Sensitivity analysis of input parameters of hybrid manipulator

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Exclusive solution can be found with the presupposition of the kinematical platform of Cartesian serial-parallel manipulator maintain the special shape, practical example carried out on the sensitivity of six input kinematical parameters to output spindle's position and pose, different importance of input kinematical parameter of upper platform and lower platform to the configuration sensitivity of output spindle obtained, the same analysis made on redundant actuated structure of this type of manipulator, also the different effects of input parameters to the configuration of output spindle based on this type.

Keywords: Hybrid manipulator; Parameter; Position; Pose; Sensitivity analysis; Parameter

1. Introduction

Serial-parallel manipulator constructed based on the advantages and disadvantages of the traditional serial robots and parallel robots, it has a series of advantages such as large working space and simple control in kinematic joints[1] of serial robots, also has high stiffness and load carrying capability and precision of movement of parallel robots, its application in production is still remained limited, precision analysis of robot less involved in the analysis and discussion on the methods of kinematics, the main problem of not widely used is the accuracy is not high enough[2]. Analysis on sensitivity of output shaft to input motion parameters[3-4] of a new type of redundant actuating cartesian serial-parallel manipulator proposed in this paper, the main factors for determining the spatial position and orientation of output shaft discussed, example carried out on the sensitivity of six input kinematical parameters to output spindle's position and pose, different importance of input kinematical parameter of upper platform and lower platform to the configuration sensitivity of output spindle obtained, which provide the basis for the elimination of source of main error and find the right way to reduce that, intuitive simulation results provided valid data as determined base.

2. Sensitivity Analysis of Redundant Actuating Cartesian Serial-parallel Manipulator

Structure of the serial-parallel manipulator mentioned in this paper shown in Figure 1, the two fixed-length rod on the upper moving platform installed on the two horizontal sliders of same level by revolutes, the other end of two fixed length rod connected to the upper moving platform by ball joints, upper moving platform is a hollow spindle, the other end of the output shaft connected to the lower moving platform through a flat-plate ball joints, the two fixed-length rod on the lower moving platform installed on the two horizontal sliders of same level by revolutes, the other end of two fixed length rod connected to the sliders on lower moving platform by revolutes, six degree of freedom(DOF) realized by the actuated of two vertical sliders and four horizontal sliders on the frame. In order to improve its property, two redundant actuating branch chains SPS added on the initial 2PPRS-S-2PPRR mechanism(shown in Fig. 2), In this paper, sensitivity analysis of input parameters to the position and pose of output shaft carried out with some constructive fruits deduced.

Inverse solution of kinematics of mechanism defined as seek for the input with given condition of the output configuration of mechanism, if the position and orientation of output shaft have been provided, each input parameters can be obtained then[5], (X_P, Y_P, Z_P) and (α, β, γ) are the position of tip point P of output shaft and orientation of output shaft respectively, α, β, γ are the direction angle of cosine between output shaft and axis X, Y, Z respectively. Two fixed-length rod on the upper moving platform of the same length, the same condition exist in the Two fixed-length rod on the lower moving platform, assuming the structure of upper and lower moving platform always keep the form of isosceles trapezoidal, there is only one solution for the inverse kinematics, name of each part shown in Figure 1, length of four fixed-length rods is l, the length of the upper moving platform expressed as $|b_1 b_2| = t_1$, the length of the lower moving platform expressed as $|b_3 b_4| = t_2$, the length of the center points between upper and lower moving platform expressed as $|O_1 O_2| = t$, the length between tip point P and the center O_2 of the lower moving platform is h, coordination of centers of upper and lower moving platform can be obtained as following,

$$\begin{cases} X_{O_1} = X_P - (t+h)\cos\alpha \\ Y_{O_1} = Y_P - (t+h)\cos\beta \\ Z_{O_1} = Z_P - (t+h)\cos\gamma \end{cases} \tag{1}$$

$$\begin{cases} X_{O_2} = X_P - h\cos\alpha \\ Y_{O_2} = Y_P - h\cos\beta \\ Z_{O_2} = Z_P - h\cos\gamma \end{cases} \tag{2}$$

Inverse solution of position of joints $b_1, b_2, b_3, b_4, d_1, d_2$ on moving platform can be obtained according to the method presented in reference [6], the position of D_1, D_2 have been known, with the the

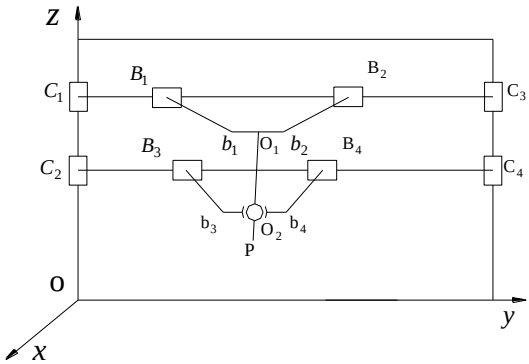


Figure 1: Coordinate and symbls of 2PPRS-S-2PPRR mechanism

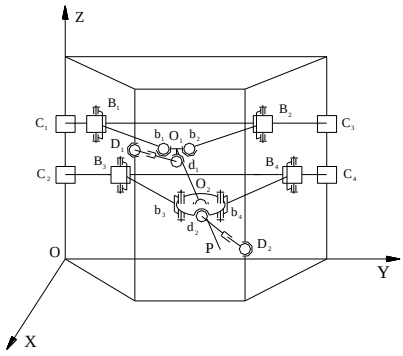


Figure 2: Coordinate and symbls of redundant actuating mechanism

Following determined conditions, there are $|B_1b_1| = |B_2b_2| = |B_3b_3| = |B_4b_4| = l$, $X_{B_1} = X_{B_2} = X_{B_3} = X_{B_4} = 0$, $Z_{B_1} = Z_{B_2} = Z_{O_1}$, $Z_{B_3} = Z_{B_4} = Z_{O_2}$, six DOF motion achieved by control the six input variables $Z_{C_{1,3}}$, $Z_{C_{2,4}}$, Y_{B_1}, Y_{B_2}, Y_{B_3}, Y_{B_4}, expression of partial derivatives of input variables as Eq. 3 to 10 to six output variables X_P,

Y_P, Z_P, α, β, γ obtained respectively as Eq. 11 to 40, general expression of sensitivity of each input variable to the position and orientation of output shaft obtained then.

$$Y_{B_1} = Y_P - (t+h)\cos\beta - \frac{t_1}{2} - \sqrt{l^2 - (X_P - (t+h)\cos\alpha)^2}, \tag{3}$$

$$Y_{B_2} = Y_P - (t+h)\cos\beta + \frac{t_1}{2} + \sqrt{l^2 - (X_P - (t+h)\cos\alpha)^2}, \tag{4}$$

$$Y_{B_3} = Y_P - h\cos\beta - \frac{t_2}{2} - \sqrt{l^2 - (X_P - h\cos\alpha)^2}, \tag{5}$$

$$Y_{B_4} = Y_P - h\cos\beta + \frac{t_1}{2} + \sqrt{l^2 - (X_P - h\cos\alpha)^2}, \tag{6}$$

$$Z_{C_{1,3}} = Z_P - (t+h)\cos\gamma, \tag{7}$$

$$Z_{C_{2,4}} = Z_P - h\cos\gamma, \tag{8}$$

$$L_{D_1d_1} = \sqrt{(X_{D_1} - X_{d_1})^2 + (Y_{D_1} - Y_{d_1})^2 + (Z_{D_1} - Z_{d_1})^2}, \tag{9}$$

$$L_{D_2d_2} = \sqrt{(X_{D_2} - X_{d_2})^2 + (Y_{D_2} - Y_{d_2})^2 + (Z_{D_2} - Z_{d_2})^2}, \tag{10}$$

$$\frac{\partial Y_{B_1}}{\partial X_P} = \frac{\partial Y_{B_2}}{\partial X_P} = -AB, \tag{11}$$

$$\frac{\partial Y_{B_1}}{\partial Y_P} = \frac{\partial Y_{B_2}}{\partial Y_P} = 1, \tag{12}$$

$$\frac{\partial Y_{B_1}}{\partial Z_P} = \frac{\partial Y_{B_2}}{\partial Z_P} = 0, \tag{13}$$

$$\frac{\partial Y_{B_3}}{\partial X_P} = \frac{\partial Y_{B_4}}{\partial X_P} = -DE, \tag{14}$$

$$\frac{\partial Y_{B_3}}{\partial Y_P} = \frac{\partial Y_{B_4}}{\partial Y_P} = 1, \tag{15}$$

$$\frac{\partial Y_{B_3}}{\partial Z_P} = \frac{\partial Y_{B_4}}{\partial Z_P} = 0, \tag{16}$$

$$\frac{\partial Z_{C_{1,3}}}{\partial X_P} = \frac{\partial Z_{C_{1,3}}}{\partial Y_P} = \frac{\partial Z_{C_{1,3}}}{\partial Z_P} = 0 \quad , \tag{17}$$

$$\frac{\partial Z_{C_{2,4}}}{\partial X_P} = \frac{\partial Z_{C_{2,4}}}{\partial Y_P} = \frac{\partial Z_{C_{2,4}}}{\partial Z_P} = 0 \quad , \tag{18}$$

$$\frac{\partial Y_{B_1}}{\partial \alpha} = \frac{\partial Y_{B_2}}{\partial \alpha} = ABC \quad , \tag{19}$$

$$\frac{\partial Y_{B_1}}{\partial \beta} = \frac{\partial Y_{B_2}}{\partial \beta} = (t+h)\sin\beta \quad , \tag{20}$$

$$\frac{\partial Y_{B_1}}{\partial \gamma} = \frac{\partial Y_{B_2}}{\partial \gamma} = 0 \quad , \tag{21}$$

$$\frac{\partial Y_{B_3}}{\partial \alpha} = \frac{\partial Y_{B_4}}{\partial \alpha} = EGH \quad , \tag{22}$$

$$\frac{\partial Y_{B_3}}{\partial \beta} = \frac{\partial Y_{B_4}}{\partial \beta} = h\sin\beta \quad , \tag{23}$$

$$\frac{\partial Y_{B_3}}{\partial \gamma} = \frac{\partial Y_{B_4}}{\partial \gamma} = 0 \quad , \tag{24}$$

$$\frac{\partial Z_{C_{1,3}}}{\partial \alpha} = \frac{\partial Z_{C_{1,3}}}{\partial \beta} = 0 \quad , \tag{25}$$

$$\frac{\partial Z_{C_{2,4}}}{\partial \alpha} = \frac{\partial Z_{C_{2,4}}}{\partial \beta} = 0 \quad , \tag{26}$$

$$\frac{\partial Z_{C_{1,3}}}{\partial \gamma} = (t+h)\sin\gamma \quad , \tag{27}$$

$$\frac{\partial Z_{C_{2,4}}}{\partial \gamma} = h\sin\gamma \quad , \tag{28}$$

$$\frac{\partial L_{D_1 d_1}}{\partial X_P} = -M_1^{-\frac{1}{2}} R_1 \quad , \tag{29}$$

$$\frac{\partial L_{D_1 d_1}}{\partial Y_P} = -M_1^{-\frac{1}{2}} S_1 \quad , \tag{30}$$

$$\frac{\partial L_{D_1d_1}}{\partial Z_P} = -M_1^{-\frac{1}{2}} U_1 , \quad (31)$$

$$\frac{\partial L_{D_1d_1}}{\partial \alpha} = -M_1^{-\frac{1}{2}} \cdot R_1 \cdot (t+h) \cdot \sin\alpha , \quad (32)$$

$$\frac{\partial L_{D_1d_1}}{\partial \beta} = -M_1^{-\frac{1}{2}} \cdot S_1 \cdot (t+h) \cdot \sin\beta , \quad (33)$$

$$\frac{\partial L_{D_1d_1}}{\partial \gamma} = -M_1^{-\frac{1}{2}} \cdot U_1 \cdot (t+h) \cdot \sin\gamma , \quad (34)$$

$$\frac{\partial L_{D_2d_2}}{\partial X_P} = -M_2^{-\frac{1}{2}} R_2 , \quad (35)$$

$$\frac{\partial L_{D_2d_2}}{\partial Y_P} = -M_2^{-\frac{1}{2}} S_2 , \quad (36)$$

$$\frac{\partial L_{D_2d_2}}{\partial Z_P} = -M_2^{-\frac{1}{2}} U_2 , \quad (37)$$

$$\frac{\partial L_{D_2d_2}}{\partial \alpha} = -M_2^{-\frac{1}{2}} \cdot R_2 \cdot h \cdot \sin\alpha , \quad (38)$$

$$\frac{\partial L_{D_2d_2}}{\partial \beta} = -M_2^{-\frac{1}{2}} \cdot S_2 \cdot h \cdot \sin\beta , \quad (39)$$

$$\frac{\partial L_{D_2d_2}}{\partial \gamma} = -M_2^{-\frac{1}{2}} \cdot U_2 \cdot h \cdot \sin\gamma , \quad . \quad (40)$$

Where,

$A = X_P - (t+h)\cos\alpha$, $B = (l^2 - (X_P - (t+h)\cos\alpha)^2)^{-\frac{1}{2}}$,

$C = -(t+h)\sin\alpha$, $D = X_P - h\cos\alpha$, $E = (l^2 - (X_P - h\cos\alpha)^2)^{-\frac{1}{2}}$,

$G = X_P - h\cos\alpha$, $H = -h\sin\alpha$,

$$R_1 = (0.5 + l \cdot \cos\varphi_1 \cdot \cos\varphi_2 - X_P + (t+h)\cos\alpha + \frac{t_1}{2} tg\xi) ,$$

$$R_2 = (0.5 + l \cdot \cos\varphi_3 \cdot \cos\varphi_4 - X_P + h \cdot \cos\alpha + \frac{t_2}{2}) ,$$

$S_1 = (0.75 - l \cdot \cos\varphi_1 \cdot \sin\varphi_2 - Y_P + (t+h)\cos\beta)$,

$S_2 = (0.75 + l \cdot \cos\varphi_3 \cdot \sin\varphi_4 - Y_P + h \cdot \cos\beta)$,

$U_1 = (0.2 - Z_P + h \cdot \cos\gamma)$, $M_1 = R_1^{\ 2} + S_1^{\ 2} + U_1^{\ 2}$;

$M_2 = R_2^{\ 2} + S_2^{\ 2} + U_2^{\ 2}$, $U_1 = (0.8 - Z_P + (t+h)\cos\gamma - \frac{t_1}{2} \cdot \sin\xi)$

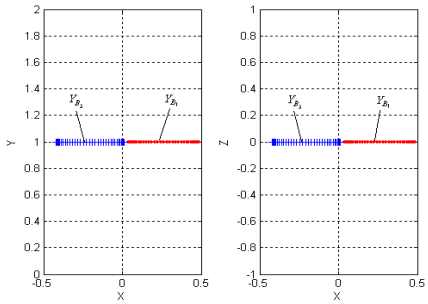
$L_{D_1 d_1} = M_1^{\frac{1}{2}}$, $L_{D_2 d_2} = M_2^{\frac{1}{2}}$,Eq. 11 to 18 , Eq. 29 to 31 , Eq. 35 to 37 are the sensitivity of input parameters to the position of output shaft, Eq. 19 to 28, Eq. 32 to 34, Eq. 38 to 40 are the sensitivity of input parameters to the orientation of output shaft.

Degree of influence of sensitivity of input parameters of different volume of Upper and lower moving platform to the output shaft is different, there are the same as Eq. 12 and 15, Eq. 13 and 16, Eq. 17 and 18, Eq. 21 and 24, Eq. 25 and 25 indicate, results of which one has greater impact on the sensitivity to position achieved after compare Eq. 11 and 14, it will play a key role in change the position of output shaft. Results of which one has greater impact on the sensitivity to orientation achieved after compare Eq. 19 and 22, Eq. 20 and 23, that will play a key role in change the orientation of output shaft. Degree of influence of sensitivity of input parameters of vertical sliders to orientation of output shaft obtained after compare Eq. 27 and 28. results of which one has greater impact on the sensitivity to position achieved after compare the sum of absolute value of Eq. 29 to 31 and that of Eq.35 to 37. results of which one has greater impact on the sensitivity to orientation achieved after compare the sum of absolute value of Eq. 32 to 34 and that of Eq.38 to 40. Through the above analysis and comparison, we can determine which set of parameters play a key role in regulating the configuration of output shaft.

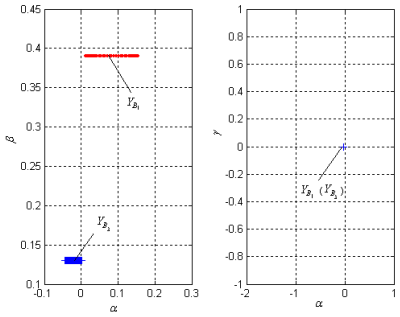
3. Example Analysis

Relative condition given as following, length of the fixed length link $l = 0.5m$, the length of upper moving platform b_1b_2 is $t_1 = 0.2m$, the length of lower moving platform b_3b_4 is $t_2 = 0.24m$, the distance O_1O_2 between the center of upper and lower moving platform is $t = 0.3m$, the distance between the tip point P and the center of lower moving platform is $h = 0.15m$, the initial position of the tip point P in fixed coordinate system is on the cycle with the center point $(0.2,0.75,0.2)$ and radius 0.1m, direction angle of cosine between the output shaft O_1O_2 and three axis of coordinate system, which defined as angle vector nP, α, β, γ are its three components respectively, take a situation of $\alpha = 60^\circ, \beta = 60^\circ, \gamma = 45^\circ$ as example, sensitivity of part input parameters to output

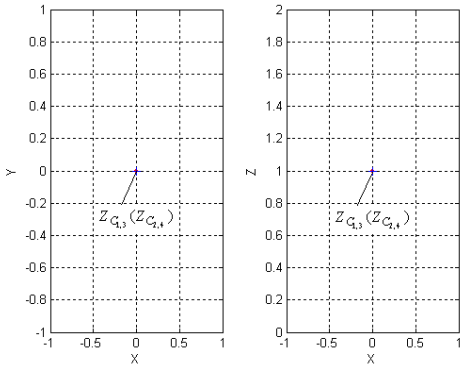
shaft can be obtained by making use of the simulation analysis, details shown in Fig.3.



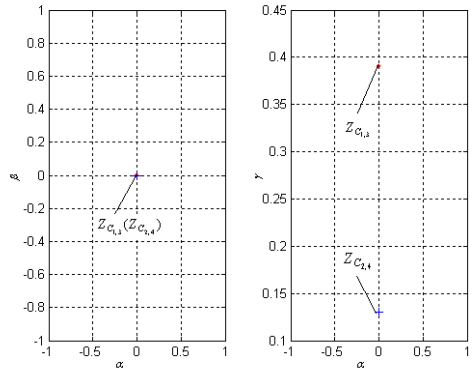
Sensitivity of horizontal sliders to position of output shaft



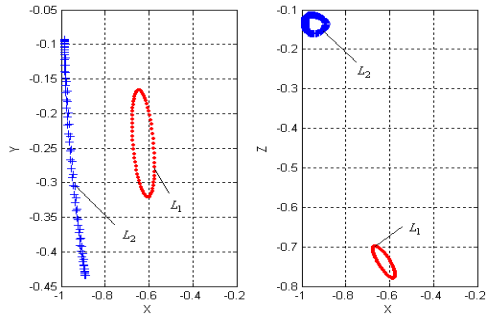
(b) Sensitivity of horizontal sliders to orientation of output shaft



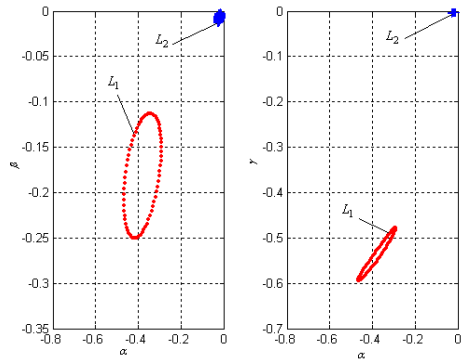
(c) Sensitivity of vertical sliders to position of output shaft



(d) Sensitivity of vertical sliders to orientation of output shaft



(e) Sensitivity of redundant actuating branch to position of output shaft



(f) Sensitivity of redundant actuating branch to orientation of output shaft

Figure 3: Comparison of Sensitivity of input parameters to configuration of output shaft

Based on the given condition of position and orientation of output shaft in space, also the structure of upper and lower moving platform always keep the form of isosceles trapezoidal, still with the Eq. 11 to 40 and data shown in Fig.3

provided. Results on sensitivity analysis of position and orientation can be drawn clearly from the individual comparison of absolute value or the comparison of sum numerical values of the same input parameters to the output shaft.

The value of sensitivity of input parameters of horizontal sliders on the upper and lower moving platforms to the position of output shaft almost the same, as Fig.3 (a) show.

The value of sensitivity of input parameters of horizontal sliders on the upper moving platform to orientation output shaft is far greater than that of horizontal sliders on the lower moving platform to output shaft, as Fig.3 (b) show.

Only sensitivity of position of input parameters of input parameters of four sliders on two vertical guide ways to the position of output shaft along the axis Z exists, the value of sensitivity is 1, no influence on the axis X and Y, the value of sensitivity is zero, as Fig.3(c) show.

The value of sensitivity of input parameters of vertical sliders on the upper moving platform to orientation of output shaft is far greater than that of vertical sliders on the lower moving platform to output shaft, as Fig.3 (d) show.

The value of sensitivity of input parameters of redundant actuating branches to the position of output shaft almost the same, as Fig.3 (e) show.

The value of sensitivity of input parameters of the first redundant actuating branch D_1d_1 (named L1 in Fig.3 (f)) to orientation output shaft is far greater than that of the second branch D_2d_2 (named L2 in Fig.3 (f)) to output shaft, as Fig.3(f) show.

Conclusion

From the above, we can get the conclusions as following.

(1) Both the upper and lower moving platforms play the same key role in position regulation of the output shaft.

(2) The upper moving platform plays a key role in orientation regulation of the output shaft.

(3) Sliders on two vertical guide ways play a key role in the position of output shaft along the axis Z exists, no influence on the axis X and Y.

(4) The vertical sliders on upper moving platform play a key role in orientation regulation of the output shaft.

(5) Both the redundant actuating branches play a key role in position regulation of the output shaft.

(6) The first redundant actuating branch plays a key role in orientation regulation of the output shaft.

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