Design of the Complex Surface of the Implicit Geometric Modeling-Based Automotive Body

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Abstract—**For improving smooth transition of the elbow and keeping geometric modeling characteristics of the elbow, the equation of blending surfaces was established by the means of implicit geometric modeling. Adjustable function of controlling parameter increases freedom of blending surfaces modeling. Sudden changes of stresses at three conjunctions, which are the conjunction of anchor ring and small circular cylinder, the conjunction of small circular cylinder and blending surfaces, the conjunction of blending surfaces and big circular cylinder, are all very small by analyzing stresses of the elbow .This illustrates that bending effect of the elbow is good. It is feasible to use the blending surfaces for the elbow.**

Keywords-Autobody; blending surface of the elbow; modeling; computer applications; design

I. INTRODUCTION

Autobody, as it is made from and fused by a variety of complex high-order curved surfaces, is quite complex in vision, drawing, expression and presentation [1]. However, the technical characteristics of the autobody itself require to use virtualization digital products and design model in the field of autobody design. As composite materials can reduce the car weight and enhance its endurability [2], they have attracted the attention of manufacturers, giving rise to a round of activities such as design, development and testing. At present, some foreign countries have begun to apply computer-controlled multi-axis winding technology to design and manufacture winding composite elbow. Interiorly, Liang Youdong[3] et al. in Zhejiang University have done some studies on the stability of geodesic and non-geodesic winding of the elbow section, which strongly promote the development of winding pattern design for elbow; Liu Rongmei [4] et al. proposed elbow surface modeling-a method by using hyperboloid of one sheet - cone - circular arc surface to achieve a smooth transition through variable diameter of straight tube. However, the blending surface in such configuration can't be adjusted, the expression is unable to be presented in a uniform way and it may be not smooth enough. These problems greatly affect the quality of the blending surface, increasing the complexity of and difficulty for elbow filament winding design and production. In filament winding technology, for the surface to be modeled, the intrinsic geometry has to be calculated such as geodesic trajectory etc., which requires that expression of the blending surface is unique and as simple as possible, and G^1 must be continual. Zhang Sanyuan [5] et al. took low order algebraic surface as a blending surface to shape tee modeling, and the generated blending surface is a low order algebraic surface with unified expression. However, the coefficients in the expression do not have intuitive geometric meaning, so the shape of the algebraic surfaces is uncontrollable. A method described below will achieve composite elbow surface modeling and its blending surface is a low order algebraic surface with simple and unified expression. What's more, the shape of the blending surface generated in this way is controllable.

II. GEOMETRIC MODELING IMPROVEMENT AND CONTINUITY

A. Overall Design of the Elbow

1) Geometric model: Geometric model discussed here is bell and spigot elbow [4,6]. *Figure 1 is the plane figure of the elbow model.*
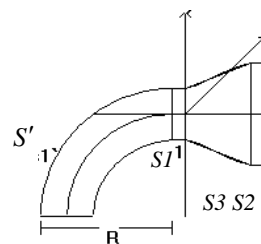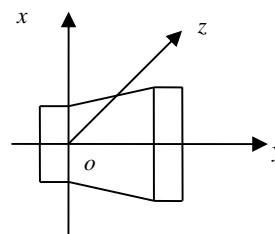

FIGURE I. BELL AND SPIGOT ELBOW
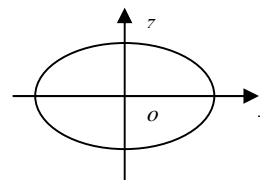

FIGURE II. BLENDING SURFACE

FIGURE III. CROSS SECTION

The source surfaces S_1', S_1 and S_2 are the anchor ring,

$$S' : f'(x,y,z) = [(x+h_1)^2 + (y+R)^2 + z^2 + R^2 - r_0^2]^2 - 4R^2[(x+h_1)^2 + (y+R)^2] = 0 \quad (1)$$

$$S_1 : \begin{cases} f(x,y,z) = x^2 + z^2 - r_0^2 = 0 \\ y_1 \le y \le y_2 \end{cases} \quad (2)$$

$$S_2 : \begin{cases} g(x,y,z) = x^2 + z^2 - R_0^2 = 0 \\ y_3 \le y \le y_4 \end{cases} \quad (3)$$

where R is the radius of curvature of the elbow centerline, r_0 the radius of the elbow and h, h_1 are the lengths of the blending surface and the small circular cylinder respectively. $y_2 = y_1 + h_1$, $y_3 = y_1 + h_1 + h$. In order to construct blending surface of the two source surfaces, a simple algebraic surface S_3 needs to be constructed first as a control surface. And S_3 has to meet two requirements: first, it must intersect with both S_1 and S_2; second, if S_3 is a centered surface, its central point should be at the origin. According to the shape of the bell and spigot elbow, hyperboloid of one sheet is selected as the controlling surface.

$$S_3 : h(x,y,z) = \frac{x^2}{a^2} + \frac{z^2}{c^2} - \frac{y^2}{b^2} - 1 = 0 \quad (4)$$

where $a, b, c > 0$ represent the length of the semi-axis respectively in the direction of x, y, z . To achieve G^1 smooth transition for these two source surfaces, the relationship among the parameters a, b, c, R_0, h upon comprehensive analysis above is as follows:

$$\begin{cases} 0 < c = a \le 1 \\ R_0 < h \le \dfrac{b}{a}\sqrt{R_0^2 - a^2} \\ R_0 > 1 \end{cases}$$

Thus, when the hyperboloid of the revolution of one sheet with circular cross section (rather than elliptical cross section) is set as a control surface, its algebraic expression is:

$$z^2 + x^2 = r_0^2 + \frac{y^2}{b^2} \quad (5)$$

To construct the blending surface, in addition to the control surfaces S_3, the two parameters n and λ_1 also have to be determined. From [6], we know that $n = 3$, so the blending surface equation is:

$$S : F(x,y,z) = h^3(x,y,z) + \lambda_1 f(x,y,z) g(x,y,z) = 0$$

namely

$$(z^2 + x^2 - \frac{y^2}{b^2} - r_0^2)^3 + \lambda_1 \left(x^2 + z^2 - r_0^2 \right)\left(R_0^2 - x^2 - z^2 \right) = 0 \quad (6)$$

2) Effect of Control Parameter λ_1 on the Blending Surface Shape

As the plane y is an ellipse (Figure 3), the cross-section resulted from using a constant to cut the blending surface (Equation (6), shown in Figure 2). When z is a constant, through analysis, it can be seen that the effect of λ_1 on x is that x is a decreasing function concerning λ_1; when λ_1 increases, blending surface becomes thinner and vice versa. Therefore, though the control surface and the source surface have the same line of intersection, the regulation of the control parameter λ_1 leads to differences in the shape of blending surface. The blending surface S achieves G^1 continuity with the source surface S_1 at the line of intersection C_1; the blending surface S achieves G^1 continuity with the source surface S_2 at the line of intersection C_2 . Therefore, blending surface is a G^1 continuous algebra blending surface. That is, the blending surface achieves smooth transition from S_1 to S_2 [3].

B. Instance Model of Elbow

In Visual C + +6.0 language environment, we can use OpenGL graphics library to build model for the blending surface of elbow, where $b = \frac{4}{\sqrt{5}}$, $\lambda_1 = 0.0001$, $r_0 = 10$, $R_0 = 20$, $R = 100$, $y_1 = 80$, $y_2 = 92$, $y_3 = 110$, $y_4 = 122$ and the length of the blending surface $h = 30$, which made the conjunctions of the blending surface and the source surface at least reach G^1 continuity and got good results.

III. STRESS ANALYSIS OF ELBOW

In filament winding technology, stress analysis of the elbow surface to be modeled is conducted. According to the formula for stress calculation proposed by Xu Zhilun

[7],
$$N_\theta = \frac{pR_2}{2} = \frac{p}{2\kappa_v}, \quad N_\varphi = \frac{pR_2}{2}(2 - \frac{R_2}{R_1}) = \frac{p}{2\kappa_v}(2 - \frac{\kappa_u}{\kappa_v}), \quad \kappa_u = \frac{L}{E}, \quad \kappa_v = \frac{N}{G},$$

where p is the internal pressure of the elbow, R_1 and R_2 are radiuses of curvature, κ_u and κ_v are the two principal curvatures of surface at one point, E and G are the first fundamental quantity and L and N the second fundamental quantity. The stresses of the conjunctions of four surfaces: the conjunction of anchor ring and small cylindrical surface, the conjunction of the small cylindrical surface and the bending surface, and the conjunction of the bending surface and the big cylindrical surface are calculated.

A. *Stress of Anchor Ring*

Equation of the elbow surface (anchor ring) is

$$r = \{(R + r_0 \cos v)\cos u, (R + r_0 \cos v)\sin u, -r_0 \sin v\} \quad (7)$$

$$0 < r_0 < R, u \in [-u_0, u_0], v \in [0, 2\pi]$$

where R is radius of curvature of the elbow centerline and r_0 is the radius of the elbow.

$$N_\theta = \frac{pr_0}{2}, N_\varphi = \frac{pr_0}{2}(2 - \frac{r_0 \cos v}{R + r_0 \cos v}) \quad (8)$$

B. *Stress of the Elbow Blending Surface*

Equation of the elbow blending surface is

$$r = \{u\cos v, b[\sqrt[3]{\lambda_1(u^2 - r_0^2)(R_0^2 - u^2)} - r_0^2 + u^2]^{\frac{1}{2}}, -u\sin v\} \quad (9)$$

$$v \in [0, 2\pi], u \in [r_0, R_0]$$

where b is the length of the semi-axis of the control surface (hyperboloid of one sheet) in the direction of y and λ_1 is a parameter.

Assume

$$\sigma(u) = b[\sqrt[3]{\lambda_1(u^2 - r_0^2)(R_0^2 - u^2)} - r_0^2 + u^2]^{\frac{1}{2}}, E = 1 + [\sigma'(u)]^2$$

$$N_\theta = \frac{pu\sqrt{E}}{2\sigma'(u)}, N_\varphi = \frac{pu\sqrt{E}}{2\sigma'(u)}(2 - \frac{u\sigma''(u)}{E\sigma'(u)}) \quad (10)$$

C. *Stress of the Straight Pipe (cylindrical surface)*

(1)Equation of the straight section (small cylindrical surface) is

$$r = \{r_0 \cos v, y, -r_0 \sin v\} \quad (11)$$

$$y_1 \le y \le y_2, v \in [0, 2\pi]$$

$$N_\theta = \frac{pr_0}{2}, N_\varphi = pr_0 \quad (12)$$

(2) Equation of the straight section (big cylindrical surface) is

$$r = \{R_0 \cos v, y, -R_0 \sin v\} \quad (13)$$

$$y_3 \le y \le y_4, v \in [0, 2\pi]$$

$$N_\theta = \frac{pR_0}{2}, N_\varphi = pR_0 \quad (14)$$

By comparing the stress of anchor ring in equation (8) with that of the small cylindrical surface in (12), it can be seen that the stress N_θ of each one is the same and the difference of N_φ is small. By comparing the stress of small cylindrical surface in equation (12) with that of the blending surface in (10), it can be seen that the differences of N_θ, N_φ in equations (10) and (12) are small as $u \in [r_0, R_0]$ and at the conjunction of the small cylindrical surface and the blending surface, $u > r_0$ and $u \to r_0$. Similarly, by comparing (10) with (14), we'll find that the stress N_θ, N_φ in (10) are also slightly different from those in (14) as $u \in [r_0, R_0]$ and at the conjunction of the blending surface and the big circular cylinder $u < R_0$ and $u \to R_0$. Thus, it can be seen that sudden changes in stress at the conjunction of anchor ring and small cylindrical surface, the conjunction of small cylindrical surface and blending surface and the conjunction of the blending surface and big cylindrical surface are all very small. This illustrates that bending effect of the elbow is good and it is feasible to use the blending surfaces for the elbow.

IV. CONCLUSIONS

For modern autobody, attention is paid to the coordination among components of the complex surfaces and sudden changes in stresses at conjunctions. The design of automobile elbow surface will directly affect its functions and appearance. This paper, taking algebraic surface as a control surface, offers the overall modeling of composite elbow and effectively improves the bell and spigot elbow proposed by Liurong Mei [4] which uses a variety of surfaces to achieve smooth transition. What's more, it avoids the problem of parameter matching at the conjunction, and contributes to the "processing" of filament winding for elbow. As a result, we find that the method proposed in this paper is characterized by simple and unified expression, controllable shape and that the overall elbow surface is at least G^1 smooth, which are particularly suitable for application in modern filament

winding technology.

Parameters in the algebraic equation of the blending surface proposed by Zhang Sanyuan[5] have no intuitive geometric meaning. However, this paper adopts hyperboloid of one sheet as a control surface to generate blending surface, semi-axis *a*, *b*, *c* of which, as we all know, have intuitive geometric meaning. Therefore, the shape of the blending surface can be controlled by changing *a*, *b*, *c* in equation of hyperboloid of one sheet.

Sudden changes of stresses at three conjunctions, which are the conjunction of anchor ring and small cylindrical surface, the conjunction of small cylindrical surface and blending surfaces and the conjunction of blending surfaces and big cylindrical surface are all very small by analyzing stresses of the elbow. This illustrates that it is feasible to use the blending surfaces for the elbow.

The overall modeling of the composite elbow improves the manufacturing quality and performance of automotive products. It also can be applied to other tubular surface modeling (Y-pipe, manifold pipe etc.), having a certain theoretical value and a wide range of applications.

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