The Temperature of the Bathtub Water in Space and Time

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Abstract. In order to find the temperature of the bathtub water in space and time, we used *the first law of thermo dynamics* *Fouriers Law* and *Calorie formula* to derive the heat conduction equation and to study the temperature of the bathtub water in space and time. Firstly, We do not change time but change space to study the temperature distribution equation by simplifying the 3D model into a two-dimensional model Then we use the PDE toolbox of MATLAB for solving the equations . Finally, We only change the time to learn about the trend of temperature changes through the simulation tool of MATLAB

Introduction

China's total freshwater resources ranked the forefront of the world, but the per capita water resources is very small, so water conservation for us is very necessary, but in our daily life, we have a huge water consumption, mostly from the water temperature Conditioning, such as water is too hot or too cold are caused by excessive water waste, so the study of bath water temperature changes with time and space is necessary for the future of intelligent equipment, automatic water supply to provide a great help.

Organization of the Text

In order to study the temperature of the bathtub water in space and time, we use *Fourier's Law* and *Calorie formula*, *the first law of thermo dynamics* for creating a model.

Table 1: Notations and Descriptions

Notatio	Descriptions
T_{in}	The temperature in the bathtub
T_{out}	The temperature in the bathroom
T_0	The temperature of hot water flowing out of the tap
c	Specific heat capacity
ρ	The dendity of the water

- Fourier's Law

$$dQ = -k(x, y, z) \frac{\partial x}{\partial y} dS dt \quad (1)$$

$k(x, y, z)$ --- the thermal transmissivity of material

- Calorie formula

$$Q = cmu \quad (2)$$

c --- the specific heat capacity

m --- quality.

- the first law of thermodynamics

$$Q = Q_1 + Q_2 \quad (3)$$

Q--- the heat absorbed by temperature change

Q1--- the heat flowing through the boundary

Q2--- the heat provided of heat reservoir

Finally, we derive the heat conduction equation as follows

$$\int_{t_1}^{t_2} \left[\iiint_{\Omega} c \rho \frac{\partial u}{\partial t} dV \right] dt = \int_{t_1}^{t_2} \left[\iiint_{\Omega} \left(\frac{\partial}{\partial x} \left(k \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial u}{\partial z} \right) \right) dV \right] dt \quad (4)$$

To simplified model, the temperature distribution equation is projected to the three plane[1] X-Y plane, Y-Z plane and X-Z plane.

- The temperature distribution equation on X-Y plane

$$\frac{\partial u}{\partial t} - k^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = f(x, y, z) \quad (5)$$

Boundary Conditions

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + hT \Big|_w = hT_{out} \\ \frac{\partial u}{\partial x} + hT \Big|_{x=\pm \frac{b}{2}, -\frac{a}{2} \leq y \leq \frac{a}{2}} = hT_{out} \\ \frac{\partial u}{\partial y} + hT \Big|_{y=\pm \frac{a}{2}, -\frac{b}{2} \leq x \leq \frac{b}{2}} = hT_{out} \end{cases} \quad (6)$$

- The temperature distribution equation on Y-Z plane

$$\frac{\partial u}{\partial t} - k^2 \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = f(x, y, z) \quad (7)$$

Boundary Conditions

$$\begin{cases} \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} + hT \Big|_w = hT_{out} \\ \frac{\partial u}{\partial y} + hT \Big|_{y=\pm \frac{c}{2}, \frac{b}{2} \leq z \leq \frac{b}{2}} = hT_{out} \\ \frac{\partial u}{\partial z} + hT \Big|_{z=\pm \frac{b}{2}, -\frac{c}{2} \leq y \leq \frac{c}{2}} = hT_{out} \end{cases} \quad (8)$$

- The temperature distribution equation on X-Z plane

$$\frac{\partial u}{\partial t} - k^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) = f(x, y, z) \quad (9)$$

Boundary Conditions

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} + hT|_w = hT_{out} \\ \frac{\partial u}{\partial x} + hT|_{x=\pm\frac{c}{2}, -\frac{a}{2} \leq z \leq \frac{a}{2}} = hT_{out} \\ \frac{\partial u}{\partial z} + hT|_{z=\pm\frac{a}{2}, -\frac{c}{2} \leq x \leq \frac{c}{2}} = hT_{out} \end{cases} \quad (10)$$

If we want to keep the temperature (T_{in}) even throughout the bathtub, we should know the relationship between the temperature of the bathtub in space and T_{in} , namely the relationship between $\sigma^2(H)$ and T_{in}

The temperature distribution of different T_{in}

Figure 1: the temperature distribution of different T_{in}

Table 2: $\sigma^2(T) - T_{in}$

T_{in}	37	38	40	42	43	44
$\sigma^2(T)$	0.850848987	0.931503596	1.060011382	2.343273425	2.876997834	3.001693311

The relationship of $\sigma^2(T)$ and T_{in} shows as Table 2, we can easily find that the $\sigma^2(T) - T_{in}$ function is an increasing function. The bigger T_{in} is the more difficult the temperature distribution evenness.

To find the temperature of the bathtub in time, we set T_{in} as a constant, based on the above analysis, We choose a middle temperature : $T_{in} = 40^\circ\text{C}$, in order to maximize even distribution of heat for the bathtub.

Table 3: $\sigma^2(T)$, $\bar{T}$ and t relationship

t	200	400	700	1000
$\bar{T}$	39.99280973	39.96742288	39.93929502	39.94171492
$\sigma^2(T)$	0.005386366	0.007572026	0.021716671	0.141813188

In Table 3, we can find that under certain time scope and temperature terms both the variance of the temperature and the variance variation are very small, the temperature distribution of the bathtub basically doesn't vary with time.

Conclusions

From the above analysis, we can know that the bigger T_{in} is the more difficult the temperature distribution evenness. Moreover, under certain time scope and temperature terms both the variance of the temperature and the variance variation are very small. Therefore, within the permissible error range, the temperature distribution does not change over time.

References

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