales channels make quantity and pricing decisions} \\
\text{\( \theta \) and \( \gamma \) are realized} & \quad \gamma \in [0,1] \quad \text{Cachon and Lariviere (2005) show that in a revenue-sharing contract, the channel shares } \phi \text{ of supply chain’s profit while the manufacturer shares the rest } 1 - \phi \text{. We assume customers are heterogeneous in their valuations of the product’s quality/utility and we use parameter } \theta \in [0, \alpha] \text{ to represent the customers’ valuations of the products. We assume } \theta \text{ is uniformly distributed between 0 and } \alpha \text{ and } \alpha \text{ is a random variable independently drawn from a probability distribution function } F(\cdot) \text{, where } F \text{ is twice differentiable}
\end{align*}
\]

Fig. 2 The sequence of events
over \([0, H]\). The quality of new product is exogenous determined by a parameter \(\rho\), where \(\rho > 1\). This implies that a customer’s valuation of a new product is \(\rho q_n\), and of an old one is \(q_o\). When the market is provided with both old products and new products, we consider the inverse demand function as follows,
\[
p^* = \alpha - q_n - q_o, \quad p^\prime = \rho(\alpha - q_n) - q_o,
\]
where \(p^*\) is the price of an old product and \(p^\prime\) is the price of a new product.

4. Analysis

To find the subgame perfect equilibrium, we solve the problem using backward induction starting with the analysis of the third stage.

**Third stage: Quantity and pricing decisions without uncertainty**

In the third stage after realization of potential market demand and successful technology adoption rate, channels make their quantity and pricing decisions. Using holdback pricing strategy with constraints of resource, the problem becomes a Cournot competition with constraints. The pricing decisions are direct outcomes of the quantity decisions. Let \(\Pi_i(q_a, q_b | y)\) represents the profit of channel \(j, j = a\) or \(b\) given manufacturer’s technology strategy \(i\), where \(i = N\) or \(Y\) or \(M\). Next we analyze the third stage decision under three technology adoption subgames.

**(I) Subgame \(N\)**

In the competitive setting, the objectives of two channels are to maximize their profit as follows,
\[
\max \Pi_i^a(q_a | y) = \phi(p^* - c_j)q_a, \ s.t. \ 0 \leq q_a \leq y
\]
\[
\max \Pi_i^b(q_b | y) = \phi(p^* - c_j)q_b, \ s.t. \ 0 \leq q_b \leq K - y
\]
where \(p^* = A - q_n - q_o\). The results are summarized in the following Proposition.

**Proposition 1**

The equilibrium quantity and pricing decisions and the corresponding profits for channel \(a\) and channel \(b\) in subgame \(N\) are summarized in table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>(q_a)</th>
<th>(q_b)</th>
<th>(p)</th>
<th>(\Pi_i^a)</th>
<th>(\Pi_i^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A + K + y + c_j, A &lt; 2K - y + c_j)</td>
<td>(y)</td>
<td>(K - y)</td>
<td>(A - K)</td>
<td>(\phi(A - K - c_j))</td>
<td>(\phi(A - K - c_j)(K - y))</td>
</tr>
<tr>
<td>(3y + c_j &lt; A &lt; 2K - y + c_j)</td>
<td>(y)</td>
<td>(A - y - c_j)</td>
<td>(\frac{A - y - c_j}{2})</td>
<td>(\frac{3}{2}(A - y - c_j))</td>
<td>(\frac{3}{2}(A - y - c_j)^2)</td>
</tr>
<tr>
<td>(3K - 3y + c_j &lt; A &lt; K + y + c_j)</td>
<td>(A - K + y - c_j)</td>
<td>(K - y)</td>
<td>(A - K + y + c_j)</td>
<td>(\frac{A - K + y - c_j}{2})</td>
<td>(\frac{3}{2}(A - K + y - c_j)^2)</td>
</tr>
<tr>
<td>(c_j &lt; A &lt; 3y + c_j, c_j &lt; A &lt; 3K - 3y + c_j)</td>
<td>((A - c_j)/3)</td>
<td>((A - c_j)/3)</td>
<td>((A + 2c_j)/3)</td>
<td>(\phi(A - c_j)^{1/3})</td>
<td>(\phi(A - c_j)^{1/9})</td>
</tr>
</tbody>
</table>

| \(0 \leq A < c_j\) | 0 | 0 | 0 | 0 | 0 |

**(II) Subgame \(Y\)**

In subgame \(Y\), if \(\theta = 0\), the result in this situation is similar to subgame \(N\), which is obtained by substituting unit production price \(c_d\) in subgame \(N\) into \(c_j\).

If \(\theta = 1\), each channel’s objective is to maximize his profit as follows,
\[
\max \Pi_i^a(q_a | y) = \phi(p^* - c_j)q_a, \ s.t. \ 0 \leq q_a \leq y
\]
\[
\max \Pi_i^b(q_b | y) = \phi(p^* - c_j)q_b, \ s.t. \ 0 \leq q_b \leq K - y
\]
where \(p^* = \rho(A - q_n - q_o)\). The result in this situation is similar to subgame \(N\), by substituting \(c_d\) in subgame \(N\) into \(c_j / \rho\), and each channel’s profit is \(\rho\) times of that in subgame \(N\).

**(III) Subgame \(M\)**

If \(\theta = 0\) each channel’s objective is to maximize their profit as follows,
\[
\max \Pi_i^a(q_a | y) = \phi(p^* - c_j)q_a, \ s.t. \ 0 \leq q_a \leq y
\]
\[
\max \Pi_i^b(q_b | y) = \phi(p^* - c_j)q_b, \ s.t. \ 0 \leq q_b \leq K - y
\]
where \(p^* = A - q_n - q_o\). The result is shown in table 2.
when decreases in or represents the expected profit of . The results are summarized in the following Proposition.

In subgame , the demand for product increases in , 's profit , and product . The expected second stage profit of manufacturer under subgame or in , where \( q_i \) and \( q_j \) are sold separately by two competitive channels. Let \( \phi \) be the expected profit of the manufacturer in the second stage, given technology adoption strategy .

Maximize the expected profit of the manufacturer under subgame as follows,

\[
\max \Pi_{2m}(q_i, q_j) = \phi(p^* - c_j) q_i, s.t. q_i \leq y
\]

(7)

where \( p^* = p(A - q_j) - q_j \), \( p^* = A - q_j - q_j \). The results are summarized in the following Proposition.

**Proposition 2** The optimal quantity and pricing decisions and the corresponding profits for channel and channel in subgame when \( \theta = 1 \) depend on the realization of potential market demand \( A \), summarized in table 3.

### Table 3. Channels' quantity and pricing decisions in subgame \( M \) when \( \theta = 1 \)

<table>
<thead>
<tr>
<th>Case</th>
<th>( q_i )</th>
<th>( q_j )</th>
<th>( p^* )</th>
<th>( \Pi_{2m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \geq K + y + c_i, A \geq 2K - y + c_i )</td>
<td>( y )</td>
<td>( K - y )</td>
<td>( A - K )</td>
<td>( \phi(A - K - c_j) )</td>
</tr>
<tr>
<td>( A \geq 3y + 2c_j - c_i, A &lt; 2K - y + c_i )</td>
<td>( y )</td>
<td>( A - y + c_j )</td>
<td>( A - K )</td>
<td>( \phi(A - K - c_j) )</td>
</tr>
<tr>
<td>( 3(K - y) + 2c_j - c_i &lt; A &lt; K + y + c_i )</td>
<td>( y )</td>
<td>( A - y + c_j )</td>
<td>( A - K + y + c_j )</td>
<td>( \phi(A - K - c_j) )</td>
</tr>
</tbody>
</table>

From proposition 2, we find that \( p^* \) increases in \( \rho \) while \( p^* \) decreases in \( \rho \), and product 's profit increases in \( \rho \) while product b 's profit decreases in \( \rho \). In competitive setting, improvement of product quality raises sales price of new product and thus increases channel a 's profit. On the other side, quality improvement of new product may decrease sales price of old product and thus reduce the profit of channel b.

**Second stage: Resource allocation with uncertainty**

In the second stage, the manufacturer allocates production resource between the two products facing the stochastic potential market demand and uncertain successful technology adoption rate. The objective of the manufacturer is to maximize \( 1 - \phi \) share of the profits generated by the two products, which are sold separately by two competitive channels. Let \( \Pi_1(y) \) represents the expected profit of manufacturer in the second stage, given technology adoption strategy \( i \), where \( i = N \) or \( Y \) or \( M \). We analyze the resource allocation decision under three technology choice subgames.

**I) Subgame N**

According to proposition1, we analyze the problem in two scenarios: in scenario \( 1, 0 \leq y < K/2 \) and in scenario \( 2, K/2 \leq y \leq K \). The expected second stage profit of manufacturer under subgame \( N \) in scenario \( i \) is represented by \( \Pi_{1i}(y) \) \( i = 1, 2 \), which are presented as follows,

\[
\Pi_{1i}(y) = \frac{2(1-\phi)}{9} \left[ \int_{c_i}^{1} (a-c_j)^2 dF(a) \right] + \frac{(1-\phi)}{4} \left[ \int_{c_j}^{1} (a-c_j)^2 dF(a) \right] + \frac{(1-\phi)}{4} \left[ \int_{c_j}^{1} (a-c_j)^2 dF(a) \right]
\]

(9)

...
The manufacturer’s optimal resource allocation decision and the corresponding expected profit under subgame $N$ is illustrated in proposition 3.

**Proposition 3**  
Manufacturer’s optimal resource allocation decision under subgame $N$ is

$$ y_N = \begin{cases} \frac{K}{2}, & \text{if } \Delta_N \geq 0; \\ \text{Kor0, otherwise.} & \end{cases} $$

and the corresponding optimal expected profit is

$$ \Pi_N = \begin{cases} \Pi_N^+ (\frac{K}{2}), & \text{if } \Delta_N \geq 0; \\ \Pi_N^+ (K) & \text{or } \Pi_N^+ (0), \text{otherwise.} \end{cases} $$

where

$$ \Delta_N = \frac{1}{2} \int_{0}^{\infty} \rho \left[ K (a - c) - \frac{4}{9} \right] \rho \left[ a - c \right] F(a) da - K \int_{0}^{\infty} F(a) da. $$

Under subgame $N$, the manufacturer allocates all resources to one product line or divides resources equally to two product lines with the same quality. Resource allocation decisions are made by trading off between profits generated by monopoly channel and duopoly channel. The manufacturer adopts ML or DL product-channel design when technology adoption strategy is $N$.

**(II) Subgame $Y$**

The optimal resource allocation decision $y$ and optimal manufacturer’s profit $\Pi_y$ in subgame $Y$ is similar to subgame $N$, where

$$ \Delta_y = (1-\rho) \int_{0}^{\infty} \rho \left[ a - c \right] F(a) da - \frac{4}{9} \int_{0}^{\infty} \rho \left[ a - c \right] F(a) da - K \int_{0}^{\infty} F(a) da. $$

Under subgame $Y$, the optimal resource allocation decision and the corresponding expected profit under subgame $Y$ is

$$ y_Y = \text{arg max} \left\{ \Pi_Y^+(y), \Pi_Y^+(y), \Pi_Y^+(y) \right\}, $$

and the corresponding optimal expected profit is

$$ \Pi_Y = \text{max} \left\{ \Pi_Y^+(y), \Pi_Y^+(y), \Pi_Y^+(y) \right\}, $$

where $y_Y$ is defined as follows, $i=1,2,3$.

Define $y'_{ii}$ as stationary points satisfy (11) with $0 \leq y'_{ii} \leq \frac{K-c_i+c_j}{2}$. For $\Pi_{ii}^+$, the optimal resource allocation quantity $y'_{ii} = \text{arg max} \left\{ \Pi_{ii}^+(0), \Pi_{ii}^+(y'), \Pi_{ii}^+(y') \right\}$.

$$ (\rho-1) \int_{0}^{\infty} \rho \left[ a - c_i - \frac{4}{9} \right] \rho \left[ a - c_i \right] F(a) da - K \int_{0}^{\infty} F(a) da \leq y'_{ii} \leq \frac{K-c_i+c_j}{2}, $$

Define $y''_{ii}$ as stationary points satisfy (12) with $\frac{K-c_i+c_j}{2} \leq y''_{ii} \leq \frac{2(\rho-1)K+c_i-c_j}{3\rho-1}$. For $\Pi_{ii}^+$, the optimal resource allocation quantity $y''_{ii} = \text{arg max} \left\{ \Pi_{ii}^+(y''), \Pi_{ii}^+(y''), \Pi_{ii}^+(y'') \right\}$.

$$ (\rho-1) \int_{0}^{\infty} \rho \left[ a - c_i - \frac{4}{9} \right] \rho \left[ a - c_i \right] F(a) da - K \int_{0}^{\infty} F(a) da \leq \frac{2(\rho-1)K+c_i-c_j}{3\rho-1}.$$
Define $y_{u,i}^3$ as stationary points satisfy (13) with

$$
(y_{u,i}^3)^2 < \frac{(2\rho-1)K + \rho c_f - c_f}{3\rho - 1} \leq y_{u,i}^3 \leq K.
$$

For $\Pi_{IM}^3$, the optimal resource allocation quantity

$$
y_{u,i}^3 = \text{arg max} \left\{ \Pi_{iM}^3 \right\}.
$$

First stage: Technology choice

Given the results presented in three subgames, technology adoption decision is made so that the expected profit of the manufacturer, denoted by $\Pi'$, is maximized. Thus,

$$
\Pi' = \max \{ \Pi_{1M}, \Pi_{2M}, \Pi_{3M} \}
$$

and the optimal technology adoption strategy is chosen accordingly.

5. Numerical studies

We study which product-channel design is optimal and explore the impact of parameters such as $\rho$, $\gamma$, $H$ and $K$ on the choice of optimal strategy. We begin by constructing base scenarios from the parameter values shown in Table 4.

Table 4. Parameter values for numerical analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_f$</td>
<td>1</td>
</tr>
<tr>
<td>$c_r$</td>
<td>20</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.2, 2.4, ..., 38.4</td>
</tr>
<tr>
<td>$H/K$</td>
<td>2.5, 5, ..., 50</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.02, 0.4, 0.6, 0.8, 1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>50, 100, ..., 800</td>
</tr>
</tbody>
</table>

Considering the constraint of $H \geq 2K + c_f$, which is implied in the analysis of previous section, we limit the ratio of $H/K$ is equal or greater than 2.5. Under three distribution functions, which are uniform distribution, normal distribution and exponential distribution, we analyze how the manufacturer designs his product line and channel structure.

We derive the optimal resource allocation decisions under subgame $N$ and $\gamma$ (table 5). We find that the optimal resource allocation decisions $y_N$ and $y_\gamma$ differ as the distribution function of $\alpha$ varies.

Table 5. $y_N$ and $y_\gamma$ in different distribution functions of $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$y_N$</th>
<th>$y_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(0, H)$</td>
<td>$\alpha - N(H/2, \sigma^2)$</td>
<td>$\alpha - \sigma(2/H)$</td>
</tr>
<tr>
<td>$\Delta_\alpha &lt; 0$</td>
<td>$\Delta_\alpha &lt; 0$</td>
<td>$\Delta_\alpha &gt; 0$</td>
</tr>
<tr>
<td>$H$</td>
<td>$0$</td>
<td>$H/2$</td>
</tr>
</tbody>
</table>

When $\alpha \sim U(0, H)$ or $\alpha \sim N(H/2, \sigma^2)$, optimal resource allocation decision is $y_{\gamma} = 0 \text{ or } K, i = N, Y$. From numerical analysis, we find that manufacturer’s profit rises as market size $H$ and manufacturer’s size $K$ increases. Technology adoption strategy $M$ becomes strategy $N$ when (1) successful technology adoption rate is low; and (2) product quality parameter is low (e.g. $0 \leq \gamma \leq 0.1, 1 < \rho < 3$); and (3) market size is small. On the contrary, technology adoption strategy $M$ becomes strategy $Y$ when (1) successful technology adoption rate is high; and (2) product quality parameter is high (e.g. $0.1 < \gamma \leq 1, \rho > 3$); and (3) market size or resource quantity is large. When $\alpha \sim \varepsilon(2/H)$, it is shown that in most cases, technology adoption strategy $M$ is optimal. When $\alpha$ follows the above three different distribution functions, manufacturer follows the same product-channel strategy. The optimal product-channel strategy of the
manufacturer is *MH* or *DM*; that is, technology adoption strategy *M* is used and resource is allocated to two product lines with different quality or devoted to product *a* only.

6. Conclusion

This paper studies the impact of downstream competition and product diversification on a manufacturer’s product line and channel design, which is decided by making technology adoption and resource allocation decisions with uncertainties of market demand and successful technology adoption rate. We characterize equilibrium quantity-pricing decisions of two channels and manufacturer’s optimal resource allocation decisions. We find that under not adoption and adoption subgames, it is optimal to allocate resource equally or devote all resource to one product line, depending on the distribution function of demand. While in mixed adoption subgame, the resource allocation decision is also affected by other factors such as quality improvement parameter, successful technology adoption rate and production cost.

References


