A New Relationship between Link Flow and Link Travel Time

Zichu Gao[1) a)] Zhang Ning[2) b)]

[1]school of Economics and Management,Beijing University of Aeronautics and Astronautics, Beijing , 100000,China

[2]school of Economics and Management, Beijing University of Aeronautics and Astronautics, Beijing , 100000,China

[a)] zhuzhusenlin@163.com,[b)] nzhang@buaa.edu.cn.

Keywords: Link flow, Link travel time

Abstract. The purpose of this paper is to explore the new relationship between the travel time and traffic flow on the road in the static state that inflow equals outflow. This paper proposes selection mechanism by a parallel two-link network model. Through computer simulation, we get the conclusion that in the static state, just with the road choice mechanism, the relationship is that when the flow on the road increases monotonously, the driving time increases first and then decreases. Congestion appears at the turning point, the hypercongestion appears after the turning point. The computer simulation proves this new relationship between the travel time and traffic flow on the road indeed exists.

1. INTRODUCTION

This article aims to propose a new relationship between the link travel flow and link travel time by simulating a comprehensive network model. In reality, when road appears congestion, vehicle travel time increases monotonously along with the increase of traffic flow on the road [1]. Under the dynamic condition, when the number of the cars driving into the road is more than the number of cars leaving the road, this road is in congestion. Namely, the inflow on this road is more than outflow of the road, road traffic jam even the bottleneck appears. At this time, cars driving time on this road is longer, and the road density is very high. Most scholars study the relationship between the traffic travel time and traffic flow based on dynamics condition [2-9]. However, the congestion in static state is not considered yet. The static state means the inflow on the road equals the outflow of the road, namely the total traffic demand remains the same value. Specifically, the number of cars driving into the road per unit time keeps unchanged. This paper constructs a network with two roads. Through computer simulation, we get the conclusion that in the static state, just with the road choice mechanism, the relationship is that when the flow on the road increases monotonously, the driving time increases first and then decreases. Congestion appears at the turning point, the hypercongestion appears after the turning point.

The next section states path choice stable equilibrium mechanism in the network model. The third section introduces model algorithm and flow chart. The fourth section presents the numerical proof, results analysis and conclusion. Finally, the fifth section is the summary of the model results in the paper.

2. Model with Path Selection Mechanism

2.1 Parallel Two-link Network

As shown in Figure 1, this paper studies the relationship of the traffic flow and travel time on link 1 in the stable state, when the alternative link 2 with infinite capacity exists. The infinite capacity means cars driving time on the link 2 keeps the same constant t2. Under stabilization mechanism of the road network and principle of Wardrop t1=t2, t1 changes when constant t2 value changes. So t2 is different related with the different network stabilization. Through road selection mechanism, we study the relationship between the flow on the road 1 and driving time t1(=t2). We study the cars driving situation by computer simulation and observe changes of the data, such as flow and travel time. The total traffic demand describes the total number of cars entering the network in a certain period of time, taking one hours usually.

Model:

Assume the weather is fine, there are two paths between the origin and the destination. The length of road 1 is L1 km. The total traffic demand in the network is M, which equals 3600pcu/h. Cars initial speed v equals 27 feet/s. Car maximum speed is vmax, which equals 100 km/h. The car length c equals 5m.The cars driving time on road 1 is t1, and the cars driving time on road 2 is t2. In the network stable state, F1 is the traffic flow on the road 1 per hour. The update step Δt equals 0.1s. Let's analyze the relationship of the link traffic flow and link travel time on the link 1 in the network stable state below.

L1=5km, v=100 km/h, c=5m, M=3600 pcu/h.

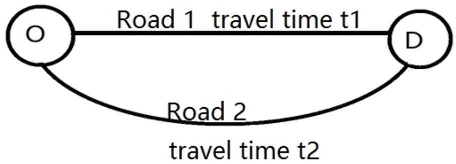

FIGURE 1. Parallel two-link network

2.2 **Multiple velocity difference Car Following Model MFVD**

Multiple velocity difference car following model enhances the stability of the traffic flow by using the velocity differences of multiple vehicles. The linear stability region of this model is expanded by using the linear stability theory compared with OV model and FVD model. The modified Korteweg-de Vrives equation of MFVD model is constructed and solved to prove the stable regional expansion. In this paper, we use the MVD model to study cars behavior on the two roads in the network and to explore the relationship between the road traffic flow and travel time on roads in the network.

The MFVD model shows below:

Acceleration:

$$a_n(t) = \alpha(V(\Delta x_n(t)) - v_n(t)) + \sum_{j=1}^{m} k_j \Delta v_{n+j-1}(t) \tag{1}$$

Parameter:

$$\alpha = 0.3\text{s}^{-1}, \quad \lambda = \begin{cases} 0.5\text{s}^{-1}, & \text{if } \Delta \text{x}_n \le 80m \\ 0\ , & \text{otherwise} \end{cases}$$

The optimal speed:

$$V(\Delta x_n) = \frac{v_{\max}}{2}(\tanh(\Delta x_n - h_c) + \tanh(h_c)) \tag{2}$$

Velocity and displacement:

$$\text{v}_\text{n}(t+\Delta t) = v_n(t) + \Delta t \cdot \frac{dv_n(t)}{dt} \tag{3}$$

$$x_n(t+\Delta t) = x_n(t) + \Delta t \cdot v_n(t) + \frac{1}{2} \cdot \frac{dv_n(t)}{dt} \cdot (\Delta t)^2 \tag{4}$$

MFVD car following model is a microscopic model. This model is used to simulate cars driving behavior on the roads in the network. We can obtain all the important data about the cars driving in the network by computer simulation, including road traffic flow and cars travel time on the road and so on. We also can realize how cars run on the road at any time by simulation.

2.3 Network Stable Equilibrium Condition and Path Selection Mechanism

The road network equilibrium is defined below:

$$f1 > 0, if \quad t1 \le t2 \tag{5}$$

$$f1 = 0, if \quad t1 > t2 \tag{6}$$

According to the principle of Wardrop, cars at the network entrance choose the shorter travel time road in the network. In this model, the road 2 has infinite capacity and t2 is the cars travel time on the road 2. N, which is the number of moving vehicles on the road 1, is used to predict cars travel time on the road 1 equivalently. When the car number on the road 1 is N*, cars travel time on the road 1 equals t2. According to the Wardrop principle, the car at the entrance of the network chooses each road equivalently in this case. When the car number on the road 1 is less than N*, cars travel time on the road 1 is less than t2. The car at the entrance of the network will choose road 1.When the car number on the road 1 is more than N*, cars travel time on the road 1 is more than t2. The car at the entrance of the network will choose road 2.

Therefore, according the definition above, equilibrium state is equivalent to:

$$f1 > 0, if \quad N_1 \le N^* \tag{7}$$

$$f1 = 0, if \quad N_1 \ge N^* \tag{8}$$

3. Algorithm and The Flow Chart

The traffic simulation in the network is a complex process, involving many different sub-modules circulation and calling .The figure 2 below is the network traffic simulation flow chart.

4. Numerical Simulation Demonstration

(1)In the stable equilibrium state, the density on the road 1 increases along with t1

When the network reach the stable state t1=t2, cars number on the road 1 equals N* at this time. The relationship between N* and t2: road 1, on which the cars number is N*, and the road 2 construct the a new different network with the different cars travel time t2, under the network equilibrium state t1=t2. Record variables value at in every stable state network, such as the cars speed, the density of the road 1, and traffic flow F1 etc. The study found that under the network stability, the density of road 1 increases monotonously along with the travel time t2 increases from 6 min to 36min by the step Δt equals 1 min.

TABLE 1.In The Network Stable Equilibrium State, The Different N* Values on the Road 1 and Different t2 Corresponding

t2	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
N*	100	180	250	330	370	460	496	580	630	690	750	797	820	840	845	857
t2	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
N*	860	862	864	866	868	871	873	875	877	880	882	884	886	888	890	

From the table 1 above, in the stable equilibrium state network, car number N* on the road 1 increases monotonously along with t2 .That means t1 increases results in the density on the road 1 increasing on one road 1. Here the space occupancy S shows below instead of the density on the road 1.

TABLE 2.In The Network Stable Equilibrium State, The space occupancy S values on the Road 1 and Different t1 Corresponding

t1	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
S	10%	18%	25%	33%	37%	46%	49.6%	58%	63%	69%	75%	79.7%	82%	84%	84.5%	85.7%
t1	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
S	86%	86.2%	86.4%	86.6%	86.8%	87.1%	87.3%	87.5%	87.7%	88%	88.2%	88.4%	88.6%	88.8%	89%	

The space occupancy S monotonically increases along with cars travel time t1 on the road 1 in Fig.2.

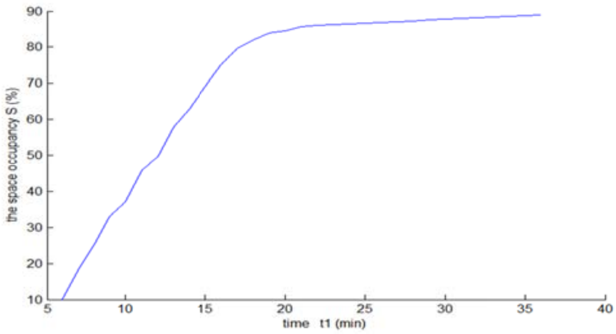

FIGURE.2.The space occupancy S increases along with cars travel time t1

(2) In the stable equilibrium state, the new relationship between the flow F1 on the road 1 and time t1

In the stable equilibrium network, record the data such as F1, t2, t1 and so on in computer simulation. According to the stabilization mechanism of the road network and principle of Wardrop t1=t2, t1 changes when constant t2 value changes. We get the flow and travel time data on the road 1, from which the flow-travel time is obtained, which is shown in Fig.4. The turning point appears at t1=17min.

(3) The same stable state on the road can be reached with the different cars initial speed

Figure 4 shows that the relationship of the flow F1 on the road 1 and travel time t1 on the road 1(and road 2) in the stable state, when cars enter the network at the low speed of 27 feet/s, the medium speed 50 feet/s and the medium speed vamx in the model. The Fig.4.above displaces that the two traffic flow--travel time curves in the stable state at the medium and high speed. The two curves both coincide with the curve of the 27 feet/s initial velocity. Fig.4 shows that when other variables keep unchanged, only when the initial speed of cars entering the network change, flow–travel time curve is not affected by the cars initial speed at the entrance. That is to say,whatever the initial speed of cars entering the network is, at last cars on the road 1 can reach the same stable state, in which all the parameters, such as the displacement between cars, cars speed and car numbers on the road 1 are all the same.

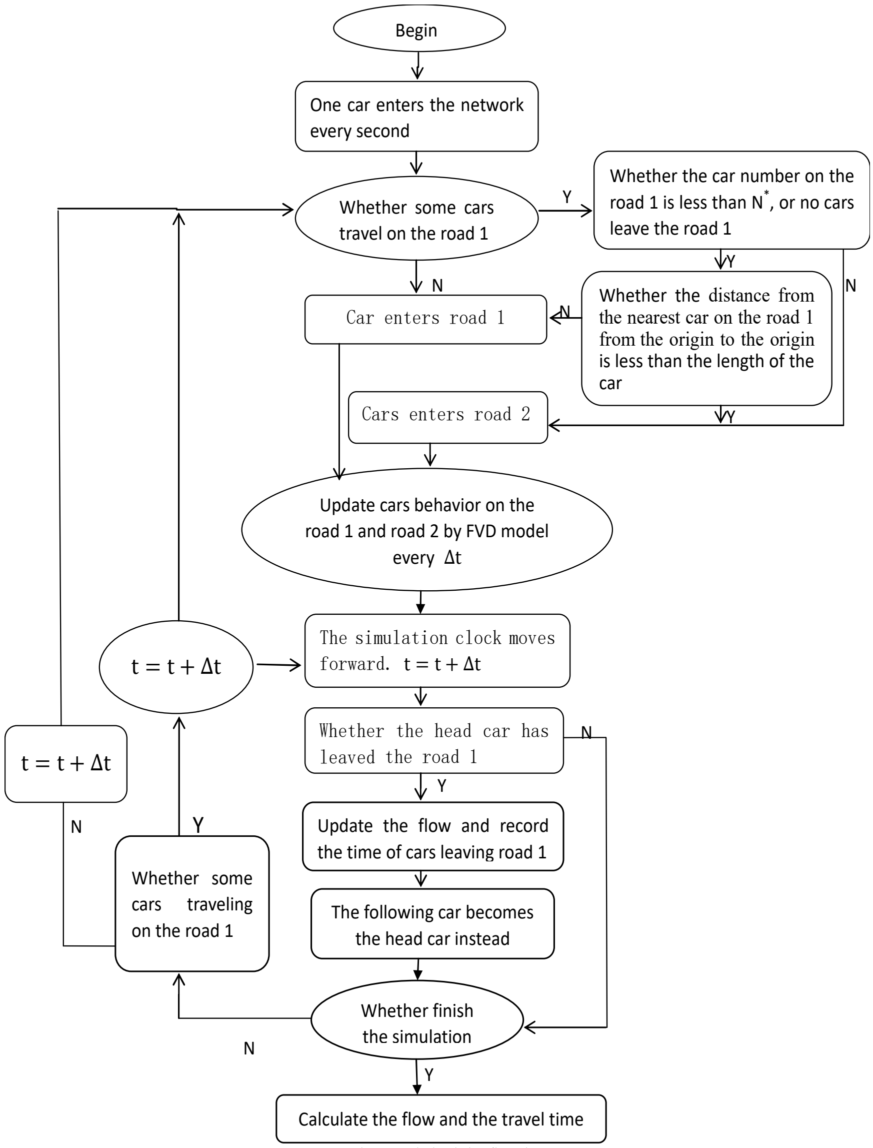

Figure 3. The network simulation flow chart

TABLE.3. In The Stable Equilibrium Network, F1, t1 and t2

F	80	130	171	193	210	228	238	255	264	273	**274**	2648	
1	0	4	6	8	5	8	7	9	8	0	**5**		
t1	6.00333	7	7.9966	8.99166	9.98833	10.9816	11.973	12.963	13.956	17.9233		12	12
	14.948	15.946	**16.923**							3		3	3
t2	6	7	8	9	10	11	12	13	14	18	64	64	64
	15	16	**17**								4	4	4
F	2522	2408	2348	2250	2160	2030	1959	1872	1836	1611			
1	1766	1706	1658										
t1	18.9183	19.9	20.97	21.96	22.93	23.9	25.0325.98		27.08	30.926		12	12
	28.03	29.071	29.992									3	3
t2	19	20	21	22	23	24	25	26	27	31	64	64	64
	28	29	30								4	4	4
F 1	1557	1516	1457	1416	1391								
t1	31.96	33.02	33.97	34.915	36.023							12	12
												3	3
t2	32	33	34	35	36						64	64	64
											4	4	4

Here we get the second conclusion of this model. The new relationship between the link flow and link travel time is that the driving time increases along with the traffic flow first and then decrease. The turning point appears the congestion. After that the hypercongestion appears.

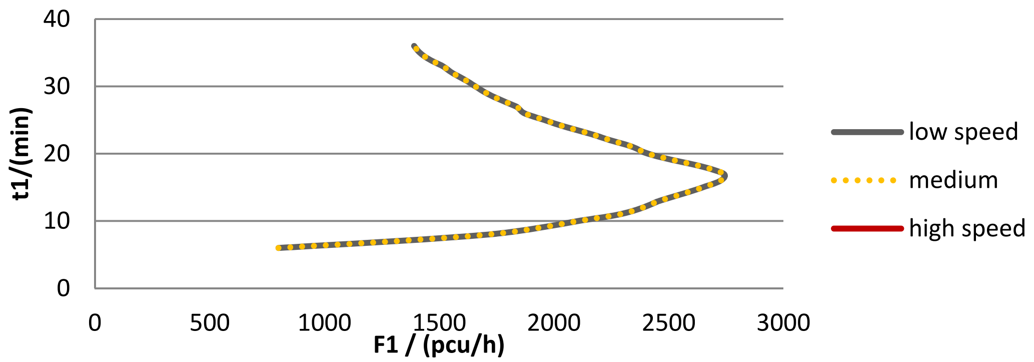

FIGURE.4. the relationship of the traffic flow on the road 1 and travel time t1

Conclusion

Most previous scholars supposed that hypercongestion just occur in the dynamic situation, and driving time on the road increases along with flow. This study aims to present a comprehensive network model with two road, which forms a road regulating mechanism. The model simulation result shows that in the static state, just with the road choice mechanism, when the flow on the road increases monotonously, the driving time increases first and then decreases. Congestion appears at the turning point, the hypercongestion appears after the turning point. The computer simulation proves this new relationship between the travel time and traffic flow on the road indeed exists.

The finding in this paper provides a new theoretical principle to describe the traffic congestion. Road traffic flow and travel time are very important concepts，road traffic flow represents city road traffic capacity, and travel time is the important index of the traffic service quality. The two concepts influence each other in the actual traffic state. This model study proposes a new theory principle about the relationship between flow and travel time on one road, and provides the scientific theory for traffic management to manage the congestion.

Acknowledgments

This paper was funded by the National Natural Science Foundation of China. We also acknowledge the editors and assessors in CDMMS 2017.

References

[1] Xiao Qiusheng, A study of the Dependency of Travel time on Traffic Flow, China Journal of highway and transport, Commun. 4(1991)41-46.

[2] Bando M, Hasebe K, Nakyaama A, Shilbata A, Sudiyama Y , Dynamical model of traffic congestion and numerical simulation, Phys.Rev.E, Commun. 51 (1995)1035-1042.

[3] Xiao Q S, Guo Z Q,Lin B L.A Study of the Dependency of Travel Time on Traffic Flow, China Journal of Highway and Transport | Chin J High Transport, Commun.4 (1991)41-46.

[4] Jiang R, Wu Q S, Zhu Z J. Full velocity difference model for a car-following theory, Phys.Rev.E Commun.64 (2001)017101-1---017101-4.

[5] Dazis D C, Herman R, Potts R B. Car-Following Theory of Steady-State Traffic Flow, Operation Research Commun. 7 , (1959)499-505.

[6] Helbing D,Tilch B. Generalized force model of traffic dynamics, Physical Review E, Commun. 58, (1998)133-138.

[7] Brackstone M, McDonld M., Car-following: a historical review, Transportation Research Part F, Commun.2, (1999)181-196.

[8] Soyoung A, Michael J C, Jorge L. Verification of A simplified Car-Following Theory, Transport Research Part B, Commun. 38 , (2004)431-440.

[9] Jiang R, Wu Q S, Zhu Z J, A new kind of traffic dynamics model, Chin.Sci.Bull. Commun. 45, (2000)1895-1899 (in Chinese) .