

Space Targets Collision Probability Model

Qingguo Zhou

School of North China Electric Power University Baoding, Baoding 071000, China

1768316219@qq.com

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Abstract. The existence of space debris limits the development of the modern space industry. So it is necessary to study space targets collision probability. Collision probability P_c can be simplified as two-dimension unequal variances PDF^[2] within circle domain integral by coordinate transformation and projection error. The integral can be turned into PDF within circle domain integral through the compression space and area circular approximation, and it can be a form of infinite series whose the first term and the recurrence formula are known. Finally, we use instance (U.S. and Russian satellite collision probability in a circular orbit) to explicit expression for validation, and the results show that the method is feasible.

Introduction

The collision warning of spacecraft and space targets, is actually a screening process, namely by judging whether they will collide with each other to rule out most space targets which won't collide with the spacecraft^[1].

About the screening of space collision between targets, we commonly use apogee - perigee screening, orbital altitude difference, time difference screening and so on..

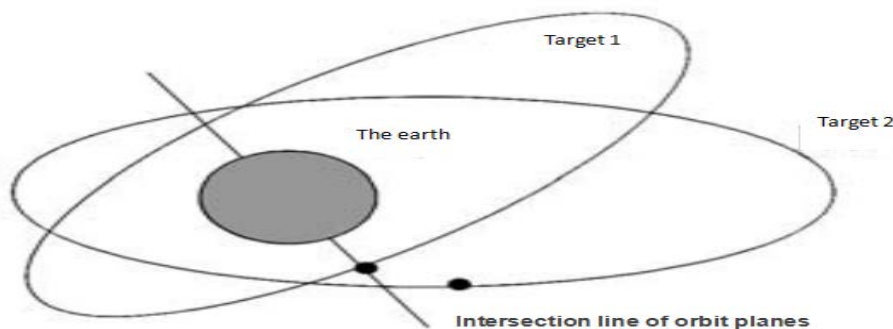


Fig.1 Geometry of space objects orbit

Figure.1

Assumption

The screening of Orbital plane intersecting line' altitude differences and time differences are based on the feature that two goals always collided near the target orbit plane intersecting line, which is represented in Fig.1. So we can carry out the following warning process:

1) Calculating Orbital plane intersecting line of two targets, judging geocentric distance when two goals go through the intersecting line, calculating the difference between two geocentric distance, namely the height difference, ruling out the target whose height difference is bigger, choosing the target whose height difference is smaller.

2) For the goals whose height difference are smaller, calculating time difference (Δt) when two goals through the intersecting line. If $\Delta t < \text{threshold}$, calculating the collision probability PC , if $PC > P_0$ (Probability threshold), we need to remove this goal.

As mentioned above, in the process of space target collision warning, we can get height difference and time difference, if they are both less than the threshold value of their own, should be further calculate collision probability.

The geometrical conditions include Δh , Δt , φ (Orbital plane angle) when two goals meet. As shown in the Fig.2, we can consider the collision probability expressed as intersection geometrical conditions and two target location prediction error variance function.

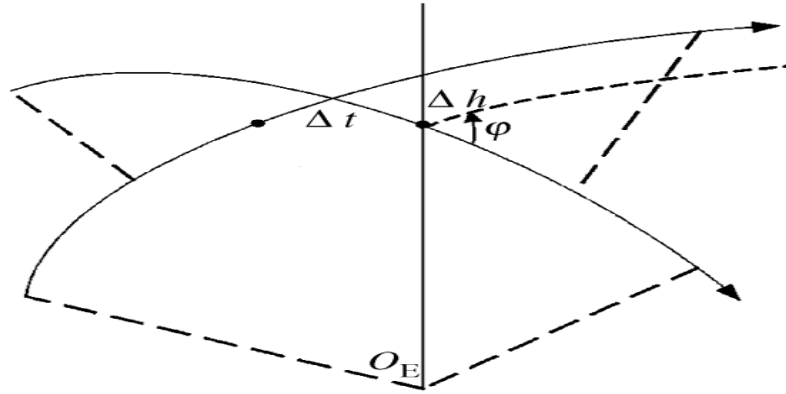


Fig.2 Encounter near the intersection of orbital plane

Figure 2

Use parameters of the coordinate system to express the collision probability

Collision probability P_c can be simplified as two-dimension unequal variances PDF^[2] within circle domain integral by coordinate transformation and projection error. The integral can be turned into PDF within circle domain integral through the compression space and area circular approximation, and it can be a form of infinite series whose the first term and the recurrence formula are known. To simplify the analysis and do not break a certain precision, we can take the first term of the infinite series as the approximation of the probability of collision^[3]:

$$p_c = e^{-v} (1 - e^{-u}) \quad (1)$$

$$\text{in which: } v = \mu_r^2 / (2\sigma^2) \quad u = R^2 / (2\sigma^2)$$

In the calculating coordinate system, before space compression, the parameter groups is $\{\mu_x, \mu_y, \sigma_x, \sigma_y, r_a\}$ and after compression space and area circular approximation, the parameter groups turn to $\{\mu_r, \sigma, R\}$.

The following are their relationship:

$$\sigma = \sigma_x \quad \mu_r^2 = \mu_x^2 + \frac{\sigma_x^2}{\sigma_y^2} \mu_y^2 \quad R^2 = \frac{\sigma_x}{\sigma_y} R_a^2 \quad (2)$$

So,

$$p_c = \exp \left[-\frac{1}{2} \left(\frac{\mu_x^2}{\sigma_x^2} + \frac{\mu_y^2}{\sigma_y^2} \right) \right] \left[1 - \exp \left(-\frac{r_a^2}{2\sigma_x \sigma_y} \right) \right] \quad (3)$$

As shown in the Fig.3, we can represent the collision probability by $\{\mu_x, \mu_z, \sigma_x, \sigma_z, r_a\}$

in the calculating coordinate system before compressing the space and integral area circular approximation.

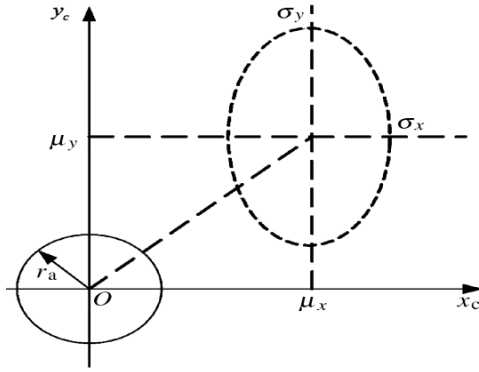


Fig.3 Parameters in integral-calculational coordinate

Figure 3

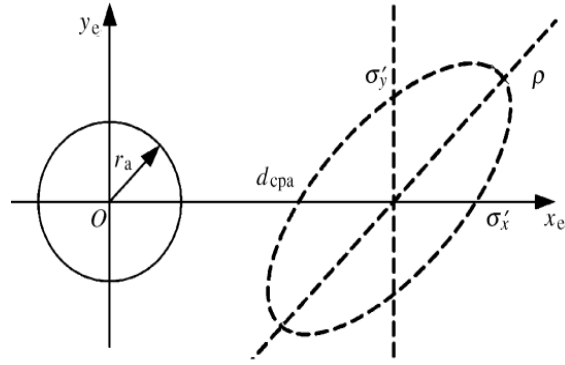


Fig.4 Parameters in encounter coordinate

Figure 4

In integral coordinate system, the joint error variance matrix is diagonal matrix, in order to calculate the probability integral easily, and we get this coordinate system by rotating original coordinate system, as shown in Fig.4 below. We define that the parameter groups is

$$\{d_{cpa}, \sigma'_x, \sigma'_z, \rho, r_a\} \quad (4)$$

in the coordinate system, according to the two same relationship (the determinant and trace of matrix remains unchanged) of covariance matrix before and after the rotating coordinates. We can get the relationship between this parameter groups and integral coordinate system parameter groups:

$$\begin{cases} d_{cpa}^2 = \mu_x^2 + \mu_z^2 \\ \sigma_x^2 \sigma_z^2 = (1 - \rho^2) \sigma_x'^2 \sigma_z'^2 \\ \sigma_x^2 + \sigma_z^2 = \sigma_x'^2 + \sigma_z'^2 \end{cases} \quad (5)$$

then, we can get the collision probability expressions:

$$p_c = \exp\left[-\frac{d_{cpa}^2}{2(1-\rho^2)\sigma_x'^2}\right] \cdot \left[1 - \exp\left(-\frac{r_a^2}{2\sqrt{1-\rho^2}\sigma_x'\sigma_z'}\right)\right] \quad (6)$$

However, parameters of these two expressions are the weight of the relative position vector and error covariance matrix of two target projected to relative plane at TCA moment, rather than the geometry of two target and their respective error variance in RSW coordinates three direction when they encounter. In order to get the collision probability with using explicit expression of geometry and error variance, It is necessary for further change.

Model test

We use instance (U.S. and Russian satellite collision probability in a circular orbit) to explicit expression for validation. According to the TLE data before the collision, by close analysis we get TCA moments is 2009-02-10 16:55:59.7958 UTC. At TCA moment two position of two targets in ECI coordinate system speed coordinates are shown in table 3.4.1. Both targets orbit is nearly circular orbit, eccentricities are $e_1 = 0.001174$ and $e_2 = 0.000660$ [4][5].

Table 1. TCA moment ECI coordinate location speed (America and Russia collision instance)

	X(km)	Y(km)	Z(km)	Vx(km/s)	Vy(km/s)	Vz(km/s)
Target 1	-1457.273246	1589.568484	6814.189959	-7.001731	-2.439512	-0.926209
Target 2	-1457.532155	1588.932671	6814.316188	3.578705	-6.172896	2.200215

Use the data above, by calculating, we get Geometry conditions:

$$h=0.698011\text{km} \quad \Delta h=0.031765\text{km} \quad d_{cpa}=0.697294\text{km}$$

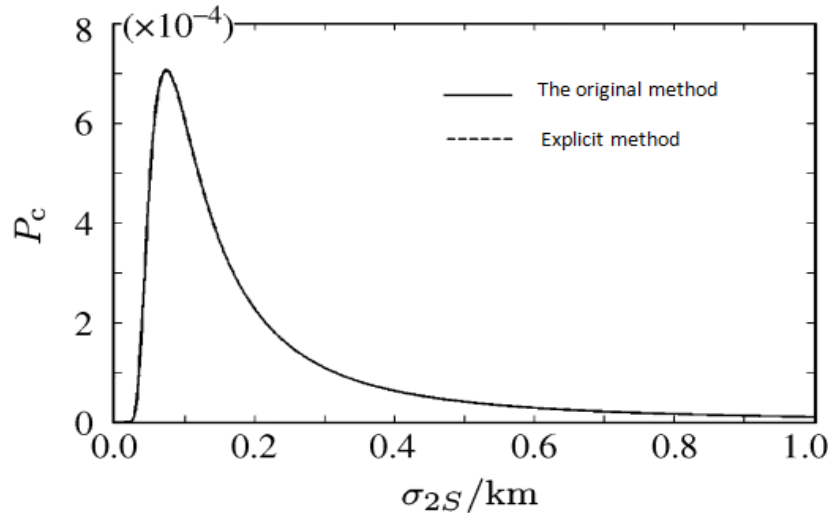
$$\Delta t=0.149075\text{s} \quad \varphi=102.458^\circ$$

Assume that two goals equivalent radius are 5 m,

$$\sigma_R : \sigma_S : \sigma_W = 1 : 5 : 1$$

$$\sigma_{1S} : \sigma_{2S} = 5 : 1$$

With explicit formula to calculate the collision probability P_c curving along with the change of σ_{2S} .



Curves of P_c with σ_{2S}

Figure 5

The Fig.5 shows, the difference between original method and the explicit formula calculating results is small which indicates that the collision probability of explicit formula is high enough, and assumptions which we put forward in the process of formula derivation are reasonable. Our model is correct.

References

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