

Modeling and Steady Holding Strategy of a Climbing Robot

Fuhua Wang^{1,2}, Yuwang Liu^{1,*} and Liangshuai Guo³

¹Shenyang Institute of Automation, Chinese Academy of Sciences, China

²University of Chinese Academy of Sciences, China

³Shenyang Ligong University, China

*Corresponding author

Abstract—Underactuated mechanism is a nonlinear system which dimension of input is less than one of state. There are still some problems to be explorations in underactuated mechanism control, especially in ones with multi-DOF. The model reference adaptive control is proposed to solve the control problem of a novel underactuated spring-coupling mechanism with multi-DOF in this paper. Firstly, dynamics model is established and linearized. Then, an adjustable gain adaptive controller based on Lyapunov stability theory is presented. Finally, simulations are presented to illustrate the superior performance of the proposed method through theoretical analysis and design. The results show that the stability, accuracy and rapid response are fine, and the adaptive steady holding problem of multi-DOF underactuated mechanism is solved.

Keywords—model reference adaptive control; multi-DOF underactuated mechanism; lyapunov stability theory; spring-coupling

I. INTRODUCTION

Full-actuated mechanism usually has many independent driving units, complex control system, larger volume and weight, so there are some limitations in the practical applications. Underactuated mechanism is a kind of system whose quantity of independent drive is less than controlled degrees of freedom. There are many advantages in it, such as adaptive target shape, self-balance internal force and strong flexibility, in addition to smaller volume and weight [1]. It has indeed become a focus of robot mechanism recently [2,3,4].

It is a challenge for the control technology that freedom of mechanism is not fully controlled, because of the quantity of independent drive less than degrees of freedom in underactuated mechanism [5,6]. Various ambitious controllers have been designed for different application scenarios by domestic and foreign scholars, and promote the development of related control theory [7,8]. In [9], nonlinear dynamic feedback method is designed for AAP (Active-Active-Passive) mechanism with rotary underactuated end joints. Motion of underactuated mechanism on the horizontal and vertical plane is implemented by that control law in the same frame. The track-interpolations easily lead to system oscillation due to adopting the high-order polynomial. In [10], a nonlinear feedback controller is proposed for gravity-assisted three-link underactuated manipulator with second-order nonholonomic constraints, through designing Lyapunov function to stabilize

the system. It has little large overshoot after addition of switch locking mechanism, and the rate of convergence can be improved. In [11], the closed-loop control strategy based on PID position control is proposed for the tendon-driven multi-finger underactuated manipulator. The optimum gripping force is calculated and predicted through the current force of tendon and grip. However, the parameters of controller base more on personal experience due to the complex structure, numerous sensors and difficult to build mathematical model. From the above literature research, the control methods of underactuated mechanism have made great progress. But there are still some shortcomings mainly in the following points:

a) Underactuated controllers are mostly the type controllers of manual-adjusted parameters currently which are generally independent of mechanical systems. They ignore their excellent characteristics of mechanical systems and cannot control high adaptability underactuated mechanism;

b) Most controllers are designed for the 2-DOF underactuated mechanisms currently. There are few theoretical and applied researches about 3-DOF underactuated mechanism with complex dynamics model and strong coupling parameter.

Model reference adaptive controller [12,13] (MRAC) is introduced into the underactuated control for the inadequacies of current control system of the underactuated mechanism in this paper. Climbing robot as the research object is composed of 3-DOF adaptive underactuated mechanism. Firstly, dynamics model constructed by Lagrange method is linearized by the Jacobian matrix and simplified the system structure; and then MRAC control with adjustable gain is designed which based on Lyapunov stability theory, so the underactuated mechanism of the climbing robot moves along the desired trajectory and asymptotic stability control is achieved. The method has advantages of steady closed-loop system, low complexity and small stability error. Finally, the simulation analysis and experimental results show that the controller can support climbing robot with holding task well done.

II. PROBLEM DESCRIPTION

A. System Structure

"Self-adaptive wheel-manipulator climbing robot with minimal drive sources (SawmBot)" combines high adaptability holding mechanism and efficient- travel wheel with principle of

underactuated manipulator and wheeled driving, shown in Figure 1. Holding mechanism of SawmBot is 3-DOF spring-coupled underactuated mechanism which consists of three limbs, namely the limb 1, limb 2 and limb 3. Each limb has three units, respectively units 1, unit 2 and unit 3. The units are connected by the joint, which is full-actuated joints on its root, underactuated joints on its middle and end.

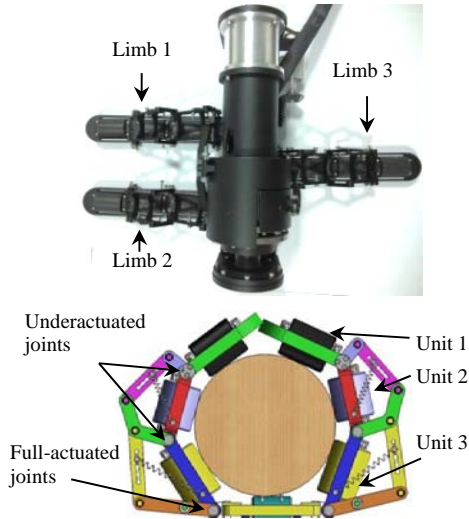


FIGURE 1. EXPERIMENT PLATFORM OF CLIMBING ROBOT

B. Construction and Linearization of State Space

Dynamics modeling of the holding mechanism is established by Lagrange method. The schematic of holding mechanism of climbing robot is shown in Figure 2. All limbs of mechanism are driven by a motor via the diversion gear train of constant speed. The left limb is analyzed since the same parameters and the output torque of each limb. It is spring-coupling mechanism (Spring-coupling Active-Passive-Passive).

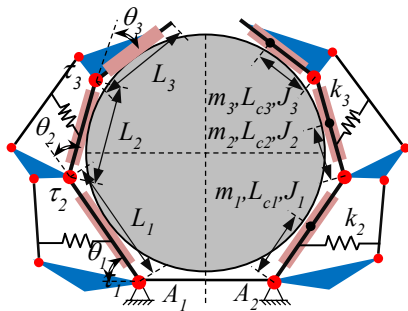


FIGURE II. SCHEMATIC OF HOLDING MECHANISM

Parameters of the hold mechanism are shown in Table 1 below.

TABLE I. PARAMETERS OF HOLDING MECHANISM

Symbol	Meaning
θ_i	Angle of i -rod ($i = 1, 2, 3$)
L_i	Length of i -rod
L_{ci}	Distance between axis of i -joint and centroid of i -rod
m_i	Mass of i -rod
J_i	Moment of inertia of i -rod
τ_i	External moment of i -joint
k_i	Equivalent elasticity coefficient of i -rod

Dynamics model of spring-coupling underactuated mechanism is

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + F_k(\theta) = \begin{bmatrix} \tau_1 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

Wherein, $M(\theta) \in \mathbb{R}^{3 \times 3}$ is an inertia matrix. $C(\theta, \dot{\theta}) \in \mathbb{R}^{3 \times 3}$ is a matrix combining centrifugal force and Coriolis forces. $F_k(\theta) \in \mathbb{R}^3$ is a elasticity matrix. We define variables $x = [\theta_1 \ \theta_2 \ \theta_3 \ \dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]^T$, the inputs of underactuated units and reaction force of target object are viewed as a perturbation of the system for nonexcitation nonlinear system $\dot{x} = f(x)$ [14]. Obviously $x_e(t) = 0$ is the equilibrium point of system, $f(x)$ is continuously and differentiable. Then the system is transformed into the following linear equation by calculating the Jacobian matrix $A = \left. \frac{\partial f}{\partial x}(x) \right|_{x=0}$,

$$\dot{x} = Ax + B\tau \quad (2)$$

Theorem 1: The system in (2) is controllable and observable.

Proof: After calculation, eigenvalues of the matrix A is $R_e(\lambda_i) = 0$ that A is not a Hurwitz matrix, and therefore it cannot determine whether the system is asymptotically steady at equilibrium point.

The actual values of parameters of holding mechanism are in Table 2.

TABLE II. PARAMETER'S VALUE OF HOLD MECHANISM

	Length $L_i(mm)$	Centroid distance $L_{ci}(mm)$	Mass $m_i(g)$	Moment of inertia $J_i(g \cdot mm^2)$	Spring stiffness $k_i(N/mm)$
Unit 1	65	30.6	121	151.5	0
Unit 2	55	25.7	98	87.98	0.49
Unit 3	50	31.1	125	163.3	0.49

Substituting the data in Table 2 into (2) we can see

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 5.4 & 3.7 & 0 & 0 & 0 \\ 0 & -19.1 & 19.2 & 0 & 0 & 0 \\ 0 & 19.2 & -60.6 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1.1 \\ -0.8 \end{bmatrix} \quad (3)$$

Its controllability matrix O_C and observability matrix O_O is

$$Q_c = \begin{bmatrix} 0 & -1 & 0 & 8.9 & 0 & -131.7 \\ 0 & 1.1 & 0 & -5.7 & 0 & -417.4 \\ 0 & 0.8 & 0 & -27.4 & 0 & 1549.5 \\ -1 & 0 & 8.9 & 0 & -131.7 & 0 \\ 1.1 & 0 & -5.65 & 0 & -417.4 & 0 \\ 0.8 & 0 & -27.4 & 0 & 1549.5 & 0 \end{bmatrix} \quad (4)$$

It can be seen that the matrix Q_C , Q_C are full rank by the above equation, so controllability and observability of the system are proved. Therefore a balance controller can be designed in steady region to asymptotically steady the system at equilibrium point.

III. CONTROLLER DESIGN

Control system model of SawmBot is shown in Figure 3. Hardware of the system is composed by control circuit, motor-driving circuit and torque measurement circuit. MRAC controller within the dashed box is shown in Figure 3.

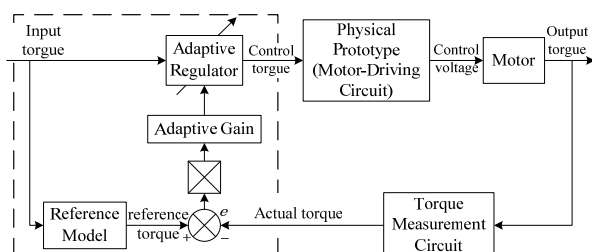


FIGURE III. CONTROL SYSTEM MODEL

In order to better exert the adaptive advantages of holding mechanism, model reference adaptive controller with adjustable gain is designed by the above derivation equations of underactuated linear system. Therefore SawmBot moves along the desired trajectory $u_d(t)$ and tend to asymptotic stability. System structure of proposed Lyapunov-MRAC controller is shown in Figure 4.

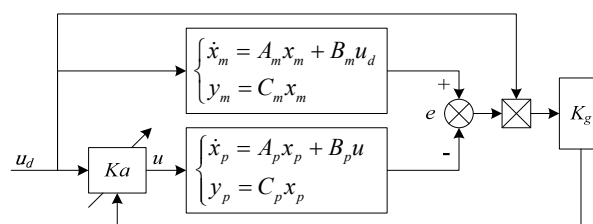


FIGURE IV. BLOCK DIAGRAM OF LYAPUNOV-MRAC CONTROL SYSTEM

Physical quantities are given in Table 3 below.

TABLE III. PARAMETERS OF INPUT AND OUTPUT

Input and output	Actual meaning	Physical meaning
Reference input, $u_d(t)$	The desired drive torque of motor	Torque
Controller input, $u(t)$	Input for controller after feedback signal	Torque
Model output, $y_m(t)$	Derived output of dynamics model	Torque
Object output, $y_o(t)$	Actually collected output of motor	Torque

Firstly error between object model and reference model adjusts controller gain $k_a(t)$ via coefficient k_g as shown in Figure 4. Then, object model is compensated by $k_a(t)$. Control law of the system is

$$u(t) = k_a(t)u_d(t) \quad (5)$$

Error $e(t)$ and gain k shown in Figure 4 can be expressed as

$$\begin{cases} e(t) = y_m(t) - k_a(t)y_p(t) = k \frac{Y(p)}{U(p)} u_d(t) \\ k = k_m - k_a(t)k_p \end{cases} \quad (6)$$

where k_p is unknown or slowly varying, i.e. the k_p can be approximated as a constant. k_m is a constant.

Equation (6) is converted into observable norm equation

$$\begin{cases} \dot{x} = Ax + kBu_d \\ e = Cx \end{cases} \quad (7)$$

Theorem 2: If adjustable gain control law is $\dot{k}_a(t) = k_g u_d(t) e(t)$, $k_g = k_v / k_p$, then the system will be asymptotically steady.

Proof: Selecting Lyapunov function

$$V = k_v x^T P x + k^2, \text{ where } k_v > 0 \quad (8)$$

$$\begin{aligned}\dot{V} &= k_v (x^T A^T + k u_d B^T) P x \\ &\quad + k_v x^T P (A x + k B u_d) + 2 k \dot{k} \\ &= -k_v x^T Q x + 2 k (\dot{k} + k_v u_d B^T P x)\end{aligned}\quad (9)$$

Linear system in (2) is fully controllable and observable by Theorem 1. Positive definite matrices P and Q are existed by quoting the *Kalman-Yakubovich* theorem, so we have $A^T P + P A = -Q$ and $B^T P = C$.

If taking adjustment law $\dot{k}_a(t) = k_v u_d(t) e(t) / k_p$,

$$\dot{k}_a = k_g u_d e(t) = \frac{k_v}{k_p} u_d C x = \frac{k_v}{k_p} u_d B^T P x \quad (10)$$

Then deriving (7) is that $\dot{k} = \dot{k}_a(t) k_p$, so

$$\dot{k} = \frac{k_v}{k_p} u_d B^T P x \cdot k_p = -k_v u_d B^T P x \quad (11)$$

Then $\dot{V} < 0$, so the system will be asymptotically steady.

IV. SIMULATION ANALYSIS

A. Simulation Analysis

Proposed model above is simulated in Matlab, taking an adaptive gain $k_g=1.1$. Simulation results are shown in Figure 5. Contrasting $u_d(t)$ and $y_m(t)$, there are hysteresis at initial interval and overshoot at the end interval due to the inertia unitin holding mechanism. Contrasting $u_d(t)$ and $y_p(t)$, the control system can drive machine well and complete holding task.

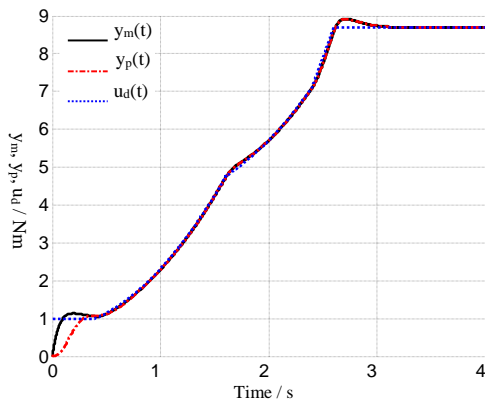


FIGURE V. CONTROL PERFORMANCE

As shown in Figure 6 is the error curve between actual output $y_p(t)$ and given input $u_d(t)$. Large change rate of the drive torque results in large errors at the beginning of each stage

when three units of each limb contact with target object in turn. But errors tend to 0 by the controller adjusting.

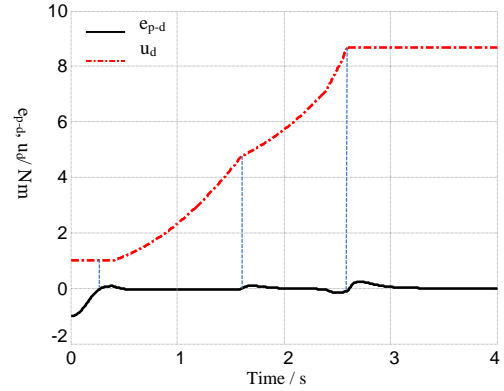


FIGURE VI. FLUCTUATION CURVE OF ERROR

V. CONCLUSION

The model reference adaptive adjustable gain control strategy is proposed for underactuated climbing robot (SawmBot). Firstly, the system dynamics model is built and approximately linearized by Lagrange equation; meanwhile reference model is constructed and the system structure is simplified. Secondly, Lyapunov-MRAC controller is constructed to ensure that the object model tracks the reference model, so climbing robot can hold the target object better. Stability of the system is proved by Lyapunov second method. The proposed strategy solves the control problem of a novel multi-DOF spring-coupling underactuated mechanism, meanwhile develops adaptive characteristic of the mechanical system. The simulation results show that the adaptive control law proposed in this paper can maintain asymptotic stability, and got steady-state holding process, even if the system parameter uncertainties and factors such as the presence of noise.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (NO: 51605474), Foundation of State Key Laboratory of Robotics (NO: 2016-Z09), Natural Science Foundation of Liaoning Province (NO: 20121060)

REFERENCES

- [1] C Cai, Y Guan, X Zhou, et al., "Joystick-based Control for a Biomimetic Biped Climbing Robot," *Robot*, vol. 34, pp. 363-368, 2012.
- [2] Y Liu, H Yu, "A survey of underactuated mechanical systems," *IET Control Theory and Applications*, vol. 7, pp. 921-935, February 2013.
- [3] Z Li, C Yang, CY Su, W Ye, "Adaptive fuzzy-based motion generation and control of mobile under-actuated manipulators," *Engineering Applications of Artificial Intelligence*, vol. 30, pp. 86-95, December 2013.
- [4] Z Yan, Z Yan, C Mou, et al., "3D Path Following Control of Underactuated UUV Based on Backstepping," *Information and Control*, vol. 41, pp. 180-192, 2012.
- [5] J She, A Zhang, X Lai, M Wu, "Global stabilization of 2-DOF underactuated mechanical systems—an equivalent-input-disturbance

- approach," *Nonlinear Dynamics*, vol. 69, pp. 495-509, 2012.
- [6] M T Ravichandran, A D Mahindrakar, "Robust stabilization of a class of underactuated mechanical systems using time scaling and Lyapunov redesign," *IEEE Transactions on Industrial Electronics*, vol. 58, pp. 4299-4313, 2011.
 - [7] A De Luca, G Oriolo, "Trajectory planning and control for planar robots with passive last joint," *The International Journal of Robotics Research*, vol. 21, pp. 575-590, 2002.
 - [8] B Roy, H H Asada, "Nonlinear feedback control of a gravity-assisted underactuated manipulator with application to aircraft assembly," *IEEE Transactions on Robotics*, vol. 25, pp. 1125-1133, 2009.
 - [9] C Cipriani, F Zaccane, G Stellin, et al., "Closed-loop controller for a bio-inspired multi-fingered underactuated prosthesis," *IEEE International Conference on Robotics and Automation*, pp. 2111-2116, 2006.
 - [10] O Begovich, E N Sanchez, M Maldonado, "Takagi-Sugeno fuzzy scheme for real-time trajectory tracking of an underactuated robot," *IEEE Transactions on Control Systems Technology*, vol. 10, pp. 14-20, 2002.
 - [11] S Yang, X Lai, M Wu, "Position control of a planar three-link underactuated mechanical system based on model reduction," *Acta Automatica Sinica*, vol. 40, pp. 1303-1310, 2014.
 - [12] N Nguyen, "Least-Squares Model-Reference Adaptive Control with Chebyshev Orthogonal Polynomial Approximation," *Journal of Aerospace Information Systems*, vol.10, pp.268-286, June 2013.
 - [13] Z Pang, H Cui, "System Identification and Adaptive Control of MATLAB simulation," Beijing University of Aeronautics and Astronautics Press, 2009.
 - [14] Khalil, K Hassan, JW Grizzle, "Nonlinear systems, the third Edition," Person Education: Prentice hall, vol. 3, pp. 88-121, 2002.