A discrete tracking-differentiator design for active vibration control in maglev systems

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Abstract. A second-order linear differentiator with small phase-delay and amplitude attenuation based on a fast synthesis function is presented for active vibration control in maglev systems. The discrete linear structure of the differentiator is given and its tracking and filtering ability is analyzed in time domain. The restraining disturbance performance with optimal control theory for bounded disturbance is discussed.

Keywords: Second-order system; Linear differentiator; Signal filter; Bounded disturbance; Optimal control

1. Introduction

In engineering practice, continuous signal and its differential signal often needs to be drawn reasonably from the discontinuous signal with random noise. However, in the traditional control engineering, it is very difficult to acquire the differential signal because the pure differentiator is unable to realize in physics. Especially, if there are some random noises in the signal, the quality of the corresponding differential signal would have trouble, and even worse it may be submerged by magnified noise [1]. Therefore, designing a simple and practical differentiator has great significance to simplify the controller design and improve the controller performance.

Generally, we often use the approximation method to gain a similar differential signal. Some scholars [2-3] have proposed many ways to obtain differential signals and linear differentiator is the simplest one. Linear differentiator with high-gain can provide $n-1$ order derivative, but this kind differentiator may contain disturbance directly in every level [4]. Linear derivative tracker has a faster convergence rate when system far from the equilibrium point, while close to equilibrium point the convergence rate will slow down [5]. The sliding mode differentiator with second order need to know in

advance that the upper bound of Lipschitz constant for the signal derivative, thus limits the type of input signal, and the chattering is inevitable [6-7]. Paper [8] studies the optimal integrated functions for double integral series system, the introduction of the switching curve in the vicinity of the linear region, and with the precise control strategy a synthesis fast discrete function is acquired. Zhang D.[9], Li H.[10], Han J.Q. [11] design a band-stop filter, a skyhook damping filter and a gap differentiator respectively, to a certain extent, which can restrain the vehicle-guideway coupled vibration. Lingling Zhang [12] proposes a hybrid nonlinear gap differentiator with global fast convergence.

Using the fast synthesis function, this paper constructs a second order linear differentiator with small phase delay and amplitude attenuation, gives the discrete linear structure, analyzes its ability to track and filter in the time domain, and discusses the ability of boundary disturbance immunity of the system based on the optimal theory and singular perturbation technical.

2. Second-Order Discrete Nonlinear Tracking Differentiator

Consider a second-order double integral series of continuous system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ru, |u| \leq 1 \end{cases} \quad (1)$$

Suppose the initial state as $x_1(0) = x_{10}, x_2(0) = x_{20}$. According to the optimal control theory, continuous system (1) has an optimal control strategy that arbitrary point $M(x_{10}, x_{20})$ in phase plane can achieve to the origin point at most one switch and the required time is the least.

Let $\Gamma(x_1(k), x_2(k), r) = x_1(k) + \frac{x_2(k) \cdot |x_2(k)|}{2 \cdot r}$, the set of switching points, that is the switch curve can be expressed as following:

$$\Gamma(x_1, x_2, r) = x_1 + \frac{1}{2 \cdot r} x_2 \cdot |x_2| = 0 \quad (2)$$

The optimal solution of the above control question is

$$u(t) = \begin{cases} -\text{sign}(\Gamma(x_1, x_2, r)), & \text{if } \Gamma(x_1, x_2, r) \neq 0 \\ \text{sign}(x_2), & \text{if } \Gamma(x_1, x_2, r) = 0, x_2 \neq 0 \end{cases} \quad (3)$$

If $M(x_{10}, x_{20})$ is not in the switch curve (2), by adopting the control law (3), the time t_A that $M(x_{10}, x_{20})$ reaches to the switch curve (2) is

$$t_A = s \cdot \frac{x_2}{r} + \sqrt{s \cdot \frac{x_1}{r} + \frac{1}{2} \frac{x_2^2}{r^2}} \quad (4)$$

If $M(x_{10}, x_{20})$ is in the switch curve (2), $\Gamma(x_1, x_2, r) = 0$, by adopting the control law (3), the time t_B that $M(x_{10}, x_{20})$ reaches to the origin point O is

$$t_B = \frac{|x_2|}{r} \quad (5)$$

In order to get the fast control function for second order discrete system which corresponds to system (1), paper [1] proposes an exact discrete control strategy that $M(x_{10}, x_{20})$ in the phase plane reaches to the origin point after a series steps ($k = 0, 1, 2, \dots$) with sample step h by fixed step method.

$$\begin{cases} [u(k), b] = \text{fast}(x_1(k), x_2(k), r, h) \\ x_1(k+1) = x_1(k) + h \cdot x_2(k) + \frac{1}{2} rh^2 \cdot u(k) - \frac{1}{3} bh^3 \\ x_2(k+1) = x_2(k) + rh \cdot u(k) - \frac{1}{2} bh^2, \quad k = 0, 1, 2, \dots \end{cases} \quad (6)$$

For system (1), the function $[u(k), b] = \text{fast}(x_1(k), x_2(k), r, h)$ can be sure by the following rules:

(1) When $\Gamma(x_1(k), x_2(k), r) > 0$, let $s = \text{sign}(\Gamma(x_1(k), x_2(k), r))$,

$$t_A = \frac{x_2(k)}{r} \cdot s + \sqrt{\frac{x_1(k)}{r} \cdot s + \frac{x_2^2(k)}{2 \cdot r^2}}, \text{ then}$$

1) if $t_A \geq h$, then

$$\begin{cases} b = 0 \\ u(k) = -s \end{cases} \quad (7)$$

2) if $t_A < h$, then

$$\begin{cases} b = 0 \\ u(k) = -\left(-\frac{1}{2} + \frac{x_2(k)s}{rh} + \frac{1}{2} \sqrt{1 + \left(\frac{4x_2(k)}{rh} + \frac{8x_1(k)}{rh^2}\right)s} \right)s \end{cases} \quad (8)$$

(2) When $\Gamma(x_1(k), x_2(k), r) = 0$, let $t_B = \frac{|x_2(k)|}{r}$, then

1) if $t_B \geq h$, then

$$\begin{cases} b = 0 \\ u(k) = -\text{sign}(x_2(k)) \end{cases} \quad (9)$$

2)if $t_B < h$, then

$$\begin{cases} b = \frac{6(x_2(k)h + 2x_1(k))}{rh^3} \\ u(k) = -\frac{2(2x_2(k)h + 3x_1(k))}{rh^2} + bh \end{cases} \quad (10)$$

Using the fast control function (6) of the second order discrete system, paper[8] insteads the corresponding variables in rules (7-10), and constructs a kind of fast and precise differentiator with small phase delay and amplitude attenuation as below,

$$\begin{cases} [u(k), b] = \text{fast}(x_1(k) - v(k), c_1 x_2(k), r, c_0 h) \\ x_1(k+1) = x_1(k) + h \cdot x_2(k) + \frac{1}{2} rh^2 \cdot u(k) - \frac{1}{3} bh^3 \\ x_2(k+1) = x_2(k) + rh \cdot u(k) - \frac{1}{2} bh^2, \quad k = 0, 1, 2 \dots \end{cases} \quad (11)$$

where, x_1 is the tracking signal of the input signal $v(k)$, x_2 is the differential approximation signal of $v(k)$. r is the speed coefficient, h is the step, that is sample time. c_1 is the damping factor, c_0 is filtering factor. The component $x_1(k)$ of system (6) will be the fastest track of the input signal $v(k)$ [8].

3. Linearized Structure

Generally r is chosen large enough to make sure reach to the switch curve (2) in time $t_A(k)$. As $v(k)$ is changing, $M_k(x_1(k) - v(k), x_2(k))$ can't reach to the switch curve (2) practically. Therefore, we only check whether $M_k(x_1(k) - v(k), x_2(k))$ is in the switch curve, including the tracking ability of the differentiator when $t_A(k) < h$.

When $\Gamma(x_1, x_2, r) \neq 0$ and $t_A(k) < h$, choose the following precise control law as formula (8)),

$$u(k) = -\left(-\frac{1}{2} + \frac{x_2(k) \cdot s(k)}{rh}\right) + \frac{1}{2} \sqrt{1 + \frac{4}{rh} \left(x_2(k) + \frac{2(x_1(k) - v(k))}{h}\right) s(k)} \cdot s(k) \quad (12)$$

Generally r is very large, (12) can be simplified as

$$u(k) \doteq -\left(-\frac{1}{2} + \frac{x_2(k)s(k)}{rh} + \frac{1}{2}\left(1 + \frac{1}{2}\left(\frac{4}{rh}\left(x_2(k) + \frac{2(x_1(k)-v(k))}{h}\right)s(k)\right)\right)\right) \cdot s(k) = -\frac{2x_2(k)}{rh} - \frac{2(x_1(k)-v(k))}{rh^2} \quad (13)$$

According to the fast control function $fast(x_1(k)-v(k), c_1x_2(k), r, c_0h)$ in paper[8], replacing the corresponding variables in formula(13) based on rules(7-10), that is $x_2(k) \rightarrow c_1x_2(k), h \rightarrow c_0h$, the other variables remain unchanged, we get

$$u(k) = -\frac{2x_2(k)}{rh} - \frac{2(x_1(k)-v(k))}{rh^2} = -\frac{2c_1x_2(k)}{rc_0h} - \frac{2(x_1(k)-v(k))}{rc_0^2h^2} \quad (14)$$

Substituting $b=0$ into formula (11), we have

$$\begin{cases} x_1(k+1) = x_1(k) + h \cdot x_2(k) + \frac{rh^2 \cdot u(k)}{2} \\ x_2(k+1) = x_2(k) + rh \cdot u(k) \end{cases} \quad (15)$$

Substituting Eq.(14) into formula (15), we get the following linearization form of nonlinear differentiator(11)

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 - \frac{1}{c_0^2} & h(1 - \frac{c_1}{c_0}) \\ -\frac{2}{c_0^2h} & 1 - \frac{2c_1}{c_0} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \frac{1}{c_0^2} \begin{bmatrix} 1 \\ 2 \\ h \end{bmatrix} v(k) \quad (16)$$

4. Filtering Ability Analysis

As the sample step is generally small, system (15) can be approximated as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \quad (17)$$

where, $u = -\frac{2c_1x_2}{c_0h} - \frac{2(x_1-v)}{c_0^2h^2}$. Suppose $v(k) = A \sin \omega t$, according to $x_2(k) \rightarrow c_1x_2(k), h \rightarrow c_0h$, (17) is

$$\ddot{x}_1 + \frac{2c_1}{c_0h} \dot{x}_1 + \frac{2}{c_0^2h^2} x_1 = \frac{2}{c_0^2h^2} v = \frac{2}{c_0^2h^2} A \sin \omega t \quad (18)$$

Using the method of undetermined coefficients, the analytical solution of $x_1(t)$ is

$$x_1(t) = e^{-\frac{t}{h}} (C_1 \cos \frac{t}{h} + C_2 \sin \frac{t}{h}) - \frac{4c_1 c_0 h \omega A}{c_0^4 h^4 \omega^4 + 4} \cos \omega t - \frac{2(c_0^2 h^2 \omega^2 - 2)A}{c_0^4 h^4 \omega^4 + 4} \sin \omega t \quad (19)$$

when $\frac{t}{h} > 4$, the first term including $e^{-\frac{t}{h}}$ in formula (19) will attenuate to 0 quickly.

According to the last two terms in formula (19), the stable solution of $x_1(t)$ is

$$\bar{x}_1 = -\frac{4c_1 c_0 h \omega A}{c_0^4 h^4 \omega^4 + 4} \cos \omega t - \frac{2(c_0^2 h^2 \omega^2 - 2)A}{c_0^4 h^4 \omega^4 + 4} \sin \omega t = \beta A \sin(\omega t + \varphi) = \beta A \sin \omega(t + \tau) \quad (20)$$

$$\text{where, } \beta = \frac{2\sqrt{c_0^4 h^4 \omega^4 + 4(c_1^2 - 1)c_0^2 h^2 \omega^2 + 4}}{c_0^4 h^4 \omega^4 + 4}. \text{ The value of } \varphi \text{ can be}$$

discussed in two cases:

$$(1) \text{ when } 2 - c_0^2 h^2 \omega^2 > 0, \varphi = -\tan^{-1} \frac{2c_1 c_0 h \omega}{2 - c_0^2 h^2 \omega^2} \quad (21)$$

$$(2) \text{ when } 2 - c_0^2 h^2 \omega^2 < 0, \varphi = \tan^{-1} \frac{2c_1 c_0 h \omega}{c_0^2 h^2 \omega^2 - 2} + \pi \quad (22)$$

This paper mainly considers case (1), that is $2 - c_0^2 h^2 \omega^2 > 0$. Suppose $c_0 h \omega \ll \sqrt{2}$, we have

$$\beta \doteq \sqrt{(c_1^2 - 1)c_0^2 h^2 \omega^2 + 1}, \tau \doteq -c_1 c_0 h \quad (23)$$

Especially, when $c_1 = 1$, there is

$$\beta \doteq 1, \tau \doteq -c_0 h \quad (24)$$

From the above formula, when $c_0 h \omega \ll \sqrt{2}$, $c_1 = 1$, this filter will track input signal with small time delay $\tau = -c_0 h$, and have hardly any amplitude loss.

5. Restraining Disturbance Performance

When disturbance $d(t)$ exists (where, $d(t)$ is bounded, that is $|d(t)| \leq h_d$), (17) can be changed as

$$\begin{cases} \dot{x}_1 = x_2 \\ h^2 \dot{x}_2 = f(x_1 - v(t) - d(t), hx_2) \end{cases} \quad (25)$$

$$\text{where, } f(x_1 - v(t) - d(t), \varepsilon x_2) = -\frac{2c_1 x_2 h}{c_0} - \frac{2(x_1 - v(t) - d(t))}{c_0^2}.$$

The error system of formula (25) and the derivatives of signal $v(t)$ is

$$\begin{cases} \dot{e}_1 = e_2 \\ h^2 \ddot{e}_2 = f(e_1 - d(t), he_2 + h \frac{dv}{dt}) - h^2 \frac{d^2 v}{dt^2} \end{cases} \quad (26)$$

The above (26) can be changed as

$$\begin{cases} \frac{de_1}{dt/h} = he_2 \\ \frac{dhe_2}{dt/h} = f(e_1 - d(t), he_2 + h \frac{dv}{dt}) - h^2 \frac{d^2 v}{dt^2} \end{cases} \quad (27)$$

Let

$$\tau = \frac{t}{h}, z_1(\tau) = e_1(t), z_2(\tau) = he_2(t), z = [z_1 \quad z_2]^T, \bar{d}(\tau) = d(t) \quad (28)$$

we get $z = \text{diag}[1 \quad h]e$. According to formula (28), system(27) can be changed as

$$\begin{cases} \frac{dz_1}{d\tau} = z_2 \\ \frac{dz_2}{d\tau} = f(z_1 - \bar{d}(\tau), z_2 + \frac{dv}{d\tau}) - \frac{d^2 v}{d\tau^2} \end{cases} \quad (29)$$

Considering the following system [9-10]

$$\begin{cases} \frac{d\tilde{z}_1}{dt} = \tilde{z}_2 \\ \frac{d\tilde{z}_2}{dt} = f(\tilde{z}_1, \tilde{z}_2) \end{cases} \quad (30)$$

Suppose the equilibrium point of this system is stable in finite time, and the settling time function T_f is continuous in the original point, where $f(\tilde{z}_1, \tilde{z}_2)$ is continuous and $f(0) = 0$. As seen, formula (29) is the perturbed system of system (30).

Choose the Lyapunov function $(V \circ z)(\tau)$ which is Lipschitz continuous and the Lipschitz constant is M . Then, the upper right derivative of $(V \circ z)(\tau)$ is [9-10,13-14]

$$D^+(V \circ z)(\tau) = \frac{\partial V}{\partial z}(z) \left[z_2 \quad f(z_1, z_2 + \frac{dv}{d\tau}) - \frac{d^2v}{d\tau^2} \right]^T \leq \dot{V} + M(\bar{a}h_d^{\rho_d} + \bar{a}h^{\rho_1}h_1^{\rho_1} + h_2h^2) \quad (31)$$

where, h_1, h_2 is the upper bound of the first and the second derivatives of signal $v(t)$ respectively. Suppose $\rho_d \in (0,1]$ and $\delta = \bar{a}h^{\rho_1}h_1^{\rho_1} + h_2$, from formula(31), we get

$$D^+(V \circ z)(\tau) \leq \dot{V} + M\bar{a}h_d^{\rho_d} + h^{\rho}M\delta \quad (32)$$

For the practical system, there is a constant $T > 0 (T > T_f)$, for $t_{j-1}/h + T(z(t_{j-1}/h)) \leq \tau < t_j/h, j=1, \dots, k+1$, which satisfied

$$\|z(\tau)\| \leq \frac{(V(z(\tau)))^{1-\theta}}{rc(1-\theta)} \leq \frac{(\frac{2M\bar{a}h_d^{\rho_d} + 2h^{\rho}M\delta}{c})^{\frac{1-\theta}{\theta}}}{rc(1-\theta)} \quad (33)$$

where, $\theta \in (0, 0.5)$, $r = \frac{1-\theta}{\theta}$, $c > 1$.

As θ can be chosen small enough, $\frac{1-\theta}{\theta}$ can become large enough.

$$\frac{2M\bar{a}h_d^{\rho_d} + 2h^{\rho}M\delta}{c} < 1 \quad (34)$$

From formula (34), we have

$$h < (\frac{h}{2M\delta})^{\frac{1}{\rho}}, h_d < (\frac{c-2h^{\rho}M\delta}{2M\bar{a}})^{\frac{1}{\rho_d}} \quad (35)$$

At this moment, the noise of system (25) can be restrained effectively.

6. Conclusions

This paper constructs a second order linear differentiator with small phase delay and amplitude attestation using the optimal fast synthesis function of double integral series system. Using the discrete linear structure, we analyze the tracking and filtering ability of this differentiator in time domain, and discuss the

restraining disturbance performance of system to boundary disturbance based on the optimal theory and singular perturbation technique.

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