Optimizing the Throughput at an Airport Security Check Point

Yirong Yang

School of North China Electric Power University Baoding, Baoding 071000, China
1241114834@qq.com

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Abstract. To optimize the throughput at an airport security check point, the given data of MCM problem B is analyzed to obtain average arrival ratio of pre-check customers and regular customers respectively, and the service ratio of the ID check and the screening. The fundamental M/M/1 model is introduced at first, and then some stuff and screening equipment have been increased into it, making the s M/M/1 model change into one M/M/S model. After that, a new way is provided to control the queue.

Introduction

Although airports take more security measures are to prevent passengers from hijacking or destroying aircraft and to keep all passengers safe during their travel, it also brought amounts of passenger’s privacy issues and increase the extra boarding time of passengers. To solve this problem, the simulated queue system is used to reflect the character of the airport security check process. Basing on the (M/M/1/∞) model and the practical calculation example, we can propose some improvement advice and analysis its affects to all of the security check progress.

Organization of the text

Notation and Meanings.
The following table provides meanings of important notations used in the solutions.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Meanings</th>
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<tbody>
<tr>
<td>λ</td>
<td>Average arrival rates.</td>
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<td>μ</td>
<td>Average service rates.</td>
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<tr>
<td>ρ</td>
<td>Service strength.</td>
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<tr>
<td>L</td>
<td>The queue length.</td>
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<tr>
<td>W</td>
<td>The average residence (waiting) time.</td>
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<tr>
<td>B, I</td>
<td>Busy time, idle time.</td>
</tr>
<tr>
<td>P</td>
<td>The probability of n individual in the system.</td>
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Model One: A Model with Single Server by Queue Theory (M/M/1/∞).
A model with single server is the most simple queuing system that sequential arrival time of customs obeys negative exponential distribution with parameter λ, the number of the server is one, the server time V obeys the negative exponential distribution with parameter μ, the system space...
is infinite and queuing unlimited is allowed.

$\rho$ is defined as the probability of at least one customer in the queuing system, which is also the probability of the busy time of the service station. As a consequence, $\rho$ is also known as the service strength, which reflects the busy degree of the system and is described as $\frac{\lambda}{\mu}$, which is less than one if the queue’s length is limited. Meanwhile, the system will obtain statistical equilibrium, which means the possibility of n people in the system $p_n = (1-\rho)^n\rho^n, n=1,2,\cdots$ when the average of customs arrival rates is less than the average of system service rates.

On this basis, according to the queue length distribution of the stationary state, the average of the queue length $L_s$ is calculated as follows[2]:

$$L_s = \sum_{n=0}^{\infty} np_n = \sum_{n=0}^{\infty} n(1-p)p^n = \frac{\lambda}{\mu-\lambda}$$  \hspace{1cm} (1)

As for the residence time in the system of customers, it obeys complex exponential distribution with $\mu - \lambda$, or $P\{T > t\} = e^{-(\mu-\lambda)t}$, $t \geq 0$. Therefore, the average residence time is shown as below:

$$W_s = \frac{1}{\mu-\lambda}$$ \hspace{1cm} (2)

To explore the service process in details, the distribution of the busy time and the idle time is complex to find out, but we can get its ratio as:

$$\frac{B}{I} = \frac{\rho}{1-\rho}$$ \hspace{1cm} (3)

When the arrival of customers can be described as the Poisson Stream, according to the memoryless of the negative exponential and the assumption that arrival and service is independent, it is easy to prove that the time interval from one of the system idle time to the next customer arrival time (i.e. the idle period) is still subject to the negative exponential with $\lambda$ and is independent to the time interval. As a result, the average idle period is $\frac{1}{\lambda}$. Then, the average busy period is:

$$B = \frac{\rho}{1-\rho} \cdot \frac{1}{\lambda} = \frac{1}{\mu-\lambda}$$ \hspace{1cm} (4)

Compared with the equation marked (2), we find that the average busy time is same with the average residence time. As far as we known, the longer the customers resident in the system, the longer the servicers are busy continuously. To sum, the average busy time of servicers is equal to the average residence time.

**Model Two: A Fundamental Model to Depict the Security Check Process.**

To depict the security check process by Queue Theory, we divided the process into four parts, including the waiting area of document check (A zone), the document check entrance (B zone), the waiting area of the baggage and body screening (C zone) and the baggage and body screening zone (D zone), as shown below.

![Fig. 1 Airport security check process](image-url)
Considering the practical airport circumstance, the capacity of the waiting area of the baggage and body screening is limited. Accordingly, with the increase of the population in the waiting area of the baggage and body screening, the security check process is divided into three stages.

First, if the population in the waiting area of the baggage and body screening is less than its capacity, the document check is a consistent process without being blocked and the average arrival rate of the system won’t be affected, which also means that during this stage, the average service rate of C zone is equal to the average arrival rate of B zone. Similarly, we can find some formulas as follows.

\[ W_s = W_{s1} + W_{s2} \]  \hspace{1cm} (5)

\[ P_0 = P_{01} + P_{02} \]  \hspace{1cm} (6)

Meanwhile, \( \bar{B}_1, \bar{B}_2 \) can be calculated by the equitation (4).

During the second stage, the population in C zone waiting area has reached its capacity. Therefore, the document check is blocked that no one can enter the waiting area of C zone until there is some people in the waiting area of C zone have been serviced. In this circumstance, the service rate of B zone is equal to the C zone’s. As a result, the security check system can be described as a system with single queue. On this basis, when \( \rho \) is more than one, the queueing system is instability, otherwise, this system can be viewed as a M/M/1/∞ model. Therefore, some inferences can be derived as below.

\[ W_s = W_{s2} \]  \hspace{1cm} (7)

\[ \bar{B}_1 < \bar{B}_2 \]  \hspace{1cm} (8)

As for the third stage, there no one will enter the B zone, which means the combination of the waiting area of C zone and C zone can represent all parts of the system. In this circumstance, all of the \( \lambda, \mu, \lambda_2 \) is equal to zero, meanwhile, \( \bar{B}_1 = 0 \). In this stage, it’s remarkable that this queue system isn’t satisfied with Poisson distribution, so the calculation of this system is distinct from the other two stages. Specifically, the average residence time of this is equal to half of the total leave time.

The throughput can also be depicted by the average service ratio of the system as:

\[ F = \frac{\mu_1 T_1 + \mu_2 T_2 + \mu_3 T_3}{T_1 + T_2 + T_3} \]  \hspace{1cm} (9)

\[ \mu_{21} = \mu_{22} = \mu_{23} = \mu_2 \]  \hspace{1cm} (10)

According to the simultaneous equation (10) and (11), we can get the answer of \( F \) is \( \mu_2 \). A conclusion is shown that the key of the throughput is the sending out flow speed. There is an example to explain the conclusion that if \( \mu_2 > \mu_1 \), \( \mu_2 \) will decrease to \( \mu_1 \) and the sending-out speed is decrease to \( \mu_1 \), or when \( \mu_2 < \mu_1 \), the the sending out flow speed will be \( \mu_2 \). We can easily come to a conclusion that the sending out flow speed is equal to the minimum value of \( \mu \). It means the throughput is decided by the minimum value of \( \mu \).

Conclusion

Extending the queue system of document check to \( (M / M / S_i / \infty) \) and the queue system of baggage and body screening to \( (M / M / S_2 / \infty) \) [3].

By calculate the minimum value of \( \mu \), we can figure out the problem of the situation. When
$S_1 \times \mu_1 > S_2 \times \mu_2$, the problem is existing in the queue system of baggage and body screening. While $S_1 \times \mu_1 < S_2 \times \mu_2$, the bottleneck is existing in the queue system of document check.

References

