

Model for the Variation Rule of Shadow Length with Multi - parameter

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Abstract: In order to obtain the variation rule of shadow length of the pole in a certain period of time, this paper establishes a model of the change of shadow length with each parameter.

First of all, according to the observation date, the solar declination is obtained. In this paper, to make the calculation simplified, several items that have little effect on the results of the solar declination are ignored, and the simplified formula for calculating the solar declination can be obtained.

Second, using the relationship between a series of sun angles, the solar elevation angle is expressed as a function of the observation time.

Then, according to the triangular corner relationship, regarding the solar elevation angle as the intermediate variable, the shadow length is obtained as a function of time.

Finally, it can be proved that the curve is a parabola. During the observation time, shadow length increase first and then decrease, which is consistent with the objective law.

1. Introduction

The rising and falling of the sun is one of the most basic phenomena in nature, and the shadows are always associated with the sun. From experience we can see that the nearer the earth is from the sun, the higher the solar elevation angle and the shorter the shadow of the object is. At the same time, the longer the object is, the longer the shadow is.

With the development of science and technology, people have more and more requirements for data analysis. In this respect, how to determine the shooting date and location of the video and is an important aspect of video data analysis. The sun shadow positioning technology is used to determine the date and location of the video through analyzing the sun shadow changes in the video.

The foundation of the technique above is to establish the mathematical model of the shadow length change, analyze the variation rule of the shadow length with each parameter,^[1] and finally get the curve of the shadow length of the vertical pole in a certain period of time. The results of this model are reliable and have significance of guiding.

2. Assumptions

The model in this paper captures the main factors, and ignores the secondary factors. All of the derivations below are based on the following three assumptions, they are:

- The sunlight is parallel, ignoring the refraction and reflection of sunlight;
- The influence of altitude on shadow length can be ignored;
- The revolution of earth won't affect the solar elevation angle at the observation point.

3. Establish the Mathematical Model

The solar elevation angle is referred to the height of the sun, expressed in h , which is numerically equal to the height of the earth in the Earth's ground plane coordinate system.^[2]

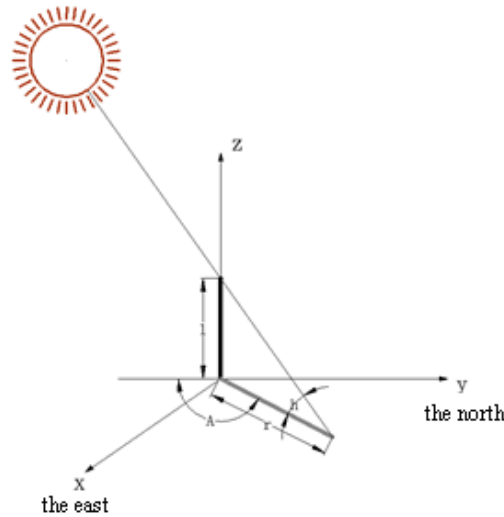


Fig.1 Geometric representation of the solar elevation angle h and the sun azimuth angle A ^[2]

As shown in Figure 1, at a certain point, the solar elevation angle satisfies the following triangular relationship: ^[3]

$$\tan h = l / r \quad (1)$$

where l is the height of the pole and r is the shadow length of the pole.

The solar elevation angle h of a place is related to the solar declination and the local time of the place. Denoting the solar declination with δ , δ satisfies the following relation:

$$\delta = 0.372 + 23.257 \sin \theta + 0.115 \sin 2\theta - 0.171 \sin 3\theta - 0.758 \cos \theta + 0.366 \cos 2\theta + 0.02 \cos 3\theta \quad (2)$$

according to the theory of series, the solar declination retains the first few items of the sine and cosine functions. Among them, θ is called the hour angle and satisfies the relation below

$$\theta = 2\pi t / 365.2422 \quad (3)$$

where t consists of two parts,

$$t = N - N_0 \quad (4)$$

in the formula above, N is called ahargana, which represents the sequence number of the date in a year. For example, the ahargana of January 1 is 1. In the common year, the ahargana of December 31 is 365, while leap year is 366. After derivation, N_0 can be expressed as follows:

$$N_0 = 79.6764 + 0.2422 \times (Y - 1985) - \text{INT}[(Y - 1985)/4] \quad (5)$$

where INT represents the rounding function and Y is the year. This formula is mainly to distinguish between common and leap year. It can be seen from the literature ^[4], after the integration of formula (2) ~ (5), ignoring several items that have little effect on the results, a simplified solar declination δ is calculated as

$$\delta = 23.45 \sin \frac{2\pi(284 + n)}{365} \quad (6)$$

where n is the number of days from January 1 to the observing date.

The geodetic latitude of a place is represented by φ . The hour angle of a place where the pole is located is indicated by Ω , and the formula for Ω is

$$\Omega = 15 \times (t_b - 12) \quad (7)$$

among them, t_b represents the observation time corresponding to Beijing time.

In conclusion, as the solar declination δ , the latitude φ and the time angle Ω have been obtained, the solar elevation angle h can be gotten. The formula is ^[5]

$$\sin h = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \Omega \quad (8)$$

Finally, by the relation

$$1 + \tan^2 h = \sec^2 h = 1 / \cos^2 h = 1 / (1 - \sin^2 h)$$

it can be proved that

$$\tanh = \frac{\sinh}{\sqrt{1 - \sin^2 h}} \quad (9)$$

From the formula (6) ~ (8) it can be seen that the solar elevation angle h satisfies

$$\sin h = \sin \varphi \sin \{23.45[\sin 2\pi(284+n)/365]\} + \cos \varphi \cos \delta \cos [15*(t_b-12)]$$

substituting formula (9), the formula of the shadow length is

$$r = l / \tan h \quad (10)$$

where l is the length of the pole.

4. Draw the Curve

From the above-mentioned model of variation rule of shadow length, any object's shadow length at any point in any time can be explored. In this paper, the model is used to analyze the change of shadow length of 3 meters high pole on October 22, 2015, Beijing time 9: 00-15: 00 in Tiananmen Square.

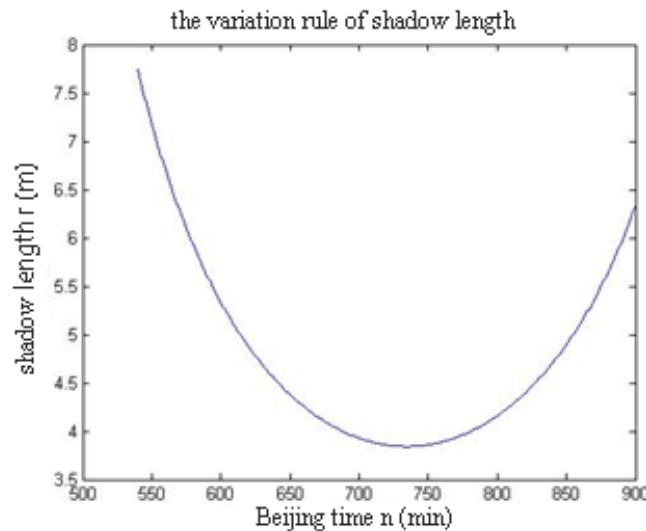


Fig.2 The curve of the shadow length of the pole

It has been obtained that pole length $l = 3\text{m}$, observation latitude $\varphi = 39$ degrees 54 minutes 26 seconds, the number of observation days $n = 295$ from January 1, 2015. Finally, using MATLAB software the following curve of shadow length on observation place can be drawn.

5. Conclusion and Analysis

Shadow length is affected by many factors in nature. This paper considers the influence of the solar declination, the latitude of the observation place, the latitude of the solar direct point (the solar elevation angle) and the height of the pole on the shadow length of the pole in a certain period of time. Then this paper applies the model to analyze the shadow length of the pole at Beijing Tiananmen Square on October 22, 2015 and finally draws the curve of shadow length of the pole shown in Figure 2.

In figure 2, to make the calculation process simple, the time is united into minutes. It can be seen that shadow length changes as a parabolic over time between 9: 00-15: 00. Between 9: 00-12: 14, the solar elevation angle gradually increases with the passage of time, thus the shadow length decreases gradually. The solar elevation angle reaches the peak at about 12:14, and the corresponding shadow length appears at the minimum value. In the range of 12: 14-15: 00, the solar elevation angle is gradually decreasing in the opposite direction, and the shadow length is gradually increasing.

In addition, the shadow length of the pole ranges from 3.5 to 8 meters in the observation time, indicating that the solar elevation angle is always less than 45 degrees. In a word, the whole curve is approximately axially symmetric to the axle at the noon time and satisfies the objective law of the shadow change in the northern hemisphere.

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