Discussion on Two Methods of Cleaning Space Junk

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Abstract. As we all know, the amount of small debris in orbit around earth has been a growing concern. The existence of debris is a major threat to the spacecraft. Meanwhile, a number of methods to remove the debris have been proposed, so our task is to determine an optimal scheme for a private firm to make profits from it. In our model, the optimal scheme means getting a higher profit, at the same cost.

Introduction

By consulting a variety of resources, we find that most of the existing cost algorithms are very complex which includes economics, aerospace, and other professional knowledge. Obviously, it is beyond our ability. So we have the simplified cost assessing model as follow.

Here we discuss two main methods to clean up space junk and compare their ability in making profits.

The simple cost model

We assume that the total cost consists of the launching cost, hardware cost and the operating cost.

Definitions:

C: the total cost

M: the payload mass of the rocket

t: the service life of the satellite

L: the cost of launching satellite

H: the hardware cost of the satellite

O: the cost of operating the satellite

k: the cost of launching one kilogram load (million dollars)

And the cost of operating a satellite one year is equal to 0.3% of its hardware cost.

$O(t) = H \times 3\% \times t$

Thus, we get the equation:

$C = L(M) + H + O(t) \quad C(M,t) = k \times M + (1 + 0.03t) \times H$

Two alternatives

High energy laser cleaning space debris.

Laser cleaning space debris technique is to make the debris surface melting, vaporization and ionization generating plasma by high energy laser irradiation, Impulse coupling to obtain the velocity increment of debris. The debris orbit changes because of the increment of debris. Debris will fall into the earth's atmosphere and eventually burn down when the height of debris orbit is lower than the edge of the dense atmosphere. The major work is to reduce velocity of debris by high energy laser irradiation eventually making the debris into ashes.

Generally considered, if the height of orbit is reduced to one hundred and thirty kilometers, space debris will gradually get burned in the atmosphere. In order to save laser energy and improve the removal efficiency, we decide to raise the height of perigee.

Definitions:
- r: the initial orbit height of space debris
- Δr: the height variation
- C_D: resistance coefficient

(At the height from 200 to 500 kilometers it is equal to 2.2~2.5.)
- ρ: atmospheric density
- s: effective resistance area
- a: the length of the semi major axis of the orbit

Taking the circular orbit as an example, the height variation in a circle obeys the following equation.

$$\triangle r = -2\pi r^2 \frac{C_D S}{m} \rho$$

The number of cycles space debris experiences, before burning up.

$$n = \frac{1000(r-130)}{\triangle r}$$

We define T as the amount of time of the debris running cycle.

$$T = 2\pi \sqrt{\frac{a^3}{\mu}}$$

In the formula, μ represents gravitational coefficient of the earth.

$$\mu = 3.986 \times 10^5 \ km^3 / s^2$$

Obviously, the total time is equal to $n \times T$

We take a five-kilogram spherical debris flying on a circular orbit at 200 kilogram altitude as an example. After the calculation, we get the total time it experienced in naturally falling down until burning up is about seven days. Therefore, we choose two hundred meters as the criterion for successful removal.

Large satellite to sweep up space debris.

As we have mentioned above, removal of the large space debris cannot be neglected. However, it is almost impossible to use the principle of laser irradiation to clean up the debris which is larger than one meter in diameter. So, we introduce the second method here.

As we all know, the satellite have the ability of actively changing its own flying orbit. Therefore, we prepare to launch a satellite with a huge space sail which will eventually capture the target debris after a few times track changes. Once the satellite and the debris are fixed together, the sails carried by the satellite will be spread in the vertical direction of flight. Although the air is very thin at the edge of the atmosphere, the resistance of the space sail still can be used, due to the rapid movement of debris. Next we will estimate the time required for the debris to fall and burn up by this method.

Definitions:
- F: the resistance of a space sail
- c: the resistance coefficient of the air
- ρ: the air density of the upper atmosphere
- s: the surface area of the space sail
- v: the velocity of the combined body
- v_0: the initial velocity of the debris
- H: the initial height of the debris orbit

According to the related document, we get the air resistance formula:

$$F = \frac{1}{2} \times c \times \rho \times s \times v^2$$

And based on the equality of universal gravitation and centripetal force, we figure out the moving speed of the object at the height of H kilometers from the ground.

$$v_0 = \sqrt{\frac{G \times M_{earth}}{(R+H)^2}}$$

So we can estimate the time needed for one time down track process.

$$d_v = \frac{F}{m}d_t = \frac{c\rho s v^2}{2m}d_t$$

$$\Rightarrow \frac{d_v}{v^2} = \frac{c\rho s}{2m}d_t$$

$$\Rightarrow \int_{v_0}^{v_1} \frac{1}{v^2}d_v = \int_0^t \frac{c\rho s}{2m}d_t$$

$$\Rightarrow T = (\frac{1}{v_0} - \frac{1}{v_1}) \times \frac{2m}{c\rho s}$$

To verify the feasibility of the scheme, we take one kilograms debris on the track at five hundred kilometers altitude as an example.

So we get the value of v_0 and v_1.

$$v_0 = 7658m/s$$

$$v_1 = 7800m/s$$

After calculation, we get the following results:

$$T = 0.5025 \quad (year)$$

So we come to the conclusion that this scheme is also a practical and effective option for commercial operation.

Comparison of two methods

Obviously, for the first scheme the longer lifetime of the satellite, the more debris will be cleaned up, and the enterprise will obtain more profits which is different from the second scheme. And longer the lifetime the satellite has the more cost of operating will be spent which is both of them need to think about.

To estimate the profits, we make several definitions as follow:
- R_1: the profit of the scheme one
- R_2: the profit of the scheme two
- t: the operating time of the satellite
- k: the cost of launching one kilogram load (million dollars)
- M_1: the mass of the satellite in the first scheme
- M_2: the mass of the satellite in the second scheme
- H_1: the cost of producing the hardware of the satellite in method one
- H_2: the cost of producing the hardware of the satellite in method two
- b: the reward for removing one piece of centimeter level debris

According to the formula, in Model5.1:

$$C(M,t) = k \times M + (1+0.03t) \times H$$

We get the function of profit on time as follow:

$$R_1(t) = 28223 \times t \times b - k \times M_1 - (1+0.03t) \times H_1$$
$$R_2(t) = 80823 \times b - k \times M_2 - (1+0.5025 \times 0.03) \times H_2$$

We make R_1 equal to R_2 and work out the key time node based on the equation as follow:

$$t = \frac{80823 \times b - k \times M_2 - 1.015075 \times H_2 + k \times M_1 + H_1}{28223 \times b - 0.03 \times H_1}$$

Obviously, when the expectancy of the lifetime of satellite is greater than the value of variable t, the profit of first method is more than the method two. If its expectancy of lifetime is less than the variable t, we suggest the second method which will make more profits.

Conclusions

A more efficient and cost-effective technology or program is bound to make the company more competitive, so as to win in the tender.

In summary, when the expectancy of the lifetime of satellite is greater than a certain value, the profit of first method is more than the method two. If its expectancy of lifetime is less than this value, we suggest the second method which is believed to make more profits.

In general, if a satellite can work for two years even longer in the space, using the space-based high energy laser to protect the specific spacecraft from the debris impact is feasible.

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