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Solar Shadow Positioning Based on Quadratic Function Fitting

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Abstract. Solar shadow positioning technology is a method through analyzing the changes of the solar shadow in the video to determine where and when the video was captured shoot. We can obtain the expression of the solar shadow over time. Set the unknown as a parameter, and perform a quadratic fitting to get the value of the unknown coefficient.

Introduction

The change of the solar shadow is caused by the rotation and revolution of the earth. The length of the shadow changes with the change of the solar position within one day, and the change of the shadow length is closely related to the position of the object. By observing the change of the shadow length of an object with time, we can determine where the object is located. And it has certain scientific significance and application value.

Using the relationship between the actual length of the object and the length of the shadow, you can determine the current position and date, which is the principle of the solar shadow positioning technology.

This article through mathematical modeling, solve the following problems:

(1) We establish a mathematical model about the change of the length of the shadow, and analyze the variation of shadow length on each parameter.

(2) According to the vertex coordinate data of the pole's solar shadow in the horizontal ground, establish a mathematical model to determine the location of the pole.

The Change of Sun Shadow

Problem Analysis.

Because the sun shadows of objects on earth are affected by the earth's rotation and revolution, the relative movement of the earth and the sun is transformed into the movement of the sun around the earth. The shadow length is related to the solar elevation angle, and it is related to the solar declination, the geographic latitude and the hour angle. The relationship between the shadow length and the various factors as follows:

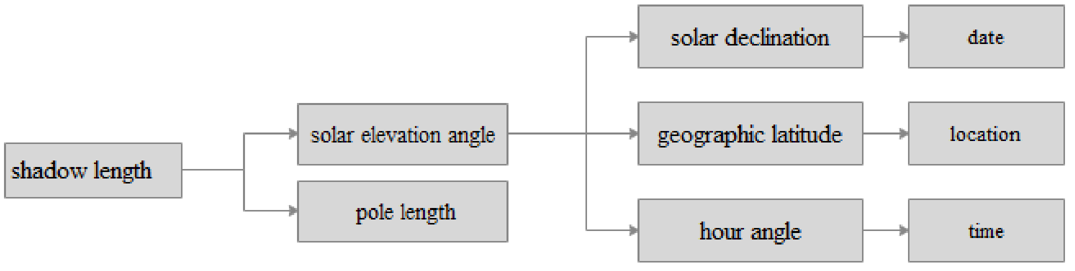

Fig. 1 The relationship between shadow length and the various factors

Modeling Process.

The study shows that the change of the shadow length is related to the solar declination, the geographic latitude and the hour angle, and when these three quantities are obtained, the solar elevation angle can be obtained, thereby getting the shadow length.

1) Calculation of Solar Elevation Angle.
According to the trigonometric function relationship and the solar elevation angle[1] formula:

$$\coth = \frac{q}{L} \quad (1)$$

$$\sinh = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos p \quad (2)$$

2) Calculation of solar declination.
Since the value of the solar declination at any time annually is strictly known, the solar declination δ is calculated as[2]:

$$\delta = 0.3723 + 23.2567\sin\theta + 0.1149\sin 2\theta - 0.1712\sin 3\theta$$
$$-0.758\cos\theta + 0.3656\cos 2\theta + 0.0201\cos 3\theta$$

$$\theta = \frac{2\pi t}{365.2422}$$

$$t = N - N_0$$

$$N_0 = 79.6764 + 0.2422 \times (year - 1985) - INT(\frac{year - 1985}{4})$$

("INT" is the integer part)

3) Calculation of Hour Angle.

$$p = \frac{(T - 12)\pi}{12}$$

p is the hour angle[3] and T is the local time.

Data Analysis.
Take October 22, 2015 Beijing time between 9: 00-15: 00 Tiananmen Square as an example, calculate the shadow length of 3-meter-high straight pole.

Latitude φ is 39 degrees 54 minutes 26 seconds that is $39.9044°$

$$N = 31 + 28 + 31 + 30 + 31 + 31 + 30 + 22 = 295$$

$$N_0 = 79.6764 + 0.2422 \times (2015 - 1985) - INT\left(\frac{2015 - 1985}{4}\right) = 80.138$$

$$t = N - N_0 = 295 - 80.138 = 214.862$$

$$\theta = \frac{2\pi t}{365.2422} = 3.696$$

$$\delta = 0.3723 + 23.2567\sin\theta + 0.1149\sin 2\theta - 0.1712\sin 3\theta$$
$$-0.758\cos\theta + 0.3656\cos 2\theta + 0.0201\cos 3\theta = -10.792°$$

So we can get the shadow length is:

$$q = L\coth = L\cot(\arcsin(\sin 39.9044\sin(-10.792) +$$
$$\cos(39.9044)\cos(-10.792)\cos(\frac{(T - 12)\pi}{12})))$$

Using matlab to draw a picture about the change curve of shadow length, as shown in Figure 2.

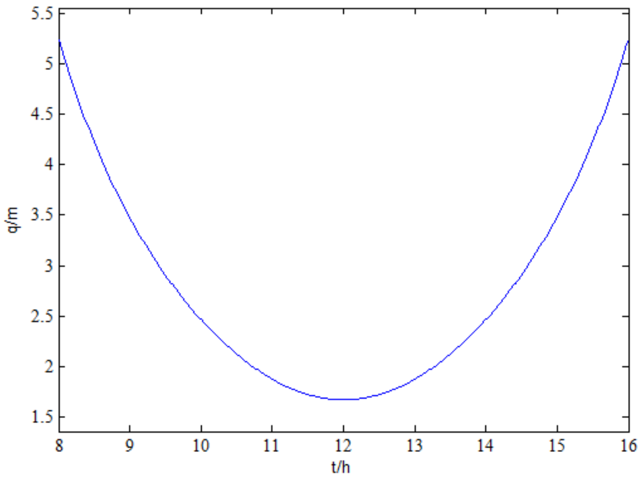

Fig. 2 The change curve of shadow length with time

Location based on the Change of Sun Shadow

Problem Analysis.

When a pole's length is unknown and its shadow is known, we can determine the location of the pole, that is to say get the longitude and latitude. First according to the shadow length get the shadow length change curve over time. The curve's lowest point is 12:00 in local time, when the shadow length is the shortest, and we can see the Beijing time from the curve. So the longitude of the pole position is known. Then by the formula in 2.2, set the latitude and the shadow length to parameters, and get the formula of shadow length changes with time. Through the quadratic function fitting, get the parameters and according to the latitude and longitude to determine the specific location.

Data Analysis.

1) Calculation of Longitude.

It is known that the length of the shadow at each time, we can get scatter distribution map, as is shown in Fig 3

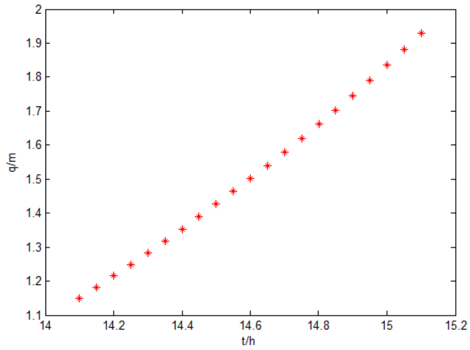

Fig. 3 scatter distribution map of shadow length in Beijing time

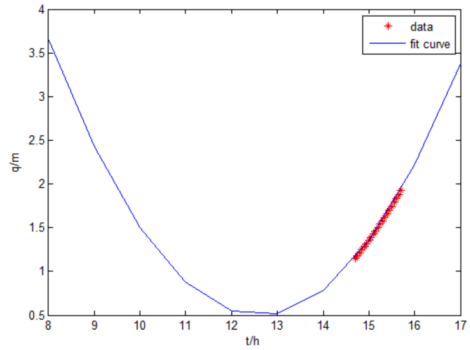

Fig. 4 Parabolic fitting curve of shadow length changes with Beijing time

According to 2.2, the change trend of the shadow length should be a part of Figure 4. Fitting with parabola [4]:

$$y=ax^2+bx+c$$

Use MATLAB to fit out of the curve about the shadow length of with Beijing time, as is shown in Figure 4. And the parameters are solved respectively:

$$a=0.1489$$
$$b=-3.7519$$
$$c=24.1275$$

The parabolic equation is

$$Y = 0.1489x^2-3.7519x + 24.1275$$

The lowest point of the parabola corresponding to the abscissa is the shortest shadow corresponding to the moment of Beijing.

$$T_0 = -b / 2a = 12.60$$

That is 12:36 in Beijing time, this time corresponding to the local 12:00. So the longitude of the position where the pole is located:

$$\varepsilon=120^\circ-0.6\times15^\circ=E111^\circ$$

2) Calculation of Latitude.

By the 2.2 formula, we can get the solar declination δ=10.6311°. According to the formula (1)(2), the shadow length is:

$$\mathrm{x} = \mathrm{L} * \cot h$$

That is: $\mathrm{x} = L * \cot(\sin^{-1}(\sin\varphi \sin 10.6311) + \cos\varphi \cos 10.6311)\cos(\frac{(T-12)\pi}{12})$

Use MATLAB to fit out of the curve about the shadow length of with Beijing time, as is shown in Figure 5[5].

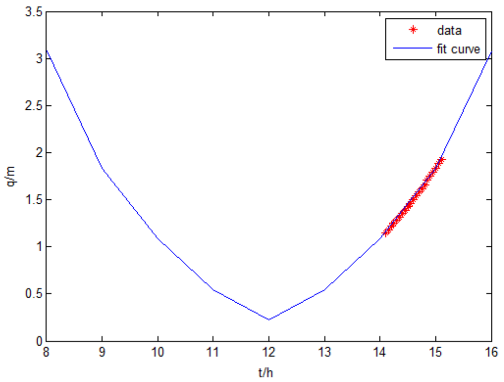

Fig. 5 Parabolic fitting curve of shadow length changes with local time

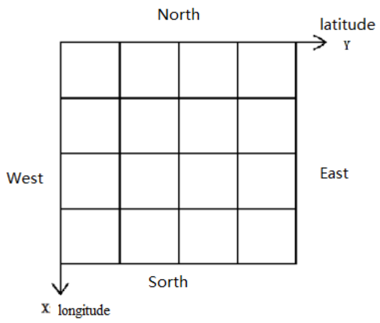

Fig. 6 Position coordinate system

Get two solutions are:

$$\begin{cases} \Phi 1=0.3048\times 180/\pi=17.46° \\ L1=1.8949 \end{cases} \quad \text{or} \quad \begin{cases} \Phi 2=0.0117\times 180/\pi=0.67° \\ L2=1.7859 \end{cases}$$

The establishment of the coordinate system is shown in Figure 6.

So we can get the location of the pole is (E111, N17.46) or (E111, N0.67), from the map we can see the possible location of the pole are Xisha Islands for the Hainan, Xisha Islands in China or West Kalimantan in Malaysia.

Conclusions

In this paper, the method of quadratic function fitting is used to research the positioning technology of shadow. A method is proposed to determine the location and date of video shooting when knowing the change of solar shadow length. Verification by example, this method is a simple and effective solar shadow positioning method.

References

[1] Information on http://baike.baidu.com/item/%E5%A4%AA%E9%98%B3%E9%AB%98%E5%BA%A6%E8%A7%92?fr=aladdin

[2] Information on http://baike.baidu.com/item/%E5%A4%AA%E9%98%B3%E8%B5%A4%E7%BA%AC?fr=aladdin

[3] Information on http://baike.baidu.com/view/823827.htm

[4] Shoukui Si, Xijing Sun, et al. Mathematical Modeling. Beijing :Nation Defense Industry Press, 2014, p. 90-91.

[5] Shoukui Si, Xijing Sun, et al. Mathematical Modeling. Beijing :Nation Defense Industry Press, 2014, p. 94.