A New Relationship between Link Flow and Link Travel Time

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Abstract. The purpose of this paper is to explore the new relationship between the travel time and traffic flow on the road in the static state that inflow equals outflow. This paper proposes selection mechanism by a parallel two-link network model. Through computer simulation, we get the conclusion that in the static state, just with the road choice mechanism, the relationship is that when the flow on the road increases monotonously, the driving time increases first and then decreases. Congestion appears at the turning point, the hypercongestion appears after the turning point. The computer simulation proves this new relationship between the travel time and traffic flow on the road indeed exists.

1. INTRODUCTION

This article aims to propose a new relationship between the link travel flow and link travel time by simulating a comprehensive network model. In reality, when road appears congestion, vehicle travel time increases monotonously along with the increase of traffic flow on the road [1]. Under the dynamic condition, when the number of the cars driving into the road is more than the number of cars leaving the road, this road is in congestion. Namely, the inflow on this road is more than outflow of the road, road traffic jam even the bottleneck appears. At this time, cars driving time on this road is longer, and the road density is very high. Most scholars study the relationship between the traffic travel time and traffic flow based on dynamics condition [2-9]. However, the congestion in static state is not considered yet. The static state means the inflow on the road equals the outflow of the road, namely the total traffic demand remains the same value. Specifically, the number of cars driving into the road per unit time keeps unchanged. This paper constructs a network with two roads. Through computer simulation, we get the conclusion that in the static state, just with the road choice mechanism, the relationship is that when the flow on the road increases monotonously, the driving time increases first and then decreases. Congestion appears at the turning point, the hypercongestion appears after the turning point.

The next section states path choice stable equilibrium mechanism in the network model. The third section introduces model algorithm and flow chart. The fourth section presents the numerical proof, results analysis and conclusion. Finally, the fifth section is the summary of the model results in the paper.

2. Model with Path Selection Mechanism

2.1 Parallel Two-link Network

As shown in Figure 1, this paper studies the relationship of the traffic flow and travel time on link 1 in the stable state, when the alternative link 2 with infinite capacity exists. The infinite capacity means cars driving time on the link 2 keeps the same constant t2. Under stabilization mechanism of the road network and principle of Wardrop t1=t2, t1 changes when constant t2 value changes. So t2 is different related with the different network stabilization. Through road selection mechanism, we study the relationship between the flow on the road 1 and driving time t1(=t2). We
study the cars driving situation by computer simulation and observe changes of the data, such as flow and travel time. The total traffic demand describes the total number of cars entering the network in a certain period of time, taking one hour usually.

Model:
Assume the weather is fine, there are two paths between the origin and the destination. The length of road 1 is L1 km. The total traffic demand in the network is M, which equals 3600 pcu/h. Cars initial speed v equals 27 feet/s. Car maximum speed is vmax, which equals 100 km/h. The car length c equals 5m. The cars driving time on road 1 is t1, and the cars driving time on road 2 is t2. In the network stable state, F1 is the traffic flow on the road 1 per hour. The update step ∆t equals 0.1s. Let's analyze the relationship of the link traffic flow and link travel time on the link 1 in the network stable state below.

L1=5km, v=100 km/h, c=5m, M=3600 pcu/h.

2.2 Multiple velocity difference Car Following Model MFVD
Multiple velocity difference car following model enhances the stability of the traffic flow by using the velocity differences of multiple vehicles. The linear stability region of this model is expanded by using the linear stability theory compared with OV model and FVD model. The modified Korteweg-de Vries equation of MFVD model is constructed and solved to prove the stable regional expansion. In this paper, we use the MVD model to study cars behavior on the two roads in the network and to explore the relationship between the road traffic flow and travel time on roads in the network.

The MFVD model shows below:

Acceleration:

\[ a_n(t) = \alpha(V(\Delta x_n(t)) - v_n(t)) + \sum_{j=1}^{m} k_j \Delta v_{n+j-1}(t) \]  

Parameter:
\[ \alpha=0.3s^{-1}, \quad \lambda = \begin{cases} 0.5s^{-1}, & \text{if } \Delta x_n \leq 80m \\ 0, & \text{otherwise} \end{cases} \]

The optimal speed:
\[ V(\Delta x_n) = \frac{\text{v}_{\text{max}}}{2} (\tanh(\Delta x_n - h_c) + \tanh(h_c)) \]

Velocity and displacement:
\[ v_n(t + \Delta t) = v_n(t) + \Delta t \cdot \frac{dv_n(t)}{dt} \]
\[ x_n(t + \Delta t) = x_n(t) + \Delta t \cdot v_n(t) + \frac{1}{2} \cdot \frac{dv_n(t)}{dt} \cdot (\Delta t)^2 \]

MFVD car following model is a microscopic model. This model is used to simulate cars driving behavior on the roads in the network. We can obtain all the important data about the cars driving in the network by computer simulation, including road traffic flow and cars travel time on the road and so on. We also can realize how cars run on the road at any time by simulation.

2.3 Network Stable Equilibrium Condition and Path Selection Mechanism
The road network equilibrium is defined below:
\[ f(t_1 > 0, \text{if } t_1 \leq t_2 \] 
\[ f(t_1 = 0, \text{if } t_1 > t_2 \] 

According to the principle of Wardrop, cars at the network entrance choose the shorter travel time road in the network. In this model, the road 2 has infinite capacity and \( t_2 \) is the cars travel time on the road 2. \( N \), which is the number of moving vehicles on the road 1, is used to predict cars travel time on the road 1 equivalently. When the car number on the road 1 is \( N^* \), cars travel time on the road 1 equals \( t_2 \). According to the Wardrop principle, the car at the entrance of the network chooses each road equivalently in this case. When the car number on the road 1 is less than \( N^* \), cars travel time on the road 1 is less than \( t_2 \). The car at the entrance of the network will choose road 1. When the car number on the road 1 is more than \( N^* \), cars travel time on the road 1 is more than \( t_2 \). The car at the entrance of the network will choose road 2.

Therefore, according the definition above, equilibrium state is equivalent to:

\[ f(t_1 > 0, \text{if } N_1 \leq N^* \] 
\[ f(t_1 = 0, \text{if } N_1 > N^* \] 

3. Algorithm and The Flow Chart

The traffic simulation in the network is a complex process, involving many different sub-modules circulation and calling. The figure 2 below is the network traffic simulation flow chart.

4. Numerical Simulation Demonstration

(1) In the stable equilibrium state, the density on the road 1 increases along with \( t_1 \)

When the network reach the stable state \( t_1 = t_2 \), cars number on the road 1 equals \( N^* \) at this time. The relationship between \( N^* \) and \( t_2 \): road 1, on which the cars number is \( N^* \), and the road 2 construct the a new different network with the different cars travel time \( t_2 \), under the network equilibrium state \( t_1 = t_2 \). Record variables value at in every stable state network, such as the cars speed, the density of the road 1, and traffic flow \( F_1 \) etc. The study found that under the network stability, the density of road 1 increases monotonously along with the travel time \( t_2 \) increases from 6 min to 36min by the step \( \Delta t \) equals 1 min.

**TABLE 1.** In the Network Stable Equilibrium State, The Different \( N^* \) Values on the Road 1 and Different \( t_2 \) Corresponding

<table>
<thead>
<tr>
<th>( t_2 )</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
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<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N^* )</td>
<td>100</td>
<td>180</td>
<td>250</td>
<td>330</td>
<td>370</td>
<td>460</td>
<td>496</td>
<td>580</td>
<td>630</td>
<td>690</td>
<td>750</td>
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<td>820</td>
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<td>882</td>
<td>884</td>
<td>886</td>
<td>888</td>
<td>890</td>
</tr>
</tbody>
</table>

From the table 1 above, in the stable equilibrium state network, car number \( N^* \) on the road 1 increases monotonously along with \( t_2 \). That means \( t_1 \) increases results in the density on the road 1 increasing on one road 1. Here the space occupancy \( S \) shows below instead of the density on the road 1.
The space occupancy S monotonically increases along with cars travel time t1 on the road 1 in Fig.2.

\[\text{FIGURE 2. The space occupancy S increases along with cars travel time t1}\]

(2) In the stable equilibrium state, the new relationship between the flow F1 on the road 1 and time t1

In the stable equilibrium network, record the data such as F1, t2, t1 and so on in computer simulation. According to the stabilization mechanism of the road network and principle of Wardrop t1=t2, t1 changes when constant t2 value changes. We get the flow and travel time data on the road 1, from which the flow-travel time is obtained, which is shown in Fig.4. The turning point appears at t1=17min.

(3) The same stable state on the road can be reached with the different cars initial speed

Figure 4 shows that the relationship of the flow F1 on the road 1 and travel time t1 on the road 1(and road 2) in the stable state, when cars enter the network at the low speed of 27 feet/s, the medium speed 50 feet/s and the medium speed vamx in the model. The Fig.4.above displaces that the two traffic flow--travel time curves in the stable state at the medium and high speed. The two curves both coincide with the curve of the 27 feet/s initial velocity. Fig.4 shows that when other variables keep unchanged, only when the initial speed of cars entering the network change, flow--travel time curve is not affected by the cars initial speed at the entrance. That is to say,whatever the initial speed of cars entering the network is, at last cars on the road 1 can reach the same stable state, in which all the parameters, such as the displacement between cars, cars speed and car numbers on the road 1 are all the same.

\[\text{TABLE 2. In The Network Stable Equilibrium State, The space occupancy S values on the Road 1 and Different t1 Corresponding}\]

<table>
<thead>
<tr>
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<th>9</th>
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<tbody>
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<td>6</td>
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<td>8</td>
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<td>17</td>
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<td>19</td>
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</tr>
<tr>
<td>S</td>
<td></td>
<td>10%</td>
<td>18%</td>
<td>25%</td>
<td>33%</td>
<td>37%</td>
<td>46%</td>
<td>49.6%</td>
<td>58%</td>
<td>63%</td>
<td>69%</td>
<td>75%</td>
<td>79.7%</td>
<td>82%</td>
<td>84%</td>
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<tr>
<td>t1</td>
<td></td>
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<tr>
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<td>86.4%</td>
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<td>87.5%</td>
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<td>88.6%</td>
<td>88.8%</td>
<td>89%</td>
<td></td>
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The space occupancy S monotonically increases along with cars travel time t1 on the road 1 in Fig.2.
Begin

One car enters the network every second

Whether some cars travel on the road 1

Whether the car number on the road 1 is less than N*, or no cars leave the road 1

Car enters road 1

Whether the distance from the nearest car on the road 1 from the origin to the origin is less than the length of the car

Cars enters road 2

Update cars behavior on the road 1 and road 2 by FVD model every $\Delta t$

The simulation clock moves forward. $t = t + \Delta t$

Whether the head car has leaved the road 1

Update the flow and record the time of cars leaving road 1

The following car becomes the head car instead

Whether finish the simulation

Calculate the flow and the travel time

Figure 3. The network simulation flow chart.
**TABLE.3. In The Stable Equilibrium Network, F1, t1 and t2**

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<thead>
<tr>
<th>F</th>
<th>80</th>
<th>130</th>
<th>171</th>
<th>193</th>
<th>210</th>
<th>228</th>
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<th>255</th>
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<td>1457</td>
<td>1416</td>
<td>1391</td>
</tr>
</tbody>
</table>

Here we get the second conclusion of this model. The new relationship between the link flow and link travel time is that the driving time increases along with the traffic flow first and then decrease. The turning point appears the congestion. After that the hypercongestion appears.

**FIGURE.4. the relationship of the traffic flow on the road 1 and travel time t1**

**Conclusion**

Most previous scholars supposed that hypercongestion just occur in the dynamic situation, and driving time on the road increases along with flow. This study aims to present a comprehensive network model with two road, which forms a road regulating mechanism. The model simulation result shows that in the static state, just with the road choice mechanism, when the flow on the road increases monotonously, the driving time increases first and then decreases. Congestion appears at the turning point, the hypercongestion appears after the turning point. The computer simulation proves this new relationship between the travel time and traffic flow on the road indeed exists.

The finding in this paper provides a new theoretical principle to describe the traffic congestion. Road traffic flow and travel time are very important concepts, road traffic flow represents city road
traffic capacity, and travel time is the important index of the traffic service quality. The two concepts influence each other in the actual traffic state. This model study proposes a new theory principle about the relationship between flow and travel time on one road, and provides the scientific theory for traffic management to manage the congestion.

Acknowledgments

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References


