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The Set SFmBDR of Fuzzy Time Series Forecasting Models

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***Abstract*—Fuzzy time series forecasting models are created by Song and Chissom in 1993. In 2012, Saxena, Sharma& Easo put forward the forecasting model which more improve the forecasting accuracy. According to this, this paper improve the set SFmBDR of fuzzy time series forecasting model based on differential rate. The forecasting model SFmBDR (0.000002,0.000004) and SFmBDR (0.000004,0.000002) of SFmBDR can gain the AFER=0% and MSE=0, during we study the problems of enrollments data of University of Alabama in 1971~1992. They should have better application potential.**

Keywords-forecasting method of fuzzy time series; fuzzy function of SFmBDR;Inverse fuzzy function of SFmBDR; forecasting function of SFmBDR

I. INTRODUCTION

Song and Chissom firstly come up with the fuzzy time series forecasting model and discuss the problems of enrollments data of University of Alabama in 1971~1992, which apply fuzzy set series[3]. Sequentially, they put forward many fuzzy times series forecasting model which can see in the references [1,2,4-6,8-17]. However, the forecasting accuracy is not high, and this problems are still puzzling people.Saxena, Sharma, Easo[9] (2012) propose the new set of forecasting fuzzy model. It gain the unprecedented forecasting accuracy, when they study the problems of enrollments data of University of Alabama. Therefore, it cause the researches of Wang, Guo, Feng, Jin, Wu[5,6,8,10-12,15-17], and they respectively present forecasting model, which have some improvement in the forecasting accuracy. This paper further excavate their research achievement, and propose the set SFmBDR of fuzzy time series forecasting model based on order differential rate. The forecasting model of SFmBDR(0.000002,0.000004) and SFmBDR(0.000004,0.000002) gain the enrollments forecasting data AFER=0% and MSE=0, when study the problems of enrollments data of University of Alabama in 1971~1992. It achieve the highest level of forecasting accuracy.

II. THE SET SFMBDR OF FUZZY TIME SERIES FORECASTING MODEL

The relative concepts see the references Saxena, Sharma, Easo[9]. When we study a problem of time series forecasting, we can assume its universe of historical data as $N=\{N_1,N_2,\cdots,N_n\}$.The formulas of differential rate of historical data is $M_t=(N_t-N_{t-1})/N_{t-1}$. The universe differential rate of historical data is $M=\{M_2,M_3,\ldots,M_n\}$. We propose the definition of fuzzy function and inverse fuzzy function and forecasting function by spread the theories of Saxena, Sharma, Easo[9].

Definition 1: If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\cdots,N_n\}$ and the universe differential rate of historical data is $M=\{M_2,M_3,\ldots,M_n\}$. If in M,we can be defined fuzzy series as $U_t(0.2,0.8)$:

$$U_t(0.2,0.8)=\begin{cases}\frac{1}{M_2}+\frac{0.8}{M_3}+\frac{0}{M_3}+\cdots+\frac{0}{M_n}, t=2,\\ \frac{0}{M_2}+\cdots+\frac{0}{M_{t-2}}+\frac{0.2}{M_{t-1}}+\frac{1}{M_t}, 3\le t\le n.\end{cases}$$

Where the inverse fuzzy series of fuzzy series $U_t(0.2,0.8)$ can be defined $S_t(0.2,0.8)$:

$$S_t(0.2,0.8)=\begin{cases}\dfrac{1+0.8}{\frac{1}{M_2}+\frac{0.8}{M_3}+\frac{0}{M_4}+\cdots+\frac{0}{M_n}}, t=2,\\ \dfrac{0.2+1}{\frac{0}{M_2}+\cdots+\frac{0}{M_{t-2}}+\frac{0.2}{M_{t-1}}+\frac{1}{M_t}}, 3\le t\le n.\end{cases}$$

Definition 2: If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\ldots,N_n\}$ and the universe differential rate of historical data is $M=\{M_2,M_3,\ldots,M_n\}$. If in M, we can be defined inverse fuzzy series function as $U_t(\mu_1,\mu_2)$

$$U_t(\mu_1,\mu_2)=\begin{cases}\frac{1}{M_2}+\frac{\mu_2}{M_3}, & if\quad t=2\\ \frac{\mu_1}{M_{t-1}}+\frac{1}{M_t}, & if\ 3\le t\le n.\end{cases}\tag{1}$$

Where independent variable $\mu_1\in[0,1)$ and $\mu_2\in[0,1)$ also call membership of $U_t(\mu_1,\mu_2)$. when $\mu_1=\mu_2=\mu$, the fuzzy function SFmBDR denoted by $U_t(\mu)$ (t = 2,3,…,n).

Definition 3: If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\cdots, N_n\}$ and the universe differential rate of historical data is $M=\{M_2,M_3,\ldots, M_n\}$. If in M,we can be defined fuzzy series function $U_t(\mu_1,\mu_2)$ of inverse fuzzy function $S_t(\mu_1,\mu_2)$:

$$S_t(\mu_1,\mu_2)=\begin{cases}\dfrac{1+\mu_2}{\dfrac{1}{M_2}+\dfrac{\mu_2}{M_3}}, & if\ t=2,\\[2ex] \dfrac{\mu_1+1}{\dfrac{\mu_1}{M_{t-1}}+\dfrac{1}{M_t}}, & if\ 3\le t\le n.\end{cases} \tag{2}$$

Where independent variable $\mu_1 \in [0,1)$ and $\mu_2 \in [0,1)$ also call membership of $S_t(\mu_1,\mu_2)$. when $\mu_1=\mu_2=\mu$, the inverse fuzzy function SFmBDR denoted by $S_t(\mu)$($t = 2,3,\ldots,n$).

Definition 4: If have a time series forecasting problem, the universe differential rate of historical data is $M=\{M_2,M_3,\ldots, M_n\}$. As every $t\in\{2,3,\cdots,n\}$, defined forecasting function

$R_t(\mu_1,\mu_2)$:

$$R_t(\mu_1,\mu_2)=N_{t-1}(1+S_t(\mu_1,\mu_2)) \tag{3}$$

Where independent variable $\mu_1 \in [0,1)$ and $\mu_2 \in [0,1)$ also call membership of forecasting function $R_t(\mu_1,\mu_2)$. N_{t-1} is the t-1 years' historical data , $S_t(\mu_1,\mu_2)$ is t years' inverse fuzzy function. when $\mu_1=\mu_2=\mu$, the inverse fuzzy function SFmBDR denoted by

$R_t(\mu)$ ($t = 2,3,\ldots,n$). When μ_1 and μ_2 get the specific membership in [0,1), $R_t(\mu_1,\mu_2)$ call the forecasting formula of SFmBDR, it can also denoted by SFmBDR(μ_1,μ_2).

If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\cdots, N_n\}$ and the universe differential rate of historical data is $M=\{M_2,M_3,\ldots, M_n\}$. If the membership μ_1 and μ_2 can certain specific figure in [0,1), we can build a forecasting model SFmBDR(μ_1,μ_2) through the formula (3), we can get the following step:

First, listing the historical sheet of time forecasting problem. Second, listing the universe of discourse historical rate N and the universe of discourse of differential rate M. Third, listing the forecasting formula SFmBDR (μ_1,μ_2). Fourth, applying SFmBDR (μ_1,μ_2) to calculate the forecasting value of historical rate. Forecasting formula SFmBDR (μ_1,μ_2) is the forecasting model SFmBDR (μ_1,μ_2), because we follow the route: fuzzy function $U_t(\mu_1,\mu_2)$ to inverse fuzzy function $S_t(\mu_1,\mu_2)$ to forecasting model SFmBDR(μ_1,μ_2) or forecasting formula SFmBDR(μ_1,μ_2). We built the forecasting model SFmBDR(μ_1,μ_2), therefore, SFmBDR(μ_1,μ_2)call as fuzzy time series forecasting model. Thus, we conclude definition 5.

Definition 5:If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\ldots, N_n\}$ and the universe differential rate of historical data is $M=\{M_2,M_3,\ldots, M_n\}$. If membership μ_1 and μ_2 get the all figures in the interval [0,1), it can get infinite fuzzy time series forecasting model SFmBDR (μ_1,μ_2). We use whole fuzzy time series forecasting model as an element, they can form set which call SFTSFMBDR(The Set of Fuzzy Time Series Forecasting Models Based on the Difference Rate), it can abbreviate as SFmBDR. Its general element as SFmBDR (μ_1,μ_2). SFmBDR (μ_1,μ_2) not only express a forecasting formula of fuzzy time series, but also show the forecasting formula of fuzzy time series forecasting models.

In table1 and table2, MSE(Mean Square Error) and AFER(Average Forecasting Error Rate) can denoted as follow:

$$MSE=\frac{1}{n}\sum_{t=1}^{n}(R_t-N_t)^2 \quad \text{and} \quad AFER=\left(\frac{1}{n}\sum_{t=1}^{n}|R_t-N_t|/N_t\right)\times 100\%.$$

TABLE I. COMPARISON OF DIFFERENT FORECASTING MODELS

Year	Enroll-ments	Wang, Guo, Feng , Jin.[5]	Feng, Guo, Wang Zhang.[6]	Wang, Guo, Feng, Jin.[8]	Saxena, Sharma, Easo. [9]	Wang, Guo, Feng, Jin.[11]	Wang, Guo, Feng, Jin.[12]
1971	13055	-	-	-	-	-	-
1972	13563	-	13563	-	13486	-	-
1973	13867	13745	13867	13845	13896	13813	13809
1974	14696	14531	14696	14729	14698	14681	14610
1975	15460	15575	15461	15412	15454	15525	15422
1976	15311	15446	15312	15317	15595	15189	15299
1977	15603	15555	15604	15620	15600	15685	15642
1978	15861	15901	15860	15895	15844	15895	15901
1979	16807	16933	16804	16786	16811	16878	16782
1980	16919	16950	16920	16961	16916	16839	16935
1981	16388	16601	16387	16334	16425	16505	16328
1982	15433	15456	15430	15461	15657	15349	15362
1983	15497	15544	15496	15497	15480	15511	15496
1984	15145	15165	15143	15094	15214	15026	15077
1985	15163	15187	15163	15133	15184	15051	15274
1986	15984	15953	15976	15972	15995	15980	15966
1987	16859	16849	16858	16805	16861	16805	16849
1988	18150	18211	18150	18183	17965	18246	18312
1989	18970	19077	18974	18990	18964	18926	18974
1990	19328	19344	19326	19338	19329	19275	19236
1991	19337	19200	19338	19346	19378	19428	19299
1992	18876	18851	18872	18822	18984	19046	18951
AFER (%)		0.462	0.0099	0.171	0.3406	0.442	0.273
MSE		8963	7	1121	9169	6825	2912

Theorem1: (the Convergence theorems of fuzzy function of SFmBDR) If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\ldots, N_n\}$ and the universe differential rate of historical data is

$M=\{M_2,M_3,\dots, M_n\}$. Thus, as every $t \in \{2,3,\dots,n\}$, when $\mu_1 \to 0, \mu_2 \to 0$, the inverse function $S_t(\mu_1,\mu_2)$ restrain difference rate M_t of historical data , thus:

$$\lim_{\mu_1 \to 0, \mu_2 \to 0} S_t(\mu_1, \mu_2) = M_t$$

Proof if every $t \in \{2,3,\dots,n\}$, ordering membership $\mu_1 \to 0$, $\mu_2 \to 0$ is limited to the calculation of inverse fuzzy function (2), it can gain the calculation form as:

$$\lim_{\mu_1 \to 0, \mu_2 \to 0} S_t(\mu_1,\mu_2) = \lim_{\mu_1 \to 0, \mu_2 \to 0} \begin{cases} \dfrac{1+\mu_2}{\dfrac{1}{M_2}+\dfrac{\mu_2}{M_3}}, t=2 \\ \dfrac{\mu_1+1}{\dfrac{\mu_1}{M_{t-1}}+\dfrac{1}{M_t}}, 3 \le t \le n \end{cases}$$

$$= \begin{cases} \dfrac{1}{\dfrac{1}{M_2}}, t=2 \\ \dfrac{1}{\dfrac{1}{M_t}}, 3 \le t \le n \end{cases} = \frac{1}{\dfrac{1}{M_t}}(2 \le t \le n) = M_t (2 \le t \le n)$$

Theorem 2: (the Convergence theorems of forecasting function of SFmBDR) If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\dots, N_n\}$and the universe differential rate of historical data is $M=\{M_2,M_3,\dots, M_n\}$. Thus, as every $t \in \{2,3,\dots,n\}$, when $\mu_1 \to 0$, $\mu_2 \to 0$, the forecasting function $R_t(\mu_1,\mu_2)$ restrain difference rate N_t of historical data , thus:

$$\lim_{\mu_1 \to 0, \mu_2 \to 0} R_t(\mu_1, \mu_2) = N_t$$

Proof if $i \in \{2,3,\dots,n\}$, according to formula of difference rate, knows:

$M_t = (N_t - N_{t-1})/N_{t-1} \Rightarrow N_t = N_{t-1}(1+ M_t)$.

Ordering membership $\mu_1 \to 0$, $\mu_2 \to 0$ is limited to the calculation of forecasting function (3) and apply to the result of theorem1, knows:

$$\lim_{\mu_1 \to 0, \mu_2 \to 0} R_t(\mu_1,\mu_2) = \lim_{\mu_1 \to 0, \mu_2 \to 0} N_{t-1}\left(1+R_t(\mu_1,\mu_2)\right)$$

$$= N_{t-1}\left(1+\lim_{\mu_1 \to 0, \mu_2 \to 0}(\mu_1,\mu_2)\right) = N_{t-1}(1+M_t) = N_t$$

Obviously, there is Theorem3.

Theorem 3 If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\dots, N_n\}$ and the universe differential rate of historical data is $M=\{M_2,M_3,\dots, M_n\}$. If every $t \in \{2,3,\dots,n\}$, thus fuzzy function U_t (μ_1,μ_2), inverse fuzzy function and forecasting function are continuous function.

As corollary, there is theorem 4:

Theorem4 If have a time series forecasting problem, its universe of discourse of historical data as $N=\{N_1,N_2,\dots, N_n\}$ and the universe differential rate of historical data is $M=\{M_2,M_3,\dots, M_n\}$. If every $t \in \{2,3,\dots,n\}$, thus, membership μ_1 1and μ_2 is enough small, the forecasting function $R_t(\mu_1,\mu_2)$ of t years' SFmBDR equal to historical data N_t. Of t years.

In this paper, we just list sectional forecasting models and get the comparison of different forecasting methods which apply inverse fuzzy, when we forecast the problems of enrollments data of University of Alabama in 1971~1992. Show in table1 and table2. Table2 shows that use SFmBDR(0.000002, 0.000004) and SFmBDR(0.000004, 0.000002) forecast the results of enrollments data of University of Alabama, it can get MSE=0 and AFER=0.0%. That is the highest accuracy. It also verify the Theorem4 is right.

III. CONCLUSION

We Provide the forecasting models SFmBDR(0.000004, 0.000002)and SFmBDR(0.000002, 0.000004) , and then we get MSE=0and AFER=0% which show in table 2, when we study the problems of enrollments data of University of Alabama in 1971~1992. We conclude the historical results of fuzzy time series forecasting models which accuracy is not high. We knows that sectional forecasting models of SFmBDR have important application potential.

TABLE II. COMPAROSON OF DIFFERENT FORECASTING MODELS

Year	Enroll-ments	Jilani, Burney, Ardil. [13]	Stevenson, Porter. [14]	Wang, Guo, Feng Zhang.[16]	Wang, Guo, Wang, Feng.[17]	SFmBDR (0.000002, 0.000004)	SFmBDR (0.000004, 0.000002)
1971	13055	13579	-	-	-	-	-
1972	13563	13798	13410	-	13563	13563	13563
1973	13867	13798	13932	13867	13867	13867	13867
1974	14696	14452	14664	14691	14695	14696	14696
1975	15460	15373	15423	15461	15460	15460	15460
1976	15311	15373	15847	15311	15311	15311	15311
1977	15603	15623	15580	15607	15603	15603	15603
1978	15861	15883	15877	15861	15861	15861	15861
1979	16807	17079	16773	16797	16796	16807	16807
1980	16919	17079	16897	16920	16919	16919	16919
1981	16388	16497	16341	16375	16388	16388	16388
1982	15433	15737	15671	15436	15432	15433	15433
1983	15497	15737	15507	15497	15497	15497	15497
1984	15145	15024	15200	15136	15144	15145	15145
1985	15163	15024	15218	15163	15163	15163	15163
1986	15984	15883	16035	15861	15982	15984	15984
1987	16859	17079	16903	16858	16859	16859	16859
1988	18150	17991	17953	18154	18150	18150	18150
1989	18970	18802	18879	18976	18970	18970	18970
1990	19328	18994	19303	19329	19327	19328	19328
1991	19337	18994	19432	19338	19337	19337	19337
1992	18876	18916	18966	18766	18874	18876	18876
AFER (%)		1.02	0.57	0.085	0.0026	0	0
MSE		41426	21575	1384	1	0	0

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