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Study on Embedded CNC System for NURBS Curves Method of Interpolation Arithmetic

Wanjun Zhang [1, 2, 3, a], Shanping Gao [1, b], Xiyan Cheng [1, c], Feng Zhang [2, d]

[1]Quanzhou Institute of Information Engineering, 36200, China

[2]School of Mechanical Engineering, Xian Jiao tong University, 741049, China

[3]Lanzhou Industry and Equipment Co. Ltd, Lanzhou 730050, China

[a]gszwj_40@163.com, [b]358391056@qq.com, [c]326593363@qq.com, [d]116543048@qq.com

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Abstract. In order to reduce the calculation amount of NURBS curves, combined with NURBS curves mathematical properties. Firstly this paper introduce on the algorithm of the NURBS curves improvement, it is the in the interpolation cycle under the condition of certain interpolation increment only and interpolation speed, by changing the interpolation increments can achieve correction curve of interpolation. Secondly, the control platform based on ARM+DSP could overcome the defects of CNC, through the design of hardware and software. Also, it realizes real-time dispatch and improves the reliability and stability. Lastly, on the basis of improvement PID numberical control arithmetic, this paper adopts Ziegler-Nichols theory to set PID parameters and uses it in embedded CNC system servo control. Also this paper users MATLAB/SIMULINk to do simulation and analyses about select. The simulation result showed that the correction by using PID controller, to servo system for real-time control, system stability improvement at the same time, response speed, feed system and raises the comprehensive performance.

1. Introduction

For complex curves and surface treatment, the calculation of NURBS curve is large, which will affect the response time of motion seriously. In order to reduce the computational complexity, the NURBS curve is mainly studied. When the feature points or points than the distance between the three and the last, the NURBS curve of the difference is very small and simple) [1], so the NURBS curve is on the curve structure method and engineering and technical personnel to solve the relevant problems of choice.

Research on NURBS curve interpolation algorithm has a large number of documents, many of which are in order to produce fast algorithm of curve on the computer and the design of the NURBS curve, most of the solutions of triangular equations, parallel algorithms such as recursive coupling, cyclic reduction method, matrix decomposition method, but for multi core computer all applicable [2]. Zhao Guoyong et al. A fast and real time interpolation algorithm for NURS curve interpolation in high speed and high precision NC machining [3]. [4-8] curve parameters adaptive interpolation algorithm, improved algorithm, parallel machine tool interpolation algorithm is proposed, which improves the approximation error of NURBS. The interpolation algorithm [9-10] given a curve improvement and correction curve, Taylor formula of first order in the expansion, the increment interpolation and interpolation speed, the derivative can achieve the purpose of curve interpolation. Although the above algorithm can obtain a constant speed, without considering the error control. For the interpolation algorithm of NURBS curve interpolation, Taylor's expansion first order and two order solution is more complicated, and the processing error is larger.

Finally, through the research on the NURBS curve interpolation algorithm, NURBS curve interpolation calculation and general computational complexity, proposed an improved interpolation algorithm, can reduce the time and improve the efficiency of interpolation interpolation. The improved method is given by using MATLAB simulation servo system PID algorithm, the results show that the interpolation algorithm can reduce interpolation computation and interpolation operation debugging time and reduce the overshoot, it has strong adaptability and versatility, provides a new method for complex dynamic control of uncertain systems.

2. Improved of NURBS curve interpolation algorithm

2.1 Mathematical foundation of NURBS curve

In this paper, NURBS curve is used to represent a parametric of an improved algorithm adaptive of NURBS curve, and it is introduced first. Supposed $p(u)$ can be represented a Improved algorithm adaptive of NURBS curve. While NURBS [3] are parametrically mathematical definition by the following Eq.(1):

$$p(u)=\frac{\sum_{i=0}^{n}\omega_i d_{i,k}(u)}{\sum_{i=0}^{n}\omega_i N_{i,k}(u)}=\sum_{i=0}^{n}p_i N_{i,k}(u) \tag{1}$$

Where u is cubic time an improved algorithm adaptive of NURBS curve each parameter, k the order of an improved algorithm adaptive of NURBS curve. p_i is the control points, ω_i is the weight vector , $N_{i,k}(u)$ is the blending function.

$$N_{i,k}(u)=\begin{cases} N_{i,k}=\begin{cases}1, u_i \le u \le u_{i+1}\\ 0, \text{other}\end{cases} \\ N_{i,k}=\dfrac{u-u_i}{u_{i+k+1}-u_i}N_{i,k-1}(u) \\ \quad +\dfrac{u_{i+k+1}-u}{u_{i+k+1}-u_{i+1}} \\ \text{define}: \dfrac{0}{0}=0 \end{cases} \tag{2}$$

By substituting U

$$U=\left\{\underbrace{c,\cdots,c}_{p+1},u_{p+1},\cdots,u_{m-p-1},\underbrace{d,\cdots,d}_{p+1}\right\} \tag{3}$$

In this mathematical model of Newton-Rapson iterative interpolation algorithm, as shown in Fig.1.If we can suppose NURBS curve with existing maximum deceleration piont $p(u_i)$ and minimum acceleration piont $p(u_{i+1})$.

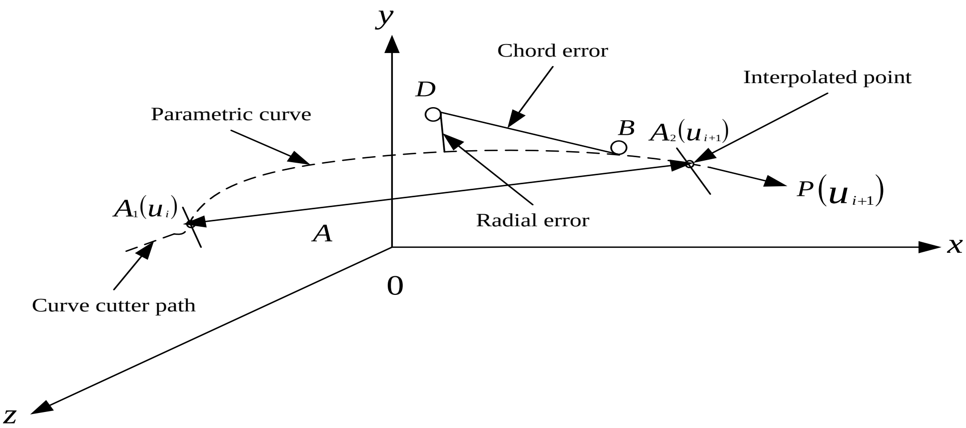

Fig. 1 This mathematical model of NURBS curve interpolation

Newton-Rapson iteration, Where

$$u_{i+1}-u_i=\frac{f(u_i)}{f'(u_i)},\quad i\ge 1 \tag{4}$$

$$u_{i+1}-u_i=\frac{f(u_i)}{\left\|\frac{dp}{du}\right\|_{u=u_i}}=\frac{V\cdot T}{\sqrt{x_i'^2+y_i'^2+z_i'^2}}-(\frac{V\cdot T}{\sqrt{x_i'^2+y_i'^2+z_i'^2}}-\frac{\left(x'_ix''_i+y'_iy''_i+z'_iz''_i\right)}{2\left(x_i^2+y_i^2+z_i^2\right)^2}V^2\cdot T^2)\frac{T^2}{2} \tag{5}$$

Newton-Rapson Cyclic iteration rule

$$\frac{|u_{i+1}-u_i|}{u_i}<\varepsilon,\qquad u_i\neq 0 \tag{6}$$

Where,

$$\frac{\left|\frac{V\cdot T}{\sqrt{x_i'^2+y_i'^2+z_i'^2}}-\frac{V\cdot T^3}{2\sqrt{x_i'^2+y_i'^2+z_i'^2}}\right|}{\frac{V\cdot T}{\sqrt{x_i'^2+y_i'^2+z_i'^2}}}-\frac{\left|\frac{\left(x'_ix''_i+y'_iy''_i+z'_iz''_i\right)}{2\left(x_i^2+y_i^2+z_i^2\right)^2}V^2\cdot T^3\right|}{\frac{V\cdot T}{\sqrt{x_i'^2+y_i'^2+z_i'^2}}}<\varepsilon \tag{7}$$

From the formula (1) ~ (10), can be found out of the coordinates of the point, From the calculation method of the above view by two order Taylor for NURBS curve interpolation coordinates expansion and Newton-Rapson iterative method, interpolation calculation and interpolation process is complex, cumbersome, long processing time, interpolation interpolation inefficient and error accuracy is not high. Therefore, it can reduce the amount of interpolation, eliminate the cumulative error in the tangent direction of the NURBS curve, save the interpolation time and improve the efficiency of interpolation.

2.2 improved interpolation algorithm

By using the Taylor formula to simplify the NURBS curve interpolation, the formula (5) (6) by Taylor two order expansions to:

$$u_{i+1}=u_i+\left.\frac{du}{dt}\right|_{t=t_i}T+\frac{1}{2}\left.\frac{d^2u}{dt^2}\right|_{t=t_i}T^2+\frac{(\Delta T)^3}{3!}f^3(\xi) \tag{8}$$

Generally very small, so the two order Taylor formula in the truncation error, ignoring it does not affect the processing.

By the formula (3) ~ (7) and (8) available, the NURBS curve of two order recursive formula

$$u_{i+1}=u_i+\frac{\Delta L_i}{\sqrt{x^2+y^2+z^2}}-\frac{\Delta L_i^2(x'x''+y'y''+z'z'')}{(x'^2+y'^2+z'^2)} \tag{9}$$

When ΔL_i compared to an hour, the available formula (13):

$$u_{i+1}\approx u_i+\frac{\Delta L_i}{\left|\frac{dp(u)}{dt}\right|_{u=u_i}}=\frac{\Delta L_i}{\sqrt{x_i'^2+y_i'^2+z_i'^2}}. \tag{10}$$

The feed increment x , y , z of each shaft is obtained by using the three coordinate axes control, and the coordinate position of the new interpolation point is obtained:

$$X'(u_i)=\frac{\Delta x}{\Delta u}\Rightarrow\Delta x_i=X'(u_i)\cdot\Delta u \tag{11}$$

$$\begin{cases}\Delta y_i=Y'(u_i)\cdot\Delta u\\ \Delta z_i=Z'(u_i)\cdot\Delta u\end{cases} \tag{12}$$

$$\Delta L=V\times T=\sqrt{\Delta x_i^2+\Delta y_i^2+\Delta z_i^2}=\Delta u\cdot\sqrt{\left[X(u_i)\right]^2+\left[Y(u_i)\right]^2+\left[Z(u_i)\right]^2}$$
$$\Rightarrow\Delta u=\frac{V\times T}{\sqrt{\left[X(u_i)\right]^2+\left[Y(u_i)\right]^2+\left[Z(u_i)\right]^2}} \tag{13}$$

So the next interpolation cycle is:

$$\begin{cases} X_{i+1} = X(u_i + \Delta u) \\ Y_{i+1} = Y(u_i + \Delta u) \\ Z_{i+1} = Z(u_i + \Delta u) \end{cases} \tag{14}$$

Can be seen from the above formula (14), and related (X_{i+1}, Y_{i+1} and Z_{i+1}). Therefore, the key of CNC honing machine interpolation is realized by interpolation, increment in interpolation period, interpolation iterative fast and interpolations of each axis. According to the method of [4] and [8], the calculation is simplified, and the interpolation value of each axis x, y, z is calculated. The interpolation method, the calculation time is stable, in line with the actual production of interpolation needs, flow chart as shown in figure 1.

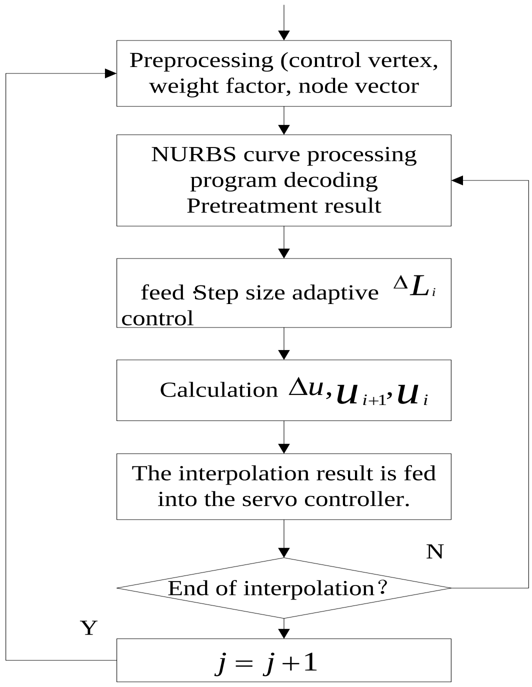

Fig. 2 Flow chart of NURBS curve interpolation algorithm

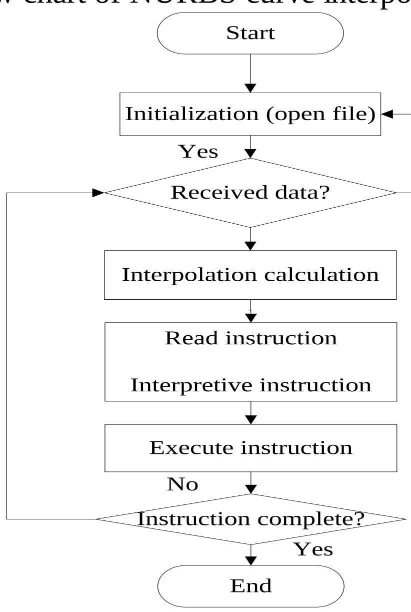

Fig. 3 Flow chart of system software design

3. Design of embedded digital control system

ARM and host computer using RS232 serial communication way to achieve, can guarantee the communication rate of 115 ~ 200 bps to transmit the data accurately [11]. When the system receives the instruction PC data transmitted over the open system file system initialization, code analysis, interpolation and data processing system for reading instruction execute the corresponding operation system software, as shown in figure 3.

4. Improved digital PID control algorithm and simulation

Using the improved digital PID control, can achieve a certain range of no error control, improve the stability of the system PID, [12] algorithm for:

$$\Delta u(k)=u(k)-u(k-1)=K_p[e(k)-e(k-1)]$$

$$K_p\left[\frac{T}{T_t}e(k)+\frac{T_d}{T}De(k)-2e(k-1)+e(k-2)\right] \quad (15)$$

By the formula (11) can be obtained:

$$\Delta u(k)=b_0e(k)+b_1e(k-1)+b_2e(k-2) \quad (16)$$

$$b_1=K_p\left(1+\frac{T}{T_t}+\frac{T_d}{T}\right)=K_p+TK_I+\frac{K_D}{T} \quad (17)$$

$$b_1=-K_p-2\frac{K_D}{T} \quad (18)$$

$$b_2=K_p\frac{K_D}{T} \quad (19)$$

The sampling period is the sampling period, and the sampling period is the output value, $k=0,1\cdots,u_k$, K_p, K_I, K_D which is expressed as the proportional coefficient, integral coefficient and differential coefficient. For systems with large fluctuation range and rapid change, the general PID control can not achieve the desired results. Therefore, the text chooses the Ziegler-Nichols setting method to get the system parameters:

$K_p=7.728$, $K_I=0.031$, $K_D=0.00731$. The transfer function of the Ziegler-Nichols tuning method is assumed to be

$$G_0(s)=\frac{4}{200s+1}e^{-100s} \quad (20)$$

The simulation parameters of Ziegler-Nichols tuning method are shown in Table 1, and the simulation results are shown in Figure 4, Figure 5 and Figure 6.

Simulation parameters	Setting parameters		
	K_P	K_I	K_D
P	$0.5k_P$		
PI	$0.45k_P$	$0.535k_P/T$	
PID	$0.6k_P$	$1.2k_P/T$	$0.07k_PT$

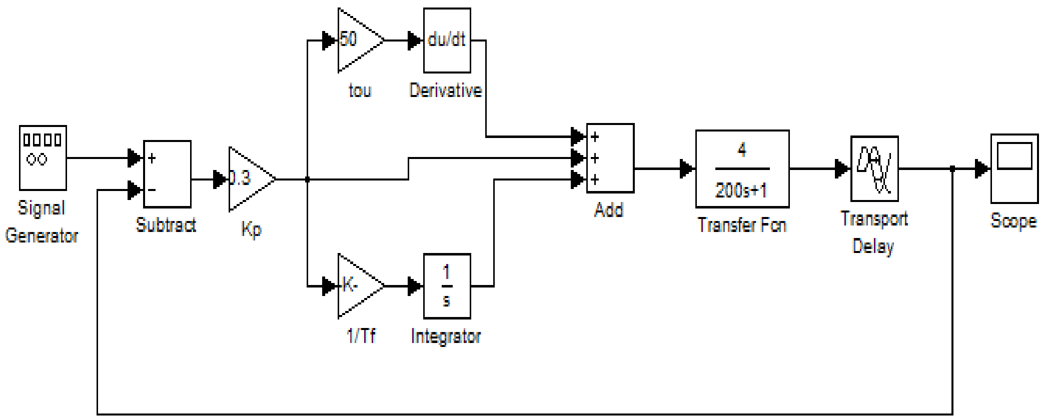

Fig. 4 Improved digital PID simulation diagram

The improved digital PID, which is able to adapt to a wide range of fluctuations, rapid changes in the servo control system, and can maintain the stability of the general system, has a strong adaptability and versatility. By using MATLAB simulation, the results show that the interpolation algorithm can reduce the amount of interpolation computation and the debugging time, and reduce the overshoot, which provides a new method for the control of complex dynamic uncertain system.

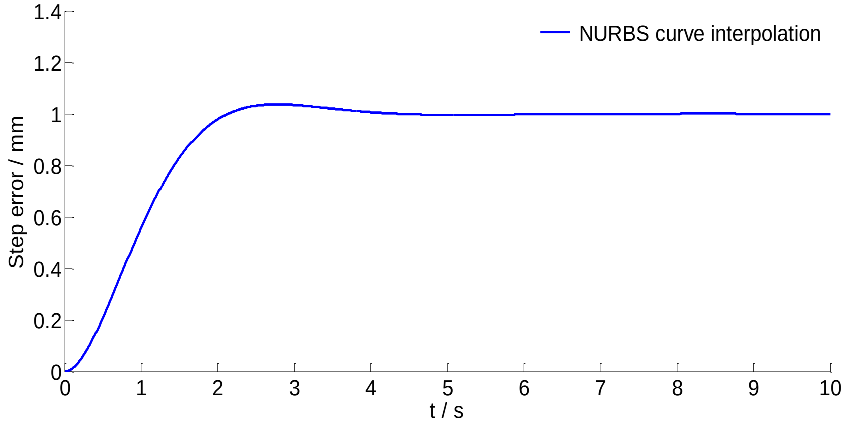

Fig. 5 NURBS curve interpolation

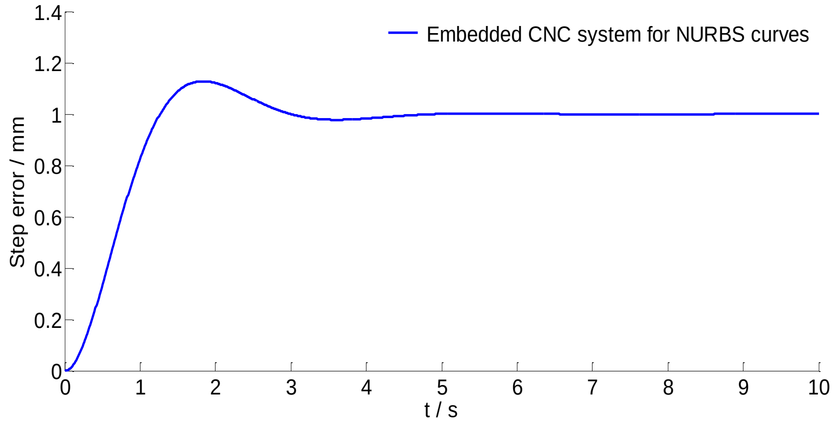

Fig. 6 Embedded CNC system for NURBS curve

5. Summary

(1) Based on NURBS curve interpolation algorithm, NURBS curve interpolation calculation and general computational complexity, proposed an improved interpolation algorithm, can reduce the time and improve the efficiency of interpolation interpolation.

(2) A new method of PID algorithm for servo system is presented, which is able to adapt to the control system with large fluctuation range and rapid change, and can keep the stability of the general system. Finally, using MATLAB simulation, the results show that the interpolation algorithm can reduce the amount of interpolation computation and the debugging time, and reduce the overshoot, which provides a new method for the control of complex dynamic uncertain system.

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